Reed, Stephen K.:
Word Problems
Research and Curriculum Reform

Lieven Verschaffel, Leuven (Belgium)
Brian Greer, Belfast (Northern Ireland)

Word problems have attracted the attention of both mathematics educators and cognitive psychologists for many years. However, while mathematics educators have investigated them primarily to assist teachers in helping their students to learn to solve these problems in more insightful and efficient ways, the motivation of cognitive psychologists is typically very different. As with other aspects of the mathematics curriculum, word problems have frequently been chosen by psychologists as an amenable subject matter for testing and applying general theories of learning, transfer, memory, text understanding, and problem solving, "with no regard for the specific nature of mathematics and mathematics instruction" (Freudenthal, 1991, p. 149).

The author of this book clearly belongs in the latter category. He has been using algebra word problems in this research over the past 15 years not, primarily, to understand how students approach and solve such tasks and to help them solve them better, but (as he acknowledges on the first page of the preface) because he was interested in "more general theoretical issues such as how students transfer from one problem to another problem". "Algebra word problems just happened to be a convenient source of problems for my research", the author explains on the same page. Looking back at these 15 years of intensive work, which has led to an impressive list of scholarly publications in the most prestigious journals of experimental psychology, the author observes – much to his regret – that his work has had little influence on mathematics education, and that work in mathematics education has had little influence on his research.

Against this background, the author decided to write this book "in which [he] could bring together some ideas from both cognitive psychology and mathematics education" (p. x), in the hope that by making people more aware of cognitively oriented research on mathematical problem solving (particularly word problems), he could help guide the ongoing curriculum reform in mathematics education in a good direction.

The introductory chapter contains a brief discussion of the major issues in the ongoing debate in the US between the old basic skills driven curriculum and new curricula based on "Curriculum and Evaluation Standards for School Mathematics" (National Council of Teachers of Mathematics, 1989, currently being revised). At the outset, Reed acknowledges that he had never paid much attention to curriculum reform in mathematics education and continues: "It was only when I was confronted with the

debate as a parent in the final stage of writing his book, that I decided that my research on mathematical problem solving would be of greater interest to my readers if I embedded it within the context of this ongoing debate" (p. 1). This personal history helps to explain why the links provided throughout the book between the author's own cognitive-psychological research and the ongoing debate about curriculum reform fail, in our view, to achieve the depth necessary to make this book a truly important contribution to the field.

The book consists of four sections. In the first, general theoretical issues that provide the necessary framework for understanding the rest of the book are presented. Chapter 2 is on rule learning. It focuses on ACT*, Anderson's rule-based theory of learning, which was constructed mainly in the 80s, as well as the computer-based tutors Anderson and his associates developed based on ACT*. The author presents this work in some detail because it is, in his view, "the most successful example we have of a long-term development effort that has used a detailed cognitive theory to develop instruction" (p. 10). However, the actual theoretical and instructional relevance of this "most detailed and extensively tested theory of learning that has been developed by cognitive psychologists" (p. 15) seem rather questionable, since Anderson's project has meanwhile gone in a different direction. As pointed out by Reed himself (p. 10), Anderson is now trying to give his computer tutors a role in supporting a curriculum that is more closely in line with the Standards, which put much more emphasis on the active involvement of students in their own work and on the promotion of understanding of concepts rather than on procedural competence.

In Chapter 3, Reed reviews the cognitive-psychological literature from the 70s and 80s on conceptual understanding and on the declarative knowledge that underlies such understanding. He explains how the notion of "schema" to describe the organisation of knowledge, originated by Bartlett, has been extended by cognitive scientists and applied in areas such as text comprehension, reasoning and problem solving.

Because these two chapters are intended to provide background information, the author suggests that the reader may skim them if familiar with the content, as will be the case for any reader with a basic knowledge of cognitive psychology. Indeed, what is presented in these two chapters is a brief, classical summary of cognitive science of the 70s and 80s similar to many overviews published in the 80s and early 90s. What is remarkable to us is that besides these two chapters on rule learning and schema theory, nothing else is given as necessary theoretical background information for a proper understanding of the (research on) learning and teaching of word problem solving. For a book published in 1999, we would have expected at least some attention to have been given to the advances in cognitive, developmental, and instructional psychology of the last decade. More specifically, what is missing in these theoretical background chapters is any recognition of the need to place cognitive functioning in a broader perspective that takes into account aspects such as affect, motivation, attitudes, beliefs, and intuitions, as well as social

and cultural factors. Stated differently, Reed's theoretical background section ends where such a section should have started, namely by recognising that since the mid-80s the first wave of cognitive science based primarily on theories of information processing has been followed by "a second wave" (Greer & Verschaffel, 1990, p. 9; De Corte, Greer, & Verschaffel, 1996, p. 497).

