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MINIMUM DOMINATING RANDIC ENERGY OF A GRAPH

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In this paper, we introduce the minimum dominating Randic energy of a graph and computed the minimum dominating Randic energy of graph. Also, obtained upper and lower bounds for the minimum dominating Randic energy of a graph.

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Key words: minimum dominating set, minimum dominating Randic eigenvalues, minimum dominating Randic energy.

1. Introduction

Let G be a simple, finite, undirected graph, The energy $E(G)$ is defined as the sum of the absolute values of the eigenvalues of its adjacency matrix. For more details on energy of graphs (see [5, 6]).

The Randic matrix $R(G) = (R_{ij})_{n\times n}$ is given by Bozkurt et al. [1–3].

$$R_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_i}}, & \text{if } v_i \sim v_j, \\ 0, & \text{otherwise.} \end{cases}$$

We can see lower and upper bounds on Randic energy in [1, 2, 4]. Some sharp upper bounds for Randic energy of graphs were obtain in [3].

2. The Minimum Dominating Randic Energy of Graph

Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and edge set E. A subset D of $V = V(G)$ is called a dominating set if every vertex in $V - D$ adjacent to a vertex of D. Minimum dominating set is called a dominating set of minimum power. Let D be a minimum dominating set of a graph G. The minimum dominating Randic matrix $R^D(G) = (R^D_{ij})_{n\times n}$ is given by

$$R^D_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_i}}, & \text{if } v_i \sim v_j, \\ 1, & \text{if } i = j \text{ and } v_i \in D, \\ 0, & \text{otherwise.} \end{cases}$$

The characteristic polynomial of $R^D(G)$ is denoted by $\phi^D_R(G, \lambda) = \det(\lambda I - R^D(G))$. Since the minimum dominating Randic Matrix is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1 > \lambda_2 > \dots \lambda_n$. The minimum dominating Randic Energy is given by

$$RE_D(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{1}$$

DEFINITION 2.1. The spectrum of a graph G is the list of distinct eigenvalues $\lambda_1 > \lambda_2 > \dots > \lambda_r$, with their multiplicities $m_1, m_2, \dots, m_r$, and we write it as

$$\text{Spec(G)} = \begin{pmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_r \\ m_1 & m_2 & \dots & m_r \end{pmatrix}.$$

This paper is organized as follows. In the Section 3, we get some basic properties of minimum dominating Randic energy of a graph. In the Section 4, minimum dominating Randic energy of some standard graphs are obtained.

3. Some Basic Properties of Minimum Dominating Randic Energy of a Graph

Let us consider

$$P = \sum_{i<j} \frac{1}{d_i d_j}.$$

Where $d_i d_j$ is the product of degrees of two vertices which are adjacent.

Proposition 3.1. *The first three coefficients of $\phi_R^D(G, \lambda)$ are given as follows:*
(i) $a_0 = 1$,
(ii) $a_1 = -|D|$,
(iii) $a_2 = |D|C_2 - P$.

$\lhd$ (i) From the definition $\Phi_R^D(G, \lambda) = \det[\lambda I - R^D(G)]$, we get $a_0 = 1$.

(ii) The sum of determinants of all 1×1 principal submatrices of $R^D(G)$ is equal to the trace of $R^D(G)$.

$\Rightarrow a_1 = (-1)^1$ trace of $[R^D(G)] = -|D|$.

(iii)

$$(-1)^2 a_2 = \sum_{1 \leqslant i<j \leqslant n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} = \sum_{1 \leqslant i<j \leqslant n} a_{ii}a_{jj} - a_{ji}a_{ij}$$
$$= \sum_{1 \leqslant i<j \leqslant n} a_{ii}a_{jj} - \sum_{1 \leqslant i<j \leqslant n} a_{ji}a_{ij} = |D|C_2 - P. \rhd$$

Proposition 3.2. *If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the minimum dominating Randic eigenvalues of $R^D(G)$, then*

$$\sum_{i=1}^{n} {\lambda_i}^2 = |D| + 2P.$$

$\lhd$ We know that

$$\sum_{i=1}^{n} \lambda_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}a_{ji} = 2\sum_{i<j}(a_{ij})^2 + \sum_{i=1}^{n}(a_{ii})^2 = 2\sum_{i<j}(a_{ij})^2 + |D| = |D| + 2P. \rhd$$

Theorem 3.1. *Let G be a graph with n vertices and then*

$$RE^D(G) \leqslant \sqrt{n(|D| + 2[P])},$$

where

$$P = \sum_{i<j} \frac{1}{d_i d_j},$$

for which $d_i d_j$ is the product of degrees of two vertices which are adjacent.

