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Common fixed point theorem for subcompatible maps of type (α) in weak non-Archimedean intuitionistic fuzzy metric space

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Abstract

In this paper, we introduce the definition of subcompatible maps and subcompatible maps of types (α) and (β), which are respectively weaker than compatible maps and compatible maps of types (α) and (β), in weak non-Archimedean intuitionistic fuzzy metric spaces and give some examples and relationship between these definitions. Thereafter, we prove common fixed point theorem for four subcompatible maps of type (α) in weak non-Archimedean intuitionistic fuzzy metric spaces.

Keywords: Weak non-Archimedean intuitionistic fuzzy metric, subcompatible maps of types (α) and (β), common fixed point.
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1. Introduction and preliminaries

The concept of fuzzy sets was introduced initially by Zadeh [17] in 1965. Since that time, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. Especially, Deng [4], Erceg [6], Kaleva and Seikkala [9], Kramosil and Michalek [10], Georege and Veeramani [7] have introduced the concept of fuzzy metric space in different ways. Grabiec [8] initiated the study of fixed point theory in fuzzy metric spaces, which is parallel to fixed point theory in probabilistic

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metric space. Many authors followed this concept by introducing and investigating the different types of contractive mappings for study of fixed point theory.

On the other hand, Atanassov [2] introduced and studied the notion of intuitionistic fuzzy set by generalizing the notion of fuzzy set [17]. Intuitionistic fuzzy set includes the degree of belongingness, degree of non-belongingness and the hesitation margin. Many applications of intuitionistic fuzzy sets are carried out using distance measures approach. Distance measure between intuitionistic fuzzy sets is an indispensable concept in fuzzy mathematics. Using the idea of intuitionistic fuzzy set, Park [12] defined the notion of intuitionistic fuzzy metric space as a generalization of fuzzy metric space due to George and Veeramani [7] and proved some known results of metric spaces for intuitionistic fuzzy metric space. Alaca et al. [1] first defined the notion of intuitionistic fuzzy metric spaces using continuous t-norm and continuous t-conorm as a generalization of fuzzy metric spaces in the sense of Kramosil and Michalek [10]. As a generalization to GV-version of this notion, Saadati et al. [13] introduced the notion of modified intuitionistic fuzzy metric spaces (modified IFMS) using continuous t-representable and defined the notion of compatible mappings in modified IFMS.

Various authors have studied results on fixed and common fixed points by using the concept of weak commutativity, compatibility and weak compatibility in different spaces. For instance Turkoglu et.al. [16] introduced compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces. Further, they proved common fixed point theorems for compatible maps. Recently, Al-Thagafi and Shahzad weakened the concept of compatibility by giving a new notion occasionally weak compatible (owc) which is more general among the commutativity concepts. After that Bouhadjera and Godet-Thobie [3] weakened the concept of occasionally weak compatibility and reciprocal continuity in the form of subcompatibility and subsequential continuity respectively and proved common fixed point theorem.

Most recently, Erduran et.al. [15] introduced the concept of weak non-Archimedean intuitionistic fuzzy metric space and proved a common fixed point theorem for a pair of generalized ψ-ϕ-contractive mappings. Also, they present that every non-Archimedean intuitionistic fuzzy metric space is itself a weak non-Archimedean intuitionistic fuzzy metric space.

In this paper, we introduce the definition of subcompatible maps and subcompatible maps of types (α) and (β), which are respectively weaker than compatible maps and compatible maps of types (α) and (β), in weak non-Archimedean intuitionistic fuzzy metric spaces and give some examples and relationship between these definitions. Thereafter, we prove a common fixed point theorem for four subcompatible maps of type (α) in weak non-Archimedean intuitionistic fuzzy metric spaces.

Now we give some definitions.

Definition 1.1 ([14]). A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,
(ii) $a * 1 = a$ for every $a \in [0,1]$,
(iii) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, for $a, b, c, d \in [0,1]$.

If in addition, $*$ is continuous, then $*$ is called a continuous t-norm. Typical examples of a continuous t-norms are $a * b = \min\{a, b\}$, $a * b = ab/\max\{a, b, \lambda\}$ for $0 < \lambda < 1$, $a * b = ab$, $a * b = \max\{a + b - 1, 0\}$.

Definition 1.2 ([14]). A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is called a t-conorm if it satisfies the following conditions:

(i) $\diamond$ is associative and commutative,
(ii) $a \diamond 0 = a$ for every $a \in [0,1]$,
(iii) $a \diamond b \le c \diamond d$ whenever $a \le c$ and $b \le d$, for $a, b, c, d \in [0,1]$.

If in addition, $\diamond$ is continuous, then $\diamond$ is called a continuous t-conorm. Typical examples of a continuous t-conorms are $a \diamond b = a + b - ab$, $a \diamond b = \max\{a, b\}$, $a \diamond b = \min\{a + b, 1\}$.

