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A new concept of (α, F_d)-contraction on quasi metric space

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Abstract

In the present paper, we introduce a new concept of (α, F_d)-contraction on quasi metric space. Then we provide some new fixed point theorems for such type mappings on left $\mathcal{K}$, left $\mathcal{M}$ and left Smyth-complete quasi metric spaces.

Keywords: Quasi metric space, left K-Cauchy sequence, left $\mathcal{K}$-completeness, fixed point.
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1. Introduction and Preliminaries

A quasi-pseudo metric on a nonempty set X is a function $d : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

- $d(x, x) = 0$,
- $d(x, y) \leq d(x, z) + d(z, y)$.

If a quasi-pseudo metric d satisfies:

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- $d(x,y) = d(y,x) = 0 \Rightarrow x = y$,

then d is said to be quasi metric, in addition if a quasi metric d satisfies

- $d(x,y) = 0 \Rightarrow x = y$,

then d is said to be T_1-quasi metric. It is clear that, every metric is a T_1-quasi metric, every T_1-quasi metric is a quasi metric and every quasi metric is a quasi-pseudo metric. In this case the pair (X,d) is said to be quasi-pseudo (resp. quasi, T_1-quasi) metric space.

Let (X,d) be a quasi-pseudo metric space. Given a point $x_0 \in X$ and a real constant $\varepsilon > 0$, the sets

$$B_d(x_0,\varepsilon) = \{y \in X : d(x_0,y) < \varepsilon\},$$

and

$$B_d[x_0,\varepsilon] = \{y \in X : d(x_0,y) \leq \varepsilon\},$$

are called open ball and closed ball, respectively, with center x_0 and radius ε.

Each quasi-pseudo metric d on X generates a topology τ_d on X which has a base the family of open balls $\{B_d(x,\varepsilon) : x \in X \text{ and } \varepsilon > 0\}$. The closure of a subset A of X with respect to τ_d is denoted by $cl_d(A)$. If d is a quasi metric on X, then τ_d is a T_0 topology, and if d is a T_1-quasi metric, then τ_d is a T_1 topology on X.

If d is a quasi metric and τ_d is T_1 topology, then d is T_1-quasi metric.

If d is a quasi-pseudo metric on X, then the functions d^{-1}, d^s and d_+ defined by

$$d^{-1}(x,y) = d(y,x),$$

$$d^s(x,y) = \max\left\{d(x,y), d^{-1}(x,y)\right\},$$

and

$$d_+(x,y) = d(x,y) + d^{-1}(x,y),$$

are also quasi-pseudo metrics on X. If d is a quasi metric, then d^s and d_+ are (equivalent) metrics on X.

Let (X,d) be a quasi metric space and $x \in X$. The convergence of a sequence $\{x_n\}$ to x with respect to τ_d called d-convergence and denoted by $x_n \overset{d}{\to} x$, is defined

$$x_n \overset{d}{\to} x \Leftrightarrow d(x,x_n) \to 0.$$

Similarly, the convergence of a sequence $\{x_n\}$ to x with respect to $\tau_{d^{-1}}$ called d^{-1}-convergence and denoted by $x_n \overset{d^{-1}}{\to} x$, is defined

$$x_n \overset{d^{-1}}{\to} x \Leftrightarrow d(x_n,x) \to 0.$$

Finally, the convergence of a sequence $\{x_n\}$ to x with respect to τ_{d^s} called d^s-convergence and denoted by $x_n \overset{d^s}{\to} x$, is defined

$$x_n \overset{d^s}{\to} x \Leftrightarrow d^s(x_n,x) \to 0.$$

It is clear that $x_n \overset{d^s}{\to} x \Leftrightarrow x_n \overset{d}{\to} x$ and $x_n \overset{d^{-1}}{\to} x$. More and detailed information about some important properties of quasi metric spaces and their topological structures can be found in [8, 16, 17].