◁ Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of $R^D(G)$. Now by Cauchy–Schwartz inequality we have

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leqslant \left(\sum_{i=1}^{n} {a_i}^2\right)\left(\sum_{i=1}^{n} {b_i}^2\right).$$

Let $a_i = 1$, $b_i = \mid \lambda_i \mid$. Then

$$\left(\sum_{i=1}^{n} |\lambda_i|\right)^2 \leqslant \left(\sum_{i=1}^{n} 1\right)\left(\sum_{i=1}^{n} |\lambda_i|^2\right),$$

$$[RE^D]^2 \leqslant n(|D| + 2P),$$

$$[RE^D] \leqslant \sqrt{n(|D| + 2P)},$$

which is upper bound. ▷

Theorem 3.2. *Let G be a graph with n vertices. If $R = \det R^D(G)$, then*

$$RE^D(G) \geqslant \sqrt{(|D| + 2P) + n(n-1)R^{\frac{2}{n}}}.$$

◁ By definition,

$$\left(RE^D(G)\right)^2 = \left(\sum_{i=1}^{n} \mid \lambda_i \mid\right)^2 = \sum_{i=1}^{n} \mid \lambda_i \mid \sum_{j=1}^{n} \mid \lambda_j \mid = \left(\sum_{i=1}^{n} \mid \lambda_i \mid^2\right) + \sum_{i \neq j} \mid \lambda_i \mid\mid \lambda_j \mid.$$

Using arithmetic mean and geometric mean inequality, we have

$$\frac{1}{n(n-1)} \sum_{i \neq j} \mid \lambda_i \mid\mid \lambda_j \mid \geqslant \left(\prod_{i \neq j} \mid \lambda_i \mid\mid \lambda_j \mid\right)^{\frac{1}{n(n-1)}}.$$

Therefore,

$$\begin{aligned}
[RE^D(G)]^2 &\geqslant \sum_{i=1}^{n} \mid \lambda_i \mid^2 + n(n-1)\left(\prod_{i \neq j} \mid \lambda_i \mid\mid \lambda_j \mid\right)^{\frac{1}{n(n-1)}} \\
&\geqslant \sum_{i=1}^{n} \mid \lambda_i \mid^2 + n(n-1)\left(\prod_{i=1}^{n} \mid \lambda_i \mid^{2(n-1)}\right)^{\frac{1}{n(n-1)}} \\
&= \sum_{i=1}^{n} \mid \lambda_i \mid^2 + n(n-1)R^{\frac{2}{n}} = (|D| + 2P) + n(n-1)R^{\frac{2}{n}}.
\end{aligned}$$

Thus,

$$RE^D(G) \geqslant \sqrt{(|D| + 2P) + n(n-1)R^{\frac{2}{n}}}. \ \triangleright$$

4. Minimum Dominating Randic Energy of Some Standard Graphs

Theorem 4.1. *The minimum dominating Randic energy of a complete graph K_n is $RE^D(K_n) = \frac{3n-5}{n-1}$.*

$\triangleleft$ Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$. The minimum dominating set $= D = \{v_1\}$. The minimum dominating Randic matrix is

$$R^D(K_n) = \begin{bmatrix} 1 & \frac{1}{n-1} & \frac{1}{n-1} & \cdots & \frac{1}{n-1} & \frac{1}{n-1} \\ \frac{1}{n-1} & 0 & \frac{1}{n-1} & \cdots & \frac{1}{n-1} & \frac{1}{n-1} \\ \frac{1}{n-1} & \frac{1}{n-1} & 0 & \cdots & \frac{1}{n-1} & \frac{1}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{n-1} & \frac{1}{n-1} & \cdots & \frac{1}{n-1} & 0 & \frac{1}{n-1} \\ \frac{1}{n-1} & \frac{1}{n-1} & \cdots & \frac{1}{n-1} & \frac{1}{n-1} & 0 \end{bmatrix}.$$

Characteristic equation is

$$\left(\lambda + \frac{1}{n-1}\right)^{n-2} \left(\lambda^2 - \frac{2n-3}{n-1}\lambda + \frac{n-3}{n-1}\right) = 0$$

and the spectrum is

$$\operatorname{Spec}_R^D(K_n) = \begin{pmatrix} \frac{(2n-3)+\sqrt{4n-3}}{2(n-1)} & \frac{(2n-3)-\sqrt{4n-3}}{2(n-1)} & \frac{-1}{n-1} \\ 1 & 1 & n-2 \end{pmatrix}.$$

