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On the intuitionistic fuzzy metric spaces and the intuitionistic fuzzy normed spaces

Xia Li^a, Meifang Guo^a, Yongfu Su^{b,*}

^aDepartment of Mathematics and Sciences, Hebei GEO University, Shijiazhuang 050031, China. ^bDepartment of Mathematics, Tianjin Polytechnic University 300387, China.

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Abstract

The purpose of this article is to evaluate the definition of a class of intuitionistic fuzzy metric space which was presented by Park [J. H. Park, Chaos Solitons Fractals, **22** (2004), 1039–1046]. This review is also appropriate to the definition of a class of intuitionistic fuzzy normed space which was presented by Saadati and Park [R. Saadati, J. H. Park, Chaos Solitons Fractals, **27** (2006), 331–344]. ©2016 All rights reserved.

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1. Introduction and preliminaries

The concept of fuzzy set was introduced by Zadeh [33] in 1965 and it is well-known that there are many viewpoints of the notion of metric space in fuzzy topology. In 1975, Kramosil and Michalek [16] introduced the concept of a fuzzy metric space, which can be regarded as a generalization of the statistical (probabilistic) metric space. Clearly, this work provides an important basis for the construction of fixed point theory in fuzzy metric spaces. Afterwards, Grabice [12] defined the completeness of the fuzzy metric space (now known as a G-complete fuzzy metric space) and extended the Banach contraction theorem to G-complete fuzzy metric spaces. Subsequently, George and Veeramani [10] modified the definition of the Cauchy sequence introduced by Grabiec. Meanwhile, they slightly modified the notion of a fuzzy metric space introduced by

 * Corresponding author

Email addresses: libinghua66@sina.com (Xia Li), 58357160@qq.com (Meifang Guo), tjsuyongfu@163.com (Yongfu Su)

Kramosil and Michalek and then defined a Hausdorff and first countable topology. Since then, the notion of a complete fuzzy metric space presented by George and Veeramani (now known as an complete fuzzy metric space) has been emerged as another characterization of completeness, and some fixed point theorems have also been constructed on the basis of this metric space. From the above analysis, we can see that there are many studies related to fixed point theory based on the above two kinds of complete fuzzy metric spaces (see [4, 5, 8, 11, 15, 18, 24, 26, 28, 30–32] and the references therein). On the other hand, the concept of intuitionistic fuzzy set was introduced by Atanassov [3] as generalization of fuzzy set. In 2004, Park introduced the notion of intuitionistic fuzzy metric space [21]. He showed that for each intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$, the topology generated by the intuitionistic fuzzy metric space and related results we refer the reader to [2, 7, 9, 13, 19, 21–23, 27]. In 2006, Saadati and Park introduced the notion of intuitionistic fuzzy normed space [25]. For more details on intuitionistic fuzzy normed space and related results we refer the reader to [1, 6, 14, 17, 20, 25, 29].

Now we shall recall some well-known definitions and results in the theory of fuzzy metric spaces which will be used in the paper. For more details, we refer the reader to [1, 2, 7, 9, 13, 14, 19–23, 25, 27, 29].

Definition 1.1. A *t*-norm * is a binary operation on [0, 1] which satisfies the following conditions:

- (a) * is associative and commutative;
- (b) * is continuous;
- (c) a * 1 = a, for all $a \in [0, 1]$;
- (d) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for each $a, b, c, d \in [0, 1]$.

The following are three basic *t*-norms:

- $a *_1 b = \max(a + b 1, 0);$
- $a *_2 b = a \cdot b;$
- $a *_3 b = \min(a, b);$

It is easy to check that,

$$a *_1 b \le a *_2 b \le a *_3 b$$

for any $a, b \in [0, 1]$.

Definition 1.2. A *t*-conorm \Diamond is a binary operation on [0, 1] which satisfies the following conditions:

- (a) \Diamond is associative and commutative;
- (b) \Diamond is continuous;
- (c) $a \diamondsuit 0 = a$ for all $a \in [0, 1]$;
- (d) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0, 1]$.

The following are three basic *t*-conorms:

- $a\Diamond_1 b = \min(a+b,1);$
- $a\Diamond_2 b = a + b ab;$
- $a \diamondsuit_3 b = \max(a, b);$

It is easy to check that,

$$a\Diamond_1 b \ge a\Diamond_2 b \ge a\Diamond_3 b,$$

for any $a, b \in [0, 1]$. It is also easy to check that,

$$a\Diamond b = 1 - (1 - a) * (1 - b),$$

and

$$a * b = 1 - (1 - a) \Diamond (1 - b)$$

for any $a, b \in [0, 1]$.