Definition 1.1 ([23])**.** Let (X,d) be a quasi metric space. A sequence $\{x_n\}$ in X is called

- left K-Cauchy (or forward Cauchy) if for every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$\forall n,k,\ n \geq k \geq n_0,\ d(x_k,x_n) < \varepsilon,$$

- right K-Cauchy (or backward Cauchy) if for every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$\forall n, k,\ n \geq k \geq n_0,\ d(x_n, x_k) < \varepsilon,$$

- d^s-Cauchy if for every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$\forall n, k \geq n_0,\ d(x_n, x_k) < \varepsilon.$$

It is clear that $\{x_n\}$ is d^s-Cauchy if and only if it is both left K-Cauchy and right K-Cauchy. If a sequence is left K-Cauchy with respect to d, then it is right K-Cauchy with respect to d^{-1}. If $\{x_n\}$ is a sequence in a quasi metric space (X, d) such that

$$\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \infty,$$

then it is left K-Cauchy sequence.

It is well known that every convergent sequence is Cauchy in a metric space. In general, this situation is not valid in a quasi metric space. That is, d-convergent or d^{-1}-convergent sequences may not be Cauchy (in the sense of d^s, left K and right K) in a quasi metric space (see [23]).

Definition 1.2 ([5, 23])**.** Let (X, d) be a quasi metric space. Then (X, d) is said to be

- bicomplete if every d^s-Cauchy sequence is d^s-convergent,
- left (right) $\mathcal{K}$-complete if every left (right) K-Cauchy sequence is d-convergent,
- left (right) $\mathcal{M}$-complete if every left (right) K-Cauchy sequence is d^{-1}-convergent,
- left (right) Smyth complete if every left (right) K-Cauchy sequence is d^s-convergent.

One can find more detailed information about some kind of Cauchyness, completeness and some important properties of quasi metric space in [5, 23, 24].

On the other hand, α-admissibility and F-contractivity of a mapping are popular concepts in recent metrical fixed point theory. The concept of α-admissibility of a mapping on a nonempty set has been introduced by Samet, et al. [25]. Let X be a nonempty set, T be a self mapping of X and $\alpha : X \times X \to [0, \infty)$ be a function. Then T is said to be α-admissible if

$$\alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, Ty) \geq 1.$$

Using α-admissibility of a mapping, Samet, et al. [25] provided some general fixed point results including many known theorems on complete metric spaces.

The concept of F-contraction was introduced by Wardowski [26]. Let $\mathcal{F}$ be the family of all functions $F : (0, \infty) \to \mathbb{R}$ satisfying the following conditions:

(F1) F is strictly increasing,

(F2) For each sequence $\{\lambda_n\}$ of positive numbers $\lim_{n\to\infty} \lambda_n = 0$ if and only if $\lim_{n\to\infty} F(\lambda_n) = -\infty$,

(F3) There exists $k \in (0, 1)$ such that $\lim_{\lambda\to 0^+} \lambda^k F(\lambda) = 0$.

Let T be a self mapping of a metric space (X, d) and $F \in \mathcal{F}$. Then, T is said to be F-contraction if for all $x, y \in X$ with $d(Tx, Ty) > 0$, there exists $\tau > 0$ such that

$$\tau + F(d(Tx, Ty)) \leq F(d(x, y)).$$

Various fixed point results for α-admissible mappings and F-contractions on complete metric space can be found in [3, 10, 12, 13, 14, 15] and [4, 6, 7, 9, 21, 22, 27], respectively.

The aim of this paper is to establish several new fixed point results on some kind of complete quasi metric spaces by taking into account α-admissibility and a quasi metric version of F-contractivity of a mapping.

We can find some recent fixed point results for single valued and multivalued mappings on quasi metric spaces in [1, 2, 11, 18, 19, 20].

2. Fixed Point Result

Let (X, d) be a quasi metric space, $T : X \to X$ be a mapping and $\alpha : X \times X \to [0, \infty)$ be a function. We will consider the following set to introduce a quasi metric version of F-contractivity of a mapping:

$$T_\alpha = \{(x, y) \in X \times X : \alpha(x, y) \geq 1 \text{ and } d(Tx, Ty) > 0\}.$$

Definition 2.1. Let (X, d) be a quasi metric space, $T : X \to X$ be a mapping satisfying

$$d(x, y) = 0 \Rightarrow d(Tx, Ty) = 0, \tag{2.1}$$

$\alpha : X \times X \to [0, \infty)$ and $F \in \mathcal{F}$ be two functions. Then T is said to be an (α, F_d)-contraction if there exists $\tau > 0$ such that

$$\tau + F(d(Tx, Ty)) \leq F(d(x, y)) \tag{2.2}$$

for all $(x, y) \in T_\alpha$.