Therefore, $RE^D(K_n) = \frac{3n-5}{n-1}$. $\triangleright$

Theorem 4.2. *The minimum dominating Randic energy of star graph $K_{1,n-1}$ is*

$$RE^D(K_{1,n-1}) = \sqrt{5}.$$

$\triangleleft$ Let $K_{1,n-1}$ be the star graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$. Here v_0 be the center. The minimum dominating set $= D = \{v_0\}$. The minimum dominating Randic matrix is

$$R^D(K_{1,n-1}) = \begin{bmatrix} 1 & \frac{1}{\sqrt{n-1}} & \frac{1}{\sqrt{n-1}} & \cdots & \frac{1}{\sqrt{n-1}} & \frac{1}{\sqrt{n-1}} \\ \frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Characteristic equation is

$$(\lambda)^{n-2}[\lambda^2 - \lambda - 1] = 0$$

spectrum is $\text{Spec}_R^D(K_{1,n-1}) = \begin{pmatrix} \frac{1+\sqrt{5}}{2} & 0 & \frac{1-\sqrt{5}}{2} \\ 1 & n-2 & 1 \end{pmatrix}$. Therefore, $RE^D(K_{1,n-1}) = \sqrt{5}$. ▷

Теорема 4.3. *The minimum dominating Randic energy of Crown graph S_n^0 is*

$$RE^D(S_n^0) = \frac{(4n-7)+\sqrt{4n^2-8n+5}}{n-1}.$$

◁ Let S_n^0 be a crown graph of order $2n$ with vertex set $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ and minimum dominating set $= D = \{u_1, v_1\}$. The minimum dominating Randic matrix is

$$R^D(S_n^0) = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 & \frac{1}{n-1} & \dots & \frac{1}{n-1} & \frac{1}{n-1} \\ 0 & 0 & 0 & \dots & 0 & \frac{1}{n-1} & 0 & \dots & \frac{1}{n-1} & \frac{1}{n-1} \\ 0 & 0 & 0 & \dots & 0 & \frac{1}{n-1} & \dots & \frac{1}{n-1} & 0 & \frac{1}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & \frac{1}{n-1} & \dots & \frac{1}{n-1} & \frac{1}{n-1} & 0 \\ 0 & \frac{1}{n-1} & \frac{1}{n-1} & \dots & \frac{1}{n-1} & 1 & 0 & \dots & 0 & 0 \\ \frac{1}{n-1} & 0 & \frac{1}{n-1} & \dots & \frac{1}{n-1} & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{n-1} & \frac{1}{n-1} & 0 & \dots & \frac{1}{n-1} & 0 & 0 & \dots & 0 & 0 \\ \frac{1}{n-1} & \frac{1}{n-1} & \frac{1}{n-1} & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

Characteristic equation is

$$\left(\lambda + \frac{1}{n-1}\right)^{n-2} \left(\lambda - \frac{1}{n-1}\right)^{n-2} \left(\lambda^2 - \frac{1}{n-1}\lambda - 1\right) \left(\lambda^2 - \frac{2n-3}{n-1}\lambda + \frac{n-3}{n-1}\right) = 0$$

spectrum is

$$\text{Spec}_R^D(S_n^0) = \begin{pmatrix} \frac{(2n-3)+\sqrt{4n-3}}{2(n-1)} & \frac{1+\sqrt{4n^2-8n+5}}{2(n-1)} & \frac{(2n-3)-\sqrt{4n-3}}{2(n-1)} & \frac{1}{n-1} & \frac{-1}{n-1} & \frac{1-\sqrt{4n^2-8n+5}}{2(n-1)} \\ 1 & 1 & 1 & n-2 & n-2 & 1 \end{pmatrix}.$$