Now we recall the definitions of intuitionistic fuzzy metric space which was introduced by Park [21] in 2004 (see also [9]).

Definition 1.3 ([21]). A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, \Diamond is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, 1)$ satisfying the following conditions: for all $x, y, z \in X, s, t > 0$,

- (IFM-1) $M(x, y, t) + N(x, y, t) \le 1;$
- (IFM-2) M(x, y, t) > 0;
- (IFM-3) M(x, y, t) = 1, if and only if x = y;
- (IFM-4) M(x, y, t) = M(y, x, t);
- (IFM-5) $M(x, z, t) * M(y, z, s) \le M(x, y, t + s);$
- (IFM-6) $M(x, z, \cdot) : (0, +\infty) \to [0, 1]$ is continuous;
- (IFM-7) N(x, y, t) < 1;
- (IFM-8) N(x, y, t) = 0, if and only if x = y;
- (IFM-9) N(x, y, t) = N(y, x, t);
- (IFM-10) $N(x, z, t) \Diamond N(y, z, s) \ge N(x, y, t+s);$
- (IFM-11) $N(x, z, \cdot) : (0, +\infty) \to [0, 1]$ is continuous.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of nonnearness between x and y with respect to t, respectively.

Definition 1.4 ([21]). Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space, and let $r \in (0, 1), t > 0$ and $x \in X$. The set

$$B_{(M,N)}(x,r,t) = \{ y \in X : M(x,y,t) > 1-r, \ N(x,y,t) < r \},\$$

is called the open ball with center x and radius r with respect to t.

There are some another definitions of intuitionistic fuzzy metric space and intuitionistic fuzzy normed space. These definitions are slightly different with corresponding to Definition 1.3.

Definition 1.5 ([17]). A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, \Diamond is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, 1)$ satisfying the following conditions: for all $x, y, z \in X, s, t > 0$,

- (i) $M(x, y, t) + N(x, y, t) \le 1;$
- (ii) M(x, y, 0) = 0;
- (iii) M(x, y, t) = 1 if and only if x = y;
- (iv) M(x, y, t) = M(y, x, t);
- (v) $M(x, z, t) * M(y, z, s) \le M(x, y, t + s);$

- (vi) $M(x, z, \cdot) : (0, +\infty) \to [0, 1]$ is left continuous;
- (vii) $\lim_{t\to\infty} M(x, y, t) = 1;$
- (viii) N(x, y, 0) = 1;
 - (ix) N(x, y, t) = 0 if and only if x = y;
 - $(\mathbf{x}) \ N(x,y,t) = N(y,x,t);$
- (xi) $N(x,z,t) \Diamond N(y,z,s) \ge N(x,y,t+s);$
- (xii) $N(x, z, \cdot) : (0, +\infty) \to [0, 1]$ is right continuous;
- (xiii) $\lim_{t\to\infty} N(x, y, t) = 0.$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of nonnearness between x and y with respect to t, respectively.

By using the notions of continuous t-norm and t-conorm, Saadati and Park [25] have recently introduced the concepts of intuitionistic fuzzy normed space and defined the convergence and Cauchy sequences in this setting as follows

Definition 1.6 ([25]). A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy normed space, if X is a vector space, * is a continuous t-norm, \diamond is a continuous t-conorm and M, N are fuzzy sets on $X \times (0, 1)$ satisfying the following conditions: for all $x, y \in X, s, t > 0$,

- (1) $M(x,t) + N(x,t) \le 1;$
- (2) M(x,t) > 0;
- (3) M(x,t) = 1, if and only if $x = \theta$;
- (4) $M(\alpha x, t) = M(\frac{x}{|\alpha|}, t)$ for each $\alpha \neq 0$;
- (5) $M(x,t) * M(y,s) \le M(x+y,t+s);$
- (6) $M(x, \cdot) : (0, +\infty) \to [0, 1]$ is continuous;
- (7) $\lim_{t\to\infty} M(x,t) = 1$, $\lim_{t\to0} M(x,t) = 0$;
- (8) N(x,t) < 1;
- (9) N(x,t) = 0, if and only if $x = \theta$;
- (10) $N(\alpha x, t) = N(\frac{x}{|\alpha|}, t)$ for each $\alpha \neq 0$;
- (11) $N(x,t) \Diamond N(y,s) \ge N(x+y,t+s);$
- (12) $N(x, \cdot) : (0, +\infty) \to [0, 1]$ is continuous;
- (13) $\lim_{t\to\infty} N(x,t) = 0$, $\lim_{t\to0} N(x,t) = 1$.