In the second section of the book, Reed describes, in three chapters, how solving word problems evolves as children progress through elementary school and high school. The first chapter deals with simple, one-step arithmetic word problems. Of course, it is an impossible task to cover fully in one chapter of 15-20 pages the research on additive and multiplicative word problems. Almost a decade ago Fuson and Greer needed much more space for their reviews of the research on additive and multiplicative structures, respectively, in the comprehensive handbook edited by Grouws (1992). Although the author does not explicitly refer to these review chapters, what is presented largely resembles a summary of them. The chapter starts with a discussion of children's superficial strategies for solving one-step problems. Then the well-known taxonomy of additive word problems developed by Riley, Greeno, and Heller is presented, followed by some empirical findings about children's performance on the problems in the taxonomy. Then Reed recapitulates the discussion about whether children's errors on additive word problems are due to comprehension or to conceptual difficulties. The (much shorter) next subsection on multiplication and division word problems contains a sketchy dimensional analysis of multiplicative structures, also based on work from the 80s by Schwartz, Vergnaud and Nesher. Surprisingly, no reference is made to empirical analyses of how these problems are currently being taught in schools or to design experiments that have taken into account the findings from the ascertaining studies on one-step word problems to develop, implement, and design new environments for teaching and learning one-step word problem solving. At the end of the chapter, the author refers to a few other recent handbook chapters that raise unanswered questions about the issues discussed in this chapter. Given the existence of several excellent overviews of the cognitive-psychological research on one-step word problems up to the early 90s, we would have preferred a chapter that took the existing reviews as a starting point for an updated review, rather than as sources to paraphrase.

In Chapter 5, Reed discusses multi-step word problems, i.e. problems in which the elementary relations underlying additive and multiplicative structures, identified in the previous chapter, are combined to form more complex solutions. Whereas the focus in Chapter 4 was, as remarked, totally on ascertaining studies, this perspective is dropped in Chapter 5, and replaced by a focus on design experiments aimed at building learning environments to assist students in solving multi-step problems, in every case taking the form of a computer-based tool and/or tutor. All three of the systems discussed enable students to construct solutions by combining the more elementary relations in a multi-step problem. The computer environments externalise the planning process by requiring students to make a series of decisions as they assemble the components, but the three systems differ in terms of the nature of the building blocks used to represent the problem and/or the solution. Although the publications used in this chapter date from the 90s, most of the conceptual analysis, the developmental work, and the evaluation studies fit within and date from the previous decade's research on word problem solving, firmly rooted in the classical information-processing theories of the first wave of the Cognitive Revolution. Accordingly, it is not surprising that the authors of these systems barely addressed the question of how these systems should be integrated within the mathematics curriculum. Moreover, taking into account new developments in (research in) mathematics education, it seems questionable whether these computerised tools or tutors will be of great help in solving the theoretical and practical problems we are faced with today (and see De Corte, Greer, & Verschaffel, 1996, p. 526). As far as we know, the constructors of these computerised tools and/or tutoring systems (like Anderson) moved on from this line of research some years ago.

In Chapter 6, the author discusses algebra word problems along lines similar to those of Kieran's chapter on the learning and teaching of school algebra in Grouws' (1992) handbook. Having described the similarities and differences between arithmetic and algebra word problems, he reviews research on the well-known "students-and-professors" problem. Thereafter, several other studies (mostly from the 80s but also some more recent) documenting the diversity of students' processes for solving algebra word problems are described. The importance of situation models in algebra word problem solving is discussed, relying on the work of Kintsch and his associates, who also developed a computer-based learning environment (called ANIMATE) to co-ordinate the situation and problem models through an equation-driven animation of the situation. Then follows a discussion of the psychological role and educational value of schemes categorising algebra word problems into classes such as distance-rate-time problems, work problems, mixture problems, etc., including an extensive discussion of the well-known studies from the late 70s by Hinsley, Hayes, and Simon documenting the important role that categorisation plays in the actual solution of (stereotyped) word problems. Reed acknowledges the critical attitude of many mathematics educators towards categorisation, on the grounds that it might reduce students' solution of a word problem to simply recognising the problem as belonging to a given type (based on a superficial scanning of the problem text), and routinely applying the equation for that particular problem type. Nevertheless, he argues for recognition of a problem as fitting a certain category and use of schematic knowledge of how to solve problems in that category as an important aspect of acquiring expertise that still merits instructional attention.

The third section contains chapters on transferring what has been learned about solving one (algebra) problem to other related (algebra) problems, the major topic of the author's research over the past 15 years. The structure of this section is based on a schema that was introduced by

the author in 1987, wherein four different possible relations between an example problem and a test problem are distinguished, based on the combination of two relational dimensions, namely whether the example problem and the test problem share a common story context and whether they share a common solution procedure (i.e., a common equation). Reed thus distinguishes between (a) equivalent problems (common story context and common solution procedure), (b) isomorphic problems (different story contexts but common solution), (c) similar problems (common story but different solution) and (d) unrelated problems (which share neither story context nor common solution). Transfer between isomorphic, similar, and unrelated problems, respectively, is discussed in the three chapters of this section.

Chapter 7 is about transfer between isomorphic problems, i.e. problems that differ in story context but nonetheless have identical solutions (operationalised as identical equations). Relying on his own research in this domain as well as related work by others, the author provides an overview of the available cognitive-psychological evidence on students' ability to see that the same solution can be used to solve two problems even though these problems are about different situations (e.g., painters painting a fence versus pipes filling a swimming pool) and to exploit such structural relationships. Generally speaking