Therefore, $RE^D(S_n^0) = \frac{(4n-7)+\sqrt{4n^2-8n+5}}{n-1}$. ▷

Theorem 4.4. *The minimum dominating Randic energy of complete bipartite graph $K_{n,n}$of order $2n$ with vertex set $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ is*

$$RE^D(K_{n,n}) = \frac{2\sqrt{n-1}}{\sqrt{n}} + 2.$$

◁ Let $K_{n,n}$ be the complete bipartite graph of order $2n$ with vertex set $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$. The minimum dominating set $= D = \{u_1, v_1\}$. The minimum dominating Randic matrix is

$$R^D(K_{n,n}) = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ 0 & 0 & 0 & 0 & \dots & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ 0 & 0 & 0 & 0 & \dots & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ 0 & 0 & 0 & 0 & \dots & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \dots & 1 & 0 & 0 & 0 \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \dots & 0 & 0 & 0 & 0 \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \dots & 0 & 0 & 0 & 0 \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \dots & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Characteristic equation is

$$\lambda^{2n-4}\left(\lambda^2 - \frac{n-1}{n}\right)\left[\lambda^2 - 2\lambda + \frac{n-1}{n}\right] = 0.$$

Hence, spectrum is $\operatorname{Spec}_R^D(K_{n,n}) = \begin{pmatrix} 1+\sqrt{\frac{1}{n}} & \frac{\sqrt{n-1}}{\sqrt{n}} & 1-\sqrt{\frac{1}{n}} & 0 & -\frac{\sqrt{n-1}}{\sqrt{n}} \\ 1 & 1 & 1 & 2n-4 & 1 \end{pmatrix}.$

Therefore, $RE^D(K_{n,n}) = \frac{2\sqrt{n-1}}{\sqrt{n}} + 2$. $\rhd$

Definition 4.1. The friendship graph, denoted by $F_3^{(n)}$, is the graph obtained by taking n copies of the cycle graph C_3 with a vertex in common. $V(F_n) = 2n+1$.

Теорема 4.5. *The minimum dominating Randic energy of Friendship graph F_n^3 is*

$$RE_D(F_n^3) = n+1.$$

$\lhd$ Let $F_3^{(n)}$ be the friendship graph with $2n+1$ vertices. Here v_1 be the center. The minimum dominating set $= D = \{v_1\}$. The minimum dominating Randic matrix is

$$R^D(F_n^3) = \begin{bmatrix} 1 & \frac{1}{2\sqrt{n}} & \frac{1}{2\sqrt{n}} & \frac{1}{2\sqrt{n}} & \frac{1}{2\sqrt{n}} & \dots & \frac{1}{2\sqrt{n}} & \frac{1}{2\sqrt{n}} \\ \frac{1}{2\sqrt{n}} & 0 & \frac{1}{2} & 0 & 0 & \dots & 0 & 0 \\ \frac{1}{2\sqrt{n}} & \frac{1}{2} & 0 & 0 & 0 & \dots & 0 & 0 \\ \frac{1}{2\sqrt{n}} & 0 & 0 & 0 & \frac{1}{2} & \dots & 0 & 0 \\ \frac{1}{2\sqrt{n}} & 0 & 0 & \frac{1}{2} & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{2\sqrt{n}} & 0 & 0 & 0 & 0 & \dots & 0 & \frac{1}{2} \\ \frac{1}{2\sqrt{n}} & 0 & 0 & 0 & 0 & \dots & \frac{1}{2} & 0 \end{bmatrix}.$$

Characteristic equation is

$$\lambda\left(\lambda + \frac{1}{2}\right)^n \left(\lambda - \frac{1}{2}\right)^{n-1}\left(\lambda - \frac{3}{2}\right) = 0.$$

Hence, spectrum is

$$\operatorname{Spec}_R^D(F_n^3) = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} & 0 & \frac{-1}{2} \\ 1 & n-1 & 1 & n \end{pmatrix}.$$

Therefore, $RE^D(F_n^3) = n+1$. $\rhd$

Теорема 4.6. *The minimum dominating Randic energy of Cocktail party graph $K_{n\times 2}$ is*

$$RE^D(K_{n\times 2}) = \frac{4n-6}{n-1}.$$

◁ Let $K_{n\times 2}$ be a Cocktail party graph of order $2n$ with vertex set $\{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$. The minimum dominating set $= D = \{u_1, v_1\}$. The minimum dominating minimum dominating Randic matrix is