Definition 1.3 ([12])**.** A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X \times X \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$, $s, t > 0$,

(IFM$_1$) $M(x, y, t) + N(x, y, t) \leq 1$,

(IFM$_2$) $M(x, y, t) > 0$,

(IFM$_3$) $M(x, y, t) = 1$ if and only if $x = y$,

(IFM$_4$) $M(x, y, t) = M(y, x, t)$,

(IFM$_5$) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$,

(IFM$_6$) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous,

(IFM$_7$) $N(x, y, t) > 0$,

(IFM$_8$) $N(x, y, t) = 0$ if and only if $x = y$,

(IFM$_9$) $N(x, y, t) = N(y, x, t)$,

(IFM$_{10}$) $N(x, z, t + s) \leq N(x, y, t) \diamond N(y, z, s)$,

(IFM$_{11}$) $N(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 1.4. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated, i.e., $x \diamond y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in X$.

Remark 1.5. In intuitionistic fuzzy metric space X, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$.

In the above definition, if the triangular inequality (IFM_5) and (IFM_{10}) are replaced by the following:

$$(NA) \quad \begin{array}{l} M(x, z, \max\{t, s\}) \geq M(x, y, t) * M(y, z, s) \\ N(x, z, \max\{t, s\}) \leq N(x, y, t) \diamond N(y, z, s) \end{array}$$

or equivalently

$$\begin{array}{l} M(x, z, t) \geq M(x, y, t) * M(y, z, t) \\ N(x, z, t) \leq N(x, y, t) \diamond N(y, z, t), \end{array}$$

then $(X, M, N, *, \diamond)$ is called non-Archimedean intuitionistic fuzzy metric space [5]. It is easy to check that the triangle inequality (NA) implies (IFM_5) and (IFM_{10}), that is, every non-Archimedean intuitionistic fuzzy metric space is itself an intuitionistic fuzzy metric space.

Example 1.6. Let X be a non-empty set with at least two elements. Define $M(x, y, t)$ by: If we define the intuitionistic fuzzy set (X, M, N) by $M(x, x, t) = 1$, $N(x, x, t) = 0$ for all $x \in X$ and $t > 0$, and $M(x, y, t) = 0$, $N(x, y, t) = 1$ for $x \neq y$ and $0 < t \leq 1$, and $M(x, y, t) = 1$, $N(x, y, t) = 0$ for $x \neq y$ and $t > 1$. Then $(X, M, N, *, \diamond)$ is a non-Archimedean intuitionistic fuzzy metric space with arbitrary continuous t-norm $*$ and t-conorm $\diamond$. Clearly $(X, M, N, *, \diamond)$ is also an intuitionistic fuzzy metric space.

Definition 1.7 ([15])**.** In Definition 1.3, if the triangular inequality (IFM_5) and (IFM_{10}) are replaced by the following:

$$(WNA) \quad \begin{array}{l} M(x,z,t) \geq \max\{M(x,y,t) * M(y,z,t/2), M(x,y,t/2) * M(y,z,t)\} \\ N(x,z,t) \leq \min\{N(x,y,t) \diamond N(y,z,t/2), N(x,y,t/2) \diamond N(y,z,t)\} \end{array}$$

for all $x,y,z \in X$ and $t > 0$, then $(X, M, N, *, \diamond)$ is said to be a weak non-Archimedean intuitionistic fuzzy metric space.

Obviously every non-Archimedean intuitionistic fuzzy metric space is itself a weak non-Archimedean intuitionistic fuzzy metric space.

The inequality (WNA) does not imply that $M(x,y,\cdot)$ is non decreasing and $N(x,y,\cdot)$ is non increasing. Thus a weak non-Archimedean intuitionistic fuzzy metric space is not necessarily an intuitionistic fuzzy metric space.

Example 1.8. Let $X = [0,\infty)$ and define $M(x,y,t)$, $N(x,y,t)$ by

$$M(x,y,t) = \begin{cases} 1, & x = y \\ \dfrac{t}{t+1}, & x \neq y \end{cases}, \quad N(x,y,t) = \begin{cases} 0, & x = y \\ \dfrac{1}{t+1}, & x \neq y \end{cases}$$

for all $t > 0$. $(X, M, N, *, \diamond)$ is a weak non-Archimedean intuitionistic fuzzy metric space with $a * b = ab$ and $a \diamond b = a + b - ab$ for every $a, b \in [0,1]$.

Now, we remind compatible maps and compatible maps of types (α) and (β) which are introduced by Turkoglu et al. in intuitionistic fuzzy metric spaces.