The following example shows the importance of condition (2.1).

Example 2.2. Let $X = \{0, 1\} \cup A$, where $A = \{2, 3, \cdots\}$ and let d be the quasi metric on X given by

$$\begin{aligned} d(n, n) &= 0 \text{ for all } n \in X \\ d(0, n) &= d(n, 1) = 0 \text{ for all } n \in X \backslash \{0\} \\ d(n, 0) &= 1 \text{ for all } n \in X \backslash \{0\} \\ d(n, m) &= \frac{1}{2^{n+1}} + \frac{1}{2^{m+1}}, \text{ otherwise.} \end{aligned}$$

Now define $T : X \to X$ as $Tn = n + 1$ for all $n \in X$ and $\alpha : X \times X \to [0, \infty)$ as

$$\alpha(n, m) = \begin{cases} 1, & (n, m) \in A \times A \\ 0, & \text{otherwise.} \end{cases}$$

Note that $T_\alpha = \{(n, m) : n, m \in A \text{ and } n \neq m\}$. Now, we show that inequality (2.2) is satisfied with $F(\lambda) = \ln \lambda$ and $\tau = \ln 2$ for all $(n, m) \in T_\alpha$. Indeed, for $n, m \in A$ with $n \neq m$, we have

$$\begin{aligned} \ln d(Tn, Tm) &= \ln d(n + 1, m + 1) \\ &= \ln \left(\frac{1}{2^{n+2}} + \frac{1}{2^{m+2}} \right) \\ &= -\ln 2 + \ln \left(\frac{1}{2^{n+1}} + \frac{1}{2^{m+1}} \right) \\ &= -\ln 2 + d(n, m). \end{aligned}$$

However, T is not (α, F_d)-contraction, since the condition (2.1) does not hold. Indeed, $d(0, 1) = 0$ but $d(T0, T1) > 0$.

Remark 2.3. If (X, d) is a T_1-quasi metric space, then every mapping $T : X \to X$ satisfies the condition (2.1).

Remark 2.4. It is clear from Definition 2.1 that if T is an (α, F_d)-contraction on a quasi metric space (X, d), then

$$d(Tx, Ty) \leq d(x, y),$$

for all $x, y \in X$ with $\alpha(x, y) \geq 1$.

Our main fixed point result is as follows:

Theorem 2.5. *Let (X, d) be a Hausdorff left $\mathcal{K}$-complete quasi metric space, $T : X \to X$ be an (α, F_d)-contraction. Assume that T is α-admissible and τ_d-continuous. If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point in X.*

Proof. Let $x_0 \in X$ be such that $\alpha(x_0, Tx_0) \geq 1$. Define a sequence $\{x_n\}$ in X by $x_n = Tx_{n-1}$ for all $n \in \mathbb{N}$. Since T is α-admissible, then $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$. Now, let $d_n = d(x_n, x_{n+1})$ for all $n \in \mathbb{N}$. If there exists $k \in \mathbb{N}$ with $d_k = d(x_k, x_{k+1}) = 0$, then x_k is a fixed point of T since d is T_1-quasi metric. Suppose $d_n > 0$ for all $n \in \mathbb{N}$. Since T is (α, F_d)-contraction, we get

$$F(d_n) \leq F(d_{n-1}) - \tau \leq F(d_{n-2}) - 2\tau \leq \cdots \leq F(d_0) - n\tau. \tag{2.3}$$

From (2.3), we get $\lim_{n\to\infty} F(d_n) = -\infty$. Thus, from (F2), we have

$$\lim_{n\to\infty} d_n = 0.$$

From (F3) there exists $k \in (0, 1)$ such that

$$\lim_{n\to\infty} d_n^k F(d_n) = 0.$$

By (2.3), the following holds for all $n \in \mathbb{N}$

$$d_n^k F(d_n) - d_n^k F(d_0) \leq -d_n^k n\tau \leq 0. \tag{2.4}$$

Letting $n \to \infty$ in (2.4), we obtain that

$$\lim_{n\to\infty} nd_n^k = 0. \tag{2.5}$$

From (2.5), there exits $n_1 \in \mathbb{N}$ such that $nd_n^k \leq 1$ for all $n \geq n_1$. So, we have, for all $n \geq n_1$