$$R^D(K_{n\times 2}) = \begin{bmatrix} 1 & \frac{1}{2n-2} & \frac{1}{2n-2} & \frac{1}{2n-2} & \cdots & 0 & \frac{1}{2n-2} & \frac{1}{2n-2} & \frac{1}{2n-2} \\ \frac{1}{2n-2} & 0 & \frac{1}{2n-2} & \frac{1}{2n-2} & \cdots & \frac{1}{2n-2} & 0 & \frac{1}{2n-2} & \frac{1}{2n-2} \\ \frac{1}{2n-2} & \frac{1}{2n-2} & 0 & \frac{1}{2n-2} & \cdots & \frac{1}{2n-2} & \frac{1}{2n-2} & 0 & \frac{1}{2n-2} \\ \frac{1}{2n-2} & \frac{1}{2n-2} & \frac{1}{2n-2} & 0 & \cdots & \frac{1}{2n-2} & \frac{1}{2n-2} & \frac{1}{2n-2} & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \frac{1}{2n-2} & \frac{1}{2n-2} & \frac{1}{2n-2} & \cdots & 1 & \frac{1}{2n-2} & \frac{1}{2n-2} & \frac{1}{2n-2} \\ \frac{1}{2n-2} & 0 & \frac{1}{2n-2} & \frac{1}{2n-2} & \cdots & \frac{1}{2n-2} & 0 & \frac{1}{2n-2} & \frac{1}{2n-2} \\ \frac{1}{2n-2} & \frac{1}{2n-2} & 0 & \frac{1}{2n-2} & \cdots & \frac{1}{2n-2} & \frac{1}{2n-2} & 0 & \frac{1}{2n-2} \\ \frac{1}{2n-2} & \frac{1}{2n-2} & \frac{1}{2n-2} & 0 & \cdots & \frac{1}{2n-2} & \frac{1}{2n-2} & \frac{1}{2n-2} & 0 \end{bmatrix}.$$

Characteristic equation is

$$\lambda^{n-1}\left(\lambda + \frac{1}{n-1}\right)^{n-2}(\lambda - 1)\left[\lambda^2 - \frac{2n-3}{n-1}\lambda + \frac{n-3}{n-1}\right] = 0.$$

Hence, spectrum is

$$\operatorname{Spec}_R^D(K_{n\times 2}) = \begin{pmatrix} \frac{2n-3+\sqrt{4n-3}}{2(n-1)} & 1 & \frac{2n-3-\sqrt{4n-3}}{2(n-1)} & 0 & \frac{-1}{n-1} \\ 1 & 1 & 1 & n-1 & n-2 \end{pmatrix}.$$

Therefore, $RE^D(K_{n\times 2}) = \frac{4n-6}{n-1}$. ▷

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References

1. *Bozkurt S. B., Gungor A. D., Gutman I., Cevik A. S.* Randic matrix and Randic energy // MATCH Commum. Math. Comput. Chem.—2010.—Vol. 64.—P. 239–250.
2. *Bozkurt S. B., Gungor A. D., Gutman I.* Randic spectral radius and Randic energy // MATCH Commum. Math. Comput. Chem.—2010.—Vol. 64.—P. 321–334.
3. *Serife Burcu Bozkurt, Durmus Bozkurt.* Sharp Upper Bounds for Energy and Randic Energy // MATCH Commum. Math. Comput. Chem.—2013.—Vol. 70.—P. 669–680.
4. *Gutman I., Furtula B., Bozkurt S. B.* On Randic energy // Linear Algebra Appl.—2014.—Vol. 442.—P. 50–57.
5. *Gutman I.* The energy of a graph // Ber. Math. Stat. Sekt. Forschungsz. Graz.—1978.—103.—P. 1–22.
6. *Gutman I.* The energy of a graph: old and new results // Combinatorics and Applications / A. Betten, A. Khoner, R. Laue and A. Wassermann, eds.—Berlin: Springer, 2001.—P. 196–211.
7. *Indulal G., Gutman I., Vijayakumar A.* On distance energy of graphs // Match Commun. Math. Comput. Chem.—2008.—Vol. 60.—P. 461–472.
8. *Rajesh Kanna M. R., Dharmendra B. N., Sridhara G.* Minimum dominating energy of a graph // Int. J. Pure and Appl. Math.—2013.—Vol. 85, № 4.—P. 707–718.

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МИНИМАЛЬНАЯ ДОМИНИРУЮЩАЯ ЭНЕРГИЯ РАНДИЧА ГРАФА

Сива Кота Редди П., Пракаша К. Н., Сиддалингасвами В. М.

В данной работе мы ввели понятие и вычислили минимальную доминирующую энергию Рандича графа. Кроме того, были найдены верхняя и нижняя границы для минимальной доминирующей энергии Рандича.

Ключевые слова: минимальный доминирующий набор, минимальные доминирующие собственные значения Рандича, минимальная доминирующая энергия Рандича.