Definition 1.9 ([16])**.** Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself. The maps A and B are said to be compatible if, for all $t > 0$,

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1, \quad \lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Definition 1.10 ([16])**.** Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself. The maps A and B are said to be compatible of type (α) if, for all $t > 0$,

$$\begin{array}{ll} \lim\limits_{n\to\infty} M(ABx_n, BBx_n, t) = 1, & \lim\limits_{n\to\infty} N(ABx_n, BBx_n, t) = 0, \\ \lim\limits_{n\to\infty} M(BAx_n, AAx_n, t) = 1, & \lim\limits_{n\to\infty} N(BAx_n, AAx_n, t) = 0, \end{array}$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Definition 1.11 ([16])**.** Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself. The maps A and B are said to be compatible of type (β) if, for all $t > 0$,

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1, \quad \lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Definition 1.12 ([11])**.** Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself. The maps A and B are said to be owc if and only if there is a point $x \in X$ which is a coincidence point of A and B at which A and B commute i.e., there is a point $x \in X$ such that $Ax = Bx$ and $ABx = BAx$.

Definition 1.13 ([11])**.** Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself. The maps A and B are said to be reciprocally continuous if $\lim_{n\to\infty} ABx_n = Ax$, $\lim_{n\to\infty} BAx_n = Bx$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

The following definition of subcompatible and subsequential continuous mappings are given by Bouhadjera et al..

Definition 1.14 ([3])**.** Two self-maps A and B on a metric space (X, d) are said to be subsequentially continuous if and only if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy $\lim_{n\to\infty} ABx_n = Ax$, $\lim_{n\to\infty} BAx_n = Bx$.

Definition 1.15 ([3])**.** Two self-maps A and B on a metric space (X, d) are said to be subcompatible iff there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy $\lim_{n\to\infty} d(ABx_n, BAx_n) = 0$.

2. Subcompatible maps and subcompatible maps of types (α) and (β) in weak non-Archimedean intuitionistic fuzzy metric space

Definition 2.1. Let $(X, M, N, *, \diamond)$ be a weak non-Archimedean intuitionistic fuzzy metric space. Self-maps A and B on X are said to be subsequentially continuous iff there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy $\lim_{n\to\infty} ABx_n = Ax$, $\lim_{n\to\infty} BAx_n = Bx$.

Clearly, if A and B are continuous or reciprocally continuous, then they are subsequentially continuous, but converse is not true in general.

Example 2.2. Let $X = [0, \infty)$ and define $M(x, y, t)$, $N(x, y, t)$ by

$$M(x,y,t) = \begin{cases} 1, & x = y \\ \dfrac{t}{t+1}, & x \neq y \end{cases}, \quad N(x,y,t) = \begin{cases} 0, & x = y \\ \dfrac{1}{t+1}, & x \neq y \end{cases}$$

for all $t > 0$. $(X, M, N, *, \diamond)$ is a weak non-Archimedean intuitionistic fuzzy metric space with $a * b = ab$ and $a \diamond b = a + b - ab$ for every $a, b \in [0, 1]$. Define A, B as follows:

$$Ax = \begin{cases} 2, & x < 3 \\ x, & x \geq 3 \end{cases}, \quad Bx = \begin{cases} 2x-4, & x \leq 3 \\ 3, & x > 3 \end{cases}.$$

Clearly A and B are discontinuous at $x = 3$. Let $\{x_n\}$ be a sequence in X defined by $x_n = 3 - \dfrac{1}{n}$ for $n = 1, 2, 3, \ldots$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 2, 2 \in X$$

and

$$\lim_{n\to\infty} ABx_n = 2 = A(2), \lim_{n\to\infty} BAx_n = 0 = B(2).$$

Therefore A and B are subsequentially continuous. Now, let $\{x_n\}$ be a sequence in X defined by $x_n = 3 + \dfrac{1}{n}$ for $n = 1, 2, 3, \ldots$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 3, 3 \in X$$

and

$$\lim_{n\to\infty} BAx_n = 3 \neq 2 = B(3).$$

Hence A and B are not reciprocally continuous.

Definition 2.3. Let $(X, M, N, *, \diamond)$ be a weak non-Archimedean intuitionistic fuzzy metric space. Self-maps A and B on X are said to be subcompatible iff there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$, $\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$.

It is easy to see that two owc maps are subcompatible, however the converse is not true in general. It is also interesting to see the following one way implication.

Commuting $\Rightarrow$ Weakly commuting $\Rightarrow$ Compatibility $\Rightarrow$ Weak compatibility $\Rightarrow$ Owc $\Rightarrow$ Subcompatibility.

Definition 2.4. Let $(X, M, N, *, \diamond)$ be a weak non-Archimedean intuitionistic fuzzy metric space. Self-maps A and B on X are said to be subcompatible of type (α) iff there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy

$$\lim_{n\to\infty} M\left(ABx_n, BBx_n, t\right) = 1, \quad \lim_{n\to\infty} N\left(ABx_n, BBx_n, t\right) = 0,$$
$$\lim_{n\to\infty} M\left(BAx_n, AAx_n, t\right) = 1, \quad \lim_{n\to\infty} N\left(BAx_n, AAx_n, t\right) = 0.$$

Clearly, if A and B are compatible of type (α), then they are subcompatible of type (α), but converse is not true in general.