$$d_n \leq \frac{1}{n^{1/k}}. \tag{2.6}$$

Therefore $\sum\limits_{n=1}^{\infty} d_n < \infty$. Now let $m, n \in \mathbb{N}$ with $m > n \geq n_1$, then we get

$$\begin{aligned} d(x_n, x_m) &\leq d(x_{n+1}, x_{n+2}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m) \\ &= d_n + d_{n+1} + \cdots + d_{m-1} \\ &\leq \sum_{k=n}^{\infty} d_k. \end{aligned}$$

Since $\sum\limits_{k=1}^{\infty} d_k$ is convergent, then we get $\{x_n\}$ is left K-Cauchy sequence in the quasi metric space (X, d). Since (X, d) left $\mathcal{K}$-complete, there exists $z \in X$ such that $\{x_n\}$ is d-converges to z, that is, $d(z, x_n) \to 0$ as $n \to \infty$. Since T is τ_d-continuous, then $d(Tz, Tx_n) = d(Tz, x_{n+1}) \to 0$ as $n \to \infty$. Since X is Hausdorff we get $z = Tz$. □

The following example shows that the Hausdorffness condition of X can not be removed in above theorem.

Example 2.6. Let $X = \{\frac{1}{n} : n \in \mathbb{N}\}$ and

$$d(x, y) = \begin{cases} 0, & x = y \\ y, & x \neq y. \end{cases}$$

Then (X, d) is a left $\mathcal{K}$-complete T_1-quasi metric space, but not Hausdorff since τ_d is cofinite topology. Define a mapping $T : X \to X$ by $Tx = \frac{x}{2}$, then T is (α, F_d)-contraction with $\alpha(x, y) = 1$, $F(\lambda) = \ln \lambda$ and $\tau = \ln 2$. Also T is α-admissible and τ_d-continuous. However, T has no fixed point.

In Theorem 2.5, if we take $\tau_{d^{-1}}$ continuity of the mapping T instead of τ_d-continuity, we can take left $\mathcal{M}$-completeness of X.

Theorem 2.7. *Let (X, d) be a Hausdorff left $\mathcal{M}$-complete quasi metric space, $T : X \to X$ be an (α, F_d)-contraction. Assume that T is α-admissible and $\tau_{d^{-1}}$-continuous. If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point in X.*

Proof. As in the proof of Theorem 2.5, we can get the iterative sequence $\{x_n\}$ is left K-Cauchy. Since (X, d) left $\mathcal{M}$-complete, there exists $z \in X$ such that $\{x_n\}$ is d^{-1}-converges to z, that is, $d(x_n, z) \to 0$ as $n \to \infty$. By $\tau_{d^{-1}}$-continuity of T, we have $d(Tx_n, Tz) = d(x_{n+1}, Tz) \to 0$ as $n \to \infty$. Since X is Hausdorff we get $z = Tz$. □

The following theorems show that if we take into account the left Smyth completeness of X, we can remove the Hausdorffness condition. However, we need the quasi metric d is still T_1-quasi metric.

Theorem 2.8. *Let (X, d) be left Smyth complete T_1-quasi metric space, $T : X \to X$ be an (α, F_d)-contraction. Assume that T is α-admissible and τ_d or $\tau_{d^{-1}}$-continuous. If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point in X.*

Proof. As in the proof of Theorem 2.5, we can get the iterative sequence $\{x_n\}$ is left K-Cauchy. Since (X, d) left Smyth complete, there exists $z \in X$ such that $\{x_n\}$ is d^s-converges to z, that is, $d^s(x_n, z) \to 0$ as $n \to \infty$.

If T is τ_d-continuous, then $d(Tz, Tx_n) = d(Tz, x_{n+1}) \to 0$ as $n \to \infty$. Therefore, we get

$$d(Tz, z) \leq d(Tz, x_{n+1}) + d(x_{n+1}, z) \to 0 \text{ as } n \to \infty.$$

If T is $\tau_{d^{-1}}$-continuous, then $d(Tx_n, Tz) = d(x_{n+1}, Tz) \to 0$ as $n \to \infty$. Therefore, we get

$$d(z, Tz) \leq d(z, x_{n+1}) + d(x_{n+1}, Tz) \to 0 \text{ as } n \to \infty.$$

Since X is T_1-quasi metric space, we obtain $z = Tz$. □

One of the following properties of the spaces X, can be considered instead of τ_d or $\tau_{d^{-1}}$-continuity of T in Theorem 2.8.