In this case, $(X, M, N, *, \Diamond)$ is called an intuitionistic fuzzy norm. For simplicity in notation, we denote the intuitionistic fuzzy normed spaces by (X, M, N) instead of $(X, M, N, *, \Diamond)$. For example, let $(X, \|\cdot\|)$ be a normed space, and let a * b = ab and $a \Diamond b = \min(a + b, 1)$ for all $a, b \in [0, 1]$. For all $x \in X$ and every t > 0, consider

$$M(x,t) = \frac{t}{t + \|x\|}, \quad N(x,t) = \frac{\|x\|}{t + \|x\|}.$$

Then (X, M, N) is an intuitionistic fuzzy normed space. The functions M(x, t) and N(x, t) denote the degree of nearness and the degree of nonnearness between x and θ with respect to t, respectively.

Based on the above definitions, many similar definitions of intuitionistic fuzzy metric space and intuitionistic fuzzy normed space were presented by some authors. But, there exist some inappropriate content in these definitions. The purpose of this article is to evaluate the definition of a class of intuitionistic fuzzy metric space which was presented by Park [21]. This review is also appropriate to the definition of a class of intuitionistic fuzzy normed space which was presented by Saadati and Park [25] and some others.

2. Main results

Theorem 2.1. Let X be a nonempty set, * be a continuous t-norm, and M be a fuzzy set on $X^2 \times (0,1)$ satisfying the conditions (IFM-2)-(IFM-6) for all $x, y, z \in X$, s, t > 0. Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space, where $a \diamond b = 1 - (1 - a) * (1 - b)$, for all $a, b \in [0,1]$ and N(x, y, t) = 1 - M(x, y, t), for all $x, y \in X$, t > 0.

Proof. It is sufficient to check the conditions (IFM-7)-(IFM-11). From the relation

$$N(x, y, t) = 1 - M(x, y, t), \quad \forall \ x, y \in X, \quad t > 0,$$

we know that, the conditions (IFM-7), (IFM-8), (IFM-9) and (IFM-11) hold. Next, we prove the condition (IFM-10). From the condition (IFM-5), we have that,

$$(1 - N(x, z, t)) * (1 - N(y, z, s)) \le (1 - N(x, y, t + s))$$

for all $x, y, z \in X$, t, s > 0.

$$1 - (1 - N(x, z, t)) * (1 - N(y, z, s)) \ge 1 - (1 - N(x, y, t + s))$$

for all $x, y, z \in X$, t, s > 0.

$$N(x,z,t)) \Diamond N(y,z,s)) \ge N(x,y,t+s))$$

for all $x, y, z \in X$, t, s > 0. This completes the proof.

In view of above conclusion, we can give the following definition of the intuitionistic fuzzy metric space.

Definition 2.2. A 3-tuple (X, M, *) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, 1)$ satisfying the following conditions: for all $x, y, z \in X, s, t > 0$,

- (IFM-i) M(x, y, t) > 0;
- (IFM-ii) M(x, y, t) = 1, if and only if x = y;
- (IFM-iii) M(x, y, t) = M(y, x, t);
- (IFM-iv) $M(x, z, t) * M(y, z, s) \le M(x, y, t + s);$
- (IFM-v) $M(x, z, \cdot) : (0, +\infty) \to [0, 1]$ is continuous.

Remark 2.3. From the condition (IFM-1), we have

$$M(x, y, t) > 1 - r \Rightarrow N(x, y, t) < r,$$

so that

$$\{y \in X : M(x, y, t) > 1 - r, N(x, y, t) < r\} = \{y \in X : M(x, y, t) > 1 - r\}$$

for any $x \in X, r \in (0, 1), t > 0$. Therefore, Definition 1.4 is equivalent to the following definition.

Definition 2.4. Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space (Definition 1.3), and let $r \in (0, 1), t > 0$ and $x \in X$. The set

$$B_{(M,N)}(x,r,t) = \{ y \in X : M(x,y,t) > 1-r \},\$$

is called the open ball with center x and radius r with respect to t.

Definition 2.5. Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space.