Example 2.5. Let $X = [0, \infty)$ and define $M(x, y, t)$, $N(x, y, t)$ by

$$M(x, y, t) = \frac{t}{t + |x - y|}, \quad N(x, y, t) = \frac{|x - y|}{t + |x - y|}$$

for all $t > 0$. $(X, M, N, *, \diamond)$ is a weak non-Archimedean intuitionistic fuzzy metric space with $a * b = ab$ and $a \diamond b = a + b - ab$ for every $a, b \in [0, 1]$. Define A, B as follows:

$$Ax = \begin{cases} x^2 + 1, & x < 1 \\ 2x - 1, & x \geq 1 \end{cases}, \quad Bx = \begin{cases} x + 1, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}.$$

Let $\{x_n\}$ be a sequence in X defined by $x_n = 1 + \dfrac{1}{n}$ for $n = 1, 2, 3, ...$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 1, 1 \in X$$

and

$$\begin{aligned}
ABx_n &= A\left(1 + \frac{3}{n}\right) = 2\left(1 + \frac{3}{n}\right) - 1 = 1 + \frac{6}{n} \\
BAx_n &= B\left(1 + \frac{2}{n}\right) = 3\left(1 + \frac{2}{n}\right) - 2 = 1 + \frac{6}{n} \\
AAx_n &= A\left(1 + \frac{2}{n}\right) = 2\left(1 + \frac{2}{n}\right) - 1 = 1 + \frac{4}{n} \\
BBx_n &= B\left(1 + \frac{3}{n}\right) = 3\left(1 + \frac{3}{n}\right) - 2 = 1 + \frac{9}{n}.
\end{aligned}$$

Therefore

$$\lim_{n\to\infty} M\left(ABx_n, BBx_n, t\right) = 1, \quad \lim_{n\to\infty} N\left(ABx_n, BBx_n, t\right) = 0,$$
$$\lim_{n\to\infty} M\left(BAx_n, AAx_n, t\right) = 1, \quad \lim_{n\to\infty} N\left(BAx_n, AAx_n, t\right) = 0,$$

that is A and B are subcompatible of type (α) but if we consider a sequence $x_n = 1 - \dfrac{1}{n}$ for $n = 1, 2, 3, ...$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 2, 2 \in X$$

and

$$\begin{aligned}
ABx_n &= A\left(2 - \frac{1}{n}\right) = 2\left(2 - \frac{1}{n}\right) - 1 = 3 - \frac{2}{n}, \\
BAx_n &= B\left(\left(1 - \frac{1}{n}\right)^2 + 1\right) = 3\left(\left(1 - \frac{1}{n}\right)^2 + 1\right) - 2, \\
AAx_n &= A\left(\left(1 - \frac{1}{n}\right)^2 + 1\right) = A\left(1 - \frac{2}{n} + \frac{1}{n^2}\right) = \left(1 - \frac{2}{n} + \frac{1}{n^2}\right)^2 + 1, \\
BBx_n &= B\left(2 - \frac{1}{n}\right) = 3\left(2 - \frac{1}{n}\right) - 2 = 4 - \frac{3}{n}.
\end{aligned}$$

Therefore

$$\lim_{n\to\infty} M(ABx_n, BBx_n, t) \neq 1, \quad \lim_{n\to\infty} N(ABx_n, BBx_n, t) \neq 0,$$
$$\lim_{n\to\infty} M(BAx_n, AAx_n, t) \neq 1, \quad \lim_{n\to\infty} N(BAx_n, AAx_n, t) \neq 0,$$

that is A and B are not compatible of type (α).

Definition 2.6. Let $(X, M, N, *, \diamond)$ be a weak non-Archimedean intuitionistic fuzzy metric space. Self maps A and B on X are said to be subcompatible of type (β) iff there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1, \quad \lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0.$$

Clearly, if A and B are compatible of type (β), then they are subcompatible of type (β), but converse is not true in general.

Example 2.7. Let $X = [0, \infty)$ and define $M(x, y, t)$, $N(x, y, t)$ by

$$M(x,y,t) = \begin{cases} 1, & x = y \\ \dfrac{t}{t+1}, & x \neq y \end{cases}, \quad N(x,y,t) = \begin{cases} 0, & x = y \\ \dfrac{1}{t+1}, & x \neq y \end{cases}$$

for all $t > 0$. $(X, M, N, *, \diamond)$ is a weak non-Archimedean intuitionistic fuzzy metric space with $a * b = ab$ and $a \diamond b = a + b - ab$ for every $a, b \in [0, 1]$. Define A, B as follows:

$$Ax = x^2, \quad Bx = \begin{cases} x + 2, & x \in [0,4] \cup (5, \infty) \\ x + 12, & x \in (4, 5] \end{cases}.$$

Let $\{x_n\}$ be a sequence in X defined by $x_n = 2 + \dfrac{1}{n}$ for $n = 1, 2, 3, ...$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 4, 4 \in X$$

and

$$AAx_n = A\left(\left(2 + \frac{1}{n}\right)^2\right) = \left(2 + \frac{1}{n}\right)^4$$
$$BBx_n = B\left(4 + \frac{1}{n}\right) = 4 + \frac{1}{n} + 12 = 16 + \frac{1}{n}.$$