It is said that the quasi metric space (X, d) has (A_d) (resp. (B_d)) property whenever $\{x_n\}$ is a sequence in X such that $x_n \xrightarrow{d} z$ and $\alpha(x_n, x_{n+1}) \geq 1$ implies $\alpha(x_n, z) \geq 1$ (resp. $\alpha(z, x_n) \geq 1$) for all $n \in \mathbb{N}$.

Theorem 2.9. *Let (X, d) be left Smyth complete T_1-quasi metric space, $T : X \to X$ be an (α, F_d)-contraction. Assume that T is α-admissible and X has (A_d), $(A_{d^{-1}})$, (B_d) or $(B_{d^{-1}})$ property. If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point in X.*

Proof. As in the proof of Theorem 2.5, we can get the iterative sequence $\{x_n\}$ is left K-Cauchy. Since (X, d) left Smyth complete, there exists $z \in X$ such that $\{x_n\}$ is d^s-converges to z, that is, $d^s(x_n, z) \to 0$ as $n \to \infty$.

If X has (A_d) or $(A_{d^{-1}})$ property, we get $\alpha(x_n, z) \geq 1$. Therefore, considering Remark 2.4, we have

$$\begin{aligned} d(z, Tz) &\leq d(z, x_{n+1}) + d(Tx_n, Tz) \\ &\leq d(z, x_{n+1}) + d(x_n, z) \to 0 \text{ as } n \to \infty. \end{aligned}$$

If X has (B_d) or $(B_{d^{-1}})$ property, we get $\alpha(z, x_n) \geq 1$. Therefore, considering Remark 2.4, we have

$$\begin{aligned} d(Tz, z) &\leq d(Tz, Tx_n) + d(x_{n+1}, z) \\ &\leq d(z, x_n) + d(x_{n+1}, z) \to 0 \text{ as } n \to \infty. \end{aligned}$$

Since X is T_1-quasi metric space, we get $z = Tz$. □

The following example emphasizes the importances of the continuity of T in Theorem 2.8 and (A_d), $(A_{d^{-1}})$, (B_d) or $(B_{d^{-1}})$ properties on X in Theorem 2.9.

Example 2.10. Let $X = [0, 1]$ and $d(x, y) = |x - y|$ for all $x, y \in X$. Now define $T : X \to X$ as

$$Tx = \begin{cases} 1, & x = 0 \\ \frac{x}{2}, & x \neq 0 \end{cases}$$

and $\alpha : X \times X \to [0, \infty)$ as

$$\alpha(x, y) = \begin{cases} 1, & xy \neq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Since (X, d) is complete metric space then it is left Smyth complete T_1-quasi metric space. Observe that T is α-admissible and also for $x_0 = 1$, we have $\alpha(x_0, Tx_0) = \alpha(1, T1) = \alpha(1, \frac{1}{2}) \geq 1$. On the other hand, since

$$\begin{aligned} T_\alpha &= \{(x, y) \in X \times X : \alpha(x, y) \geq 1 \text{ and } d(Tx, Ty) > 0\} \\ &= \{(x, y) \in X \times X : xy \neq 0 \text{ and } x \neq y\}, \end{aligned}$$

then

$$d(Tx, Ty) = \frac{1}{2} d(x, y),$$

for all $(x, y) \in T_\alpha$. That is, T is (α, F_d)-contraction with $F(\lambda) = \ln \lambda$ and $\tau = \ln 2$.

Note that T is not τ_d (and $\tau_{d^{-1}}$)-continuous. Also note that X has not (A_d) (and (B_d)) property. To see this we can consider the sequence $x_n = \frac{1}{n}$.

Remark 2.11. Taking into account the right completeness (in the sense of $\mathcal{K}$, $\mathcal{M}$ and Smyth), we can provide similar fixed point results on quasi metric space.

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