- (1) A sequence $\{x_n\}$ in X is said to be converges to $x \in X$, if $\lim_{n \to \infty} M(x_n, x, t) = 1$ for any given t > 0.
- (2) A sequence $\{x_n\} \subset X$ is said to be Cauchy, if $\lim_{n,m\to\infty} M(x_n, x_m, t) = 1$ for any given t > 0.
- (3) $(X, M, N, *, \Diamond)$ is called complete if every Cauchy sequence is convergent.

Theorem 2.6. Let X be a nonempty set, * be a continuous t-norm and M be a fuzzy set on $X^2 \times (0,1)$ satisfying the following conditions: for all $x, y, z \in X$, s, t > 0,

- (I) M(x, y, 0) = 0;
- (II) M(x, y, t) = 1, if and only if x = y;
- (III) M(x, y, t) = M(y, x, t);
- (IV) $M(x, z, t) * M(y, z, s) \le M(x, y, t + s);$
- (V) $M(x, z, \cdot) : (0, +\infty) \to [0, 1]$ is continuous;
- (VI) $\lim_{t\to\infty} M(x, y, t) = 1.$

Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space (Definition 1.5), where $a \diamond b = 1 - (1 - a) * (1 - b)$ for all $a, b \in [0, 1]$ and N(x, y, t) = 1 - M(x, y, t) for all $x, y \in X, t > 0$.

Proof. It is sufficient to check the conditions (viii)-(xiii). From the relation

$$N(x, y, t) = 1 - M(x, y, t), \quad \forall x, y \in X, \quad t > 0,$$

we know that, the conditions (viii), (ix), (x), (xii) and (xiii) hold. Next, we prove the condition (xi). From the condition (v), we have that,

$$(1 - N(x, z, t)) * (1 - N(y, z, s)) \le (1 - N(x, y, t + s))$$

for all $x, y, z \in X, t, s > 0$.

$$1 - (1 - N(x, z, t)) * (1 - N(y, z, s)) \ge 1 - (1 - N(x, y, t + s))$$

for all $x, y, z \in X, t, s > 0$.

$$N(x, z, t)) \Diamond N(y, z, s)) \ge N(x, y, t + s))$$

for all $x, y, z \in X$, t, s > 0. This completes the proof.

In view of above conclusion, we can give the following definition of the intuitionistic fuzzy metric space.

Definition 2.7. A 3-tuple (X, M, *) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, 1)$ satisfying the above conditions (I)-(IV).

Theorem 2.8. Let X be vector space, * be a continuous t-norm and M be a fuzzy set on $X \times (0, 1)$ satisfying the following conditions: for all $x, y \in X$, s, t > 0,

- (a) M(x,t) > 0;
- (b) M(x,t) = 1, if and only if $x = \theta$;
- (c) $M(\alpha x, t) = M(\frac{x}{|\alpha|}, t)$, for each $\alpha \neq 0$;
- (d) $M(x,t) * M(y,s) \le M(x+y,t+s);$
- (e) $M(x, \cdot) : (0, +\infty) \to [0, 1]$ is continuous;
- (f) $\lim_{t\to\infty} M(x,t) = 1$, $\lim_{t\to0} M(x,t) = 0$;

Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy normed space (Definition 1.6), where $a \diamond b = 1 - (1-a) * (1-b)$ for all $a, b \in [0, 1]$ and N(x, t) = 1 - M(x, t) for all $x \in X, t > 0$.

Proof. It is sufficient to check the conditions (8)-(13). From the relation

$$N(x, y, t) = 1 - M(x, y, t), \quad \forall \ x, y \in X, \quad t > 0,$$

we know that, the conditions (8), (9), (10), (12) and (13) hold. Next, we prove the condition (11). From the condition (5), we have that,

$$(1 - N(x, t)) * (1 - N(y, s)) \le (1 - N(x + y, t + s))$$

for all $x, y \in X, t, s > 0$.

$$1 - (1 - N(x, t)) * (1 - N(y, s)) \ge 1 - (1 - N(x + y, t + s))$$

for all $x, y, z \in X, t, s > 0$.

$$N(x,t)) \Diamond N(y,s)) \ge N(x+y,t+s))$$

for all $x, y, z \in X$, t, s > 0. This completes the proof.

In view of above conclusion, we can give the following definition of the intuitionistic fuzzy normed space.

Definition 2.9. A 3-tuple (X, M, *) is said to be an intuitionistic fuzzy normed space if X is a vector space, * is a continuous t-norm and M is a fuzzy set on $X \times (0, 1)$ satisfying the above conditions (a)-(f).

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