Therefore

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0,$$

that is A and B are subcompatible of type (β) but if we consider a sequence $x_n = 2 - \dfrac{1}{n}$ for $n = 1, 2, 3, ...$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 4, 4 \in X$$

and

$$AAx_n = A\left(\left(2 - \frac{1}{n}\right)^2\right) = \left(2 - \frac{1}{n}\right)^4,$$
$$BBx_n = B\left(4 - \frac{1}{n}\right) = 4 - \frac{1}{n} + 2 = 6 - \frac{1}{n}.$$

Therefore

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) \neq 1 \text{ and } \lim_{n\to\infty} N(AAx_n, BBx_n, t) \neq 0,$$

that is A and B are not compatible of type (β).

Proposition 2.8. *Let $(X, M, N, *, \diamond)$ be a weak non-Archimedean intuitionistic fuzzy metric space and $A, B : X \to X$ are subsequentially continuous mappings. A and B are subcompatible maps if and only if they are subcompatible of type (α).*

Proof. Suppose A and B are subcompatible, then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy

$$\lim_{n\to\infty} M\left(ABx_n, BAx_n, t\right) = 1, \quad \lim_{n\to\infty} N\left(ABx_n, BAx_n, t\right) = 0.$$

Since A and B are subsequentially continuous, we have

$$\lim_{n\to\infty} ABx_n = Ax = \lim_{n\to\infty} AAx_n, \quad \lim_{n\to\infty} BAx_n = Bx = \lim_{n\to\infty} BBx_n.$$

Thus from the inequality (WNA),

$$M(ABx_n, BBx_n, t) \geq M(ABx_n, BAx_n, t) * M(BAx_n, BBx_n, t/2)$$

and

$$N(ABx_n, BBx_n, t) \leq N(ABx_n, BAx_n, t) \diamond N(BAx_n, BBx_n, t/2)$$

for all $t > 0$, it follows that

$$\lim_{n\to\infty} M\left(ABx_n, BBx_n, t\right) \geq 1 * 1 = 1$$

and

$$\lim_{n\to\infty} N\left(ABx_n, BBx_n, t\right) \leq 0 \diamond 0 = 0,$$

that is

$$\lim_{n\to\infty} M\left(ABx_n, BBx_n, t\right) = 1 \text{ and } \lim_{n\to\infty} N\left(ABx_n, BBx_n, t\right) = 0$$

for all $t > 0$. By the same way, we obtain

$$\lim_{n\to\infty} M\left(BAx_n, AAx_n, t\right) = 1 \text{ and } \lim_{n\to\infty} N\left(BAx_n, AAx_n, t\right) = 0.$$

Consequently A and B are subcompatible of type (α).

Conversely, suppose that A and B are subcompatible of type (α), then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy

$$\begin{aligned} \lim_{n\to\infty} M\left(ABx_n, BBx_n, t\right) = 1 \quad \lim_{n\to\infty} N\left(ABx_n, BBx_n, t\right) = 0, \\ \lim_{n\to\infty} M\left(BAx_n, AAx_n, t\right) = 1 \quad \lim_{n\to\infty} N\left(BAx_n, AAx_n, t\right) = 0. \end{aligned}$$

Since A and B are subsequentially continuous, we have

$$\lim_{n\to\infty} ABx_n = Ax = \lim_{n\to\infty} AAx_n, \quad \lim_{n\to\infty} BAx_n = Bx = \lim_{n\to\infty} BBx_n.$$

Now, from the inequality (WNA), we have

$$M(ABx_n, BAx_n, t) \geq M(ABx_n, BBx_n, t) * M(BBx_n, BAx_n, t/2)$$

and

$$N(ABx_n, BAx_n, t) \leq N(ABx_n, BBx_n, t) \diamond N(BBx_n, BAx_n, t/2)$$

for all $t > 0$, it follows that

$$\lim_{n\to\infty} M\left(ABx_n, BAx_n, t\right) \geq 1 * 1 = 1$$

and

$$\lim_{n\to\infty} N\left(ABx_n, BAx_n, t\right) \leq 0 \diamond 0 = 0$$

for all $t > 0$, which implies that

$$\lim_{n\to\infty} M\left(ABx_n, BAx_n, t\right) = 1 \text{ and } \lim_{n\to\infty} N\left(ABx_n, BAx_n, t\right) = 0.$$

Therefore, A and B are subcompatible. This completes the proof. $\square$

Proposition 2.9. *Let $(X, M, N, *, \diamond)$ be a weak non-Archimedean intuitionistic fuzzy metric space and $A, B : X \to X$ are subsequentially continuous mappings. A and B are subcompatible maps if and only if they are subcompatible of type (β).*

Proof. Suppose A and B are subcompatible, then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1, \quad \lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0.$$

Since A and B are subsequentially continuous, we have

$$\lim_{n\to\infty} ABx_n = Ax = \lim_{n\to\infty} AAx_n, \quad \lim_{n\to\infty} BAx_n = Bx = \lim_{n\to\infty} BBx_n.$$

Thus from the inequality (WNA),

$$\begin{aligned} M(AAx_n, BBx_n, t) &\geq M(AAx_n, ABx_n, t) * M(ABx_n, BBx_n, t/2) \\ &\geq M(AAx_n, ABx_n, t) * M(ABx_n, BAx_n, t/2) * M(BAx_n, BBx_n, t/4) \end{aligned}$$

and

$$\begin{aligned} N(AAx_n, BBx_n, t) &\leq N(AAx_n, ABx_n, t) \diamond N(ABx_n, BBx_n, t/2) \\ &\leq N(AAx_n, ABx_n, t) \diamond N(ABx_n, BAx_n, t/2) \diamond N(BAx_n, BBx_n, t/4) \end{aligned}$$

for all $t > 0$, it follows that

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) \geq 1 * 1 * 1 = 1$$

and

$$\lim_{n\to\infty} N(AAx_n, BBx_n, t) \leq 0 \diamond 0 \diamond 0 = 0$$

for all $t > 0$, which implies that

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0.$$

Consequently A and B are subcompatible of type (β).

Conversely, suppose that A and B are subcompatible of type (β), then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1, \quad \lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0.$$

Now, from the inequality (WNA), we have

$$\begin{aligned} M(ABx_n, BAx_n, t) &\geq M(ABx_n, AAx_n, t) * M(AAx_n, BAx_n, t/2) \\ &\geq M(ABx_n, BAx_n, t) * M(AAx_n, BBx_n, t/2) * M(BBx_n, BAx_n, t/4) \end{aligned}$$

and

$$\begin{aligned} N(ABx_n, BAx_n, t) &\leq N(ABx_n, AAx_n, t) \diamond N(AAx_n, BAx_n, t/2) \\ &\leq N(ABx_n, BAx_n, t) \diamond N(AAx_n, BBx_n, t/2) \diamond N(BBx_n, BAx_n, t/4) \end{aligned}$$

it follows that

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) \geq 1 * 1 * 1 = 1$$

and

$$\lim_{n\to\infty} N(ABx_n, BAx_n, t) \leq 0 \diamond 0 \diamond 0 = 0$$

for all $t > 0$, which implies that

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0.$$

Therefore, A and B are subcompatible. □

Proposition 2.10. *Let $(X, M, N, *, \diamond)$ be a weak non-Archimedean intuitionistic fuzzy metric space and $A, B : X \to X$ are subsequentially continuous mappings. A and B are subcompatible maps of type (α) if and only if they are subcompatible of type (β).*

Proof. Suppose that A and B are subcompatible of type (α), then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy

$$\lim_{n\to\infty} M(ABx_n, BBx_n, t) = 1 \quad \lim_{n\to\infty} N(ABx_n, BBx_n, t) = 0,$$
$$\lim_{n\to\infty} M(BAx_n, AAx_n, t) = 1 \quad \lim_{n\to\infty} N(BAx_n, AAx_n, t) = 0.$$

Since A and B are subsequentially continuous, we have

$$\lim_{n\to\infty} ABx_n = Ax = \lim_{n\to\infty} AAx_n, \quad \lim_{n\to\infty} BAx_n = Bx = \lim_{n\to\infty} BBx_n.$$

Thus from the inequality (WNA),

$$M(AAx_n, BBx_n, t) \geq M(AAx_n, ABx_n, t) * M(ABx_n, BBx_n, t/2)$$

and

$$N(AAx_n, BBx_n, t) \leq M(AAx_n, ABx_n, t) \diamond N(ABx_n, BBx_n, t/2),$$

it follows that

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) \geq 1 * 1 = 1$$

and

$$\lim_{n\to\infty} N(AAx_n, BBx_n, t) \leq 0 \diamond 0 = 0,$$

which implies that

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0.$$

Therefore A and B are subcompatible of type (β).

Conversely, suppose that A and B are subcompatible of type (β), then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x, x \in X$ and satisfy

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1, \quad \lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0.$$

Now, from the inequality (WNA), we have

$$M(ABx_n, BBx_n, t) \geq M(ABx_n, AAx_n, t) * M(AAx_n, BBx_n, t/2)$$

and

$$N(ABx_n, BBx_n, t) \leq N(ABx_n, AAx_n, t) \diamond N(AAx_n, BBx_n, t/2),$$

it follows that

$$\lim_{n\to\infty} M(ABx_n, BBx_n, t) \geq 1 * 1 = 1$$

and

$$\lim_{n\to\infty} N(ABx_n, BBx_n, t) \leq 0 \diamond 0 = 0,$$

which implies that

$$\lim_{n\to\infty} M(ABx_n, BBx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(ABx_n, BBx_n, t) = 0$$

for all $t > 0$. By the same way, we obtain

$$\lim_{n\to\infty} M(BAx_n, AAx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(BAx_n, AAx_n, t) = 0.$$

Therefore A and B are subcompatible of type (α). □

3. A common fixed point theorem

Theorem 3.1. *Let A, B, S and T be self-maps of a weak non-Archimedean intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ and let the pairs (A, S) and (B, T) are subcompatible maps of type (α) and subsequentially continuous. If*

$$\begin{aligned} M(Ax, By, t) &\geq \psi\left(\min\left\{\begin{array}{c} M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), \\ \frac{1}{2}\left[M(By, Sx, t) + M(Ax, Ty, t)\right] \end{array}\right\}\right), \\ N(Ax, By, t) &\leq \phi\left(\max\left\{\begin{array}{c} N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t), \\ \frac{1}{2}\left[N(By, Sx, t) + N(Ax, Ty, t)\right] \end{array}\right\}\right) \end{aligned} \tag{3.1}$$

for all $x, y \in X$, $t > 0$, where $\psi, \phi : [0,1] \to [0,1]$ are continuous functions such that $\psi(s) > s$ and $\phi(s) < s$ for each $s \in (0,1)$. Then A, B, S and T have a unique common fixed point in X.

Proof. Since the pairs (A, S) and (B, T) are subcompatible maps of type (α) and subsequentially continuous, then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, z \in X$ and satisfy

$$\begin{aligned} &\lim_{n\to\infty} M\left(ASx_n, SSx_n, t\right) = M(Az, Sz, t) = 1, \quad \lim_{n\to\infty} N\left(ASx_n, SSx_n, t\right) = N(Az, Sz, t) = 0, \\ &\lim_{n\to\infty} M\left(SAx_n, AAx_n, t\right) = M(Sz, Az, t) = 1, \quad \lim_{n\to\infty} N\left(SAx_n, AAx_n, t\right) = N(Sz, Az, t) = 0, \end{aligned}$$

$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w, w \in X$ and satisfy

$$\begin{aligned} &\lim_{n\to\infty} M\left(BTy_n, TTy_n, t\right) = M(Bw, Tw, t) = 1, \quad \lim_{n\to\infty} N\left(BTy_n, TTy_n, t\right) = N(Bw, Tw, t) = 0, \\ &\lim_{n\to\infty} M\left(TBx_n, BBy_n, t\right) = M(Tw, Bw, t) = 1, \quad \lim_{n\to\infty} N\left(TBx_n, BBy_n, t\right) = N(Tw, Bw, t) = 0. \end{aligned}$$

Therefore, $Az = Sz$ and $Bw = Tw$, that is z is a coincidence point of A and S; w is a coincidence point of B and T.

Now, we prove that $z = w$. By using (3.1) for $x = x_n$ and $y = y_n$, we get

$$\begin{aligned} M(Ax_n, By_n, t) &\geq \psi\left(\min\left\{\begin{array}{c} M(Sx_n, Ty_n, t), M(Ax_n, Sx_n, t), M(By_n, Ty_n, t), \\ \frac{1}{2}\left[M(By_n, Sx_n, t) + M(Ax_n, Ty_n, t)\right] \end{array}\right\}\right), \\ N(Ax_n, By_n, t) &\leq \phi\left(\max\left\{\begin{array}{c} N(Sx_n, Ty_n, t), N(Ax_n, Sx_n, t), N(By_n, Ty_n, t), \\ \frac{1}{2}\left[N(By_n, Sx_n, t) + N(Ax_n, Ty_n, t)\right] \end{array}\right\}\right). \end{aligned}$$

Taking the limit as $n \to \infty$, we have

$$\begin{aligned} M(z, w, t) &\geq \psi\left(\min\left\{M(z, w, t), M(z, z, t), M(w, w, t), \frac{1}{2}\left[M(w, z, t) + M(z, w, t)\right]\right\}\right), \\ N(z, w, t) &\leq \phi\left(\max\left\{N(z, w, t), N(z, z, t), N(w, w, t), \frac{1}{2}\left[N(w, z, t) + N(z, w, t)\right]\right\}\right), \end{aligned}$$

that is

$$\begin{aligned} M(z, w, t) &\geq \psi\left(M(z, w, t)\right) > M(z, w, t), \\ N(z, w, t) &\leq \phi\left(N(z, w, t)\right) < N(z, w, t), \end{aligned}$$

which yield $z = w$.

Again using (3.1) for $x = z$ and $y = y_n$, we obtain

$$\begin{aligned} M(Az, By_n, t) &\geq \psi\left(\min\left\{\begin{array}{c} M(Sz, Ty_n, t), M(Az, Sz, t), M(By_n, Ty_n, t), \\ \frac{1}{2}\left[M(By_n, Sz, , t) + M(Az, Ty_n, t)\right] \end{array}\right\}\right), \\ N(Az, By_n, t) &\leq \phi\left(\max\left\{\begin{array}{c} N(Sz, Ty_n, t), N(Az, Sz, t), N(By_n, Ty_n, t), \\ \frac{1}{2}\left[N(By_n, Sz, , t) + N(Az, Ty_n, t)\right] \end{array}\right\}\right). \end{aligned}$$

Taking the limit as $n \to \infty$, we have

$$M(Az,w,t) \geq \psi\left(\min\left\{M(Sz,w,t), M(Az,Sz,t), M(w,w,t), \frac{1}{2}\left[M(w,Sz,t)+M(Az,w,t)\right]\right\}\right),$$
$$N(Az,w,t) \leq \phi\left(\max\left\{N(Sz,w,t), N(Az,Sz,t), N(w,w,t), \frac{1}{2}\left[N(w,Sz,t)+N(Az,w,t)\right]\right\}\right),$$

that is

$$M(Az,w,t) \geq \psi\left(M(Az,w,t)\right) > M(Az,w,t),$$
$$N(Az,w,t) \leq \phi\left(N(Az,w,t)\right) < N(Az,w,t),$$

which yield $Az = w = z$. Therefore $z = w$ is a common fixed point of A, B, S and T.

For uniqueness, suppose that there exist another fixed point u of A, B, S and T. Then from (3.1), we have

$$\begin{aligned} M(Az,Bu,t) &\geq \psi\left(\min\left\{\begin{array}{c} M(Sz,Tu,t), M(Az,Sz,t), M(Bu,Tu,t), \\ \frac{1}{2}\left[M(Bu,Sz,,t)+M(Az,Tu,t)\right] \end{array}\right\}\right) \\ &= \psi\left(\min\left\{M(Az,Bu,t), 1, 1, M(Az,Bu,t), \frac{1}{2}\left[M(Bu,Az,,t)+M(Az,Bu,t)\right]\right\}\right) \\ &= \psi(M(Az,Bu,t)) \\ &> M(Az,Bu,t) \end{aligned}$$

and

$$\begin{aligned} N(Az,Bu,t) &\leq \phi\left(\max\left\{\begin{array}{c} N(Sz,Tu,t), N(Az,Sz,t), N(Bu,Tu,t), \\ \frac{1}{2}\left[N(Bu,Sz,,t)+N(Az,Tu,t)\right] \end{array}\right\}\right) \\ &= \phi\left(\max\left\{N(Az,Bu,t), 0, 0, N(Az,Bu,t), \frac{1}{2}\left[N(Bu,Az,,t)+N(Az,Bu,t)\right]\right\}\right) \\ &= \phi(N(Az,Bu,t)) \\ &< N(Az,Bu,t), \end{aligned}$$

which yield $z = u$. Therefore uniqueness follows. □

If we put $S = T$ in Theorem 3.1, we get the following result.

Corollary 3.2. *Let A, B and S be self maps of a weak non-Archimedean intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ and let the pairs (A, S) and (B, S) are subcompatible maps of type (α) and subsequentially continuous. If*

$$M(Ax,By,t) \geq \psi\left(\min\left\{\begin{array}{c} M(Sx,Sy,t), M(Ax,Sx,t), M(By,Sy,t), \\ \frac{1}{2}\left[M(By,Sx,t)+M(Ax,Sy,t)\right] \end{array}\right\}\right),$$
$$N(Ax,By,t) \leq \phi\left(\max\left\{\begin{array}{c} N(Sx,Sy,t), N(Ax,Sx,t), N(By,Sy,t), \\ \frac{1}{2}\left[N(By,Sx,t)+N(Ax,Sy,t)\right] \end{array}\right\}\right)$$

for all $x, y \in X$, $t > 0$, where $\psi, \phi : [0,1] \to [0,1]$ are continuous functions such that $\psi(s) > s$ and $\phi(s) < s$ for each $s \in (0,1)$. Then A, B and S have a unique common fixed point in X.

If we put $A = B$ and $S = T$ in Theorem 3.1, we get the following result.

Corollary 3.3. *Let A and S be self maps of a weak non-Archimedean intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ and let the pairs (A, S) is subcompatible maps of type (α) and subsequentially continuous. If*

$$M(Ax, Ay, t) \geq \psi \left(\min \left\{ \begin{array}{c} M(Sx, Sy, t), M(Ax, Sx, t), M(Ay, Sy, t), \\ \frac{1}{2}\left[M(Ay, Sx, t) + M(Ax, Sy, t)\right] \end{array} \right\} \right),$$
$$N(Ax, Ay, t) \leq \phi \left(\max \left\{ \begin{array}{c} N(Sx, Sy, t), N(Ax, Sx, t), N(Ay, Sy, t), \\ \frac{1}{2}\left[N(Ay, Sx, t) + N(Ax, Sy, t)\right] \end{array} \right\} \right)$$

for all $x, y \in X$, $t > 0$, where $\psi, \phi : [0, 1] \to [0, 1]$ are continuous functions such that $\psi(s) > s$ and $\phi(s) < s$ for each $s \in (0, 1)$. Then A and S have a unique common fixed point in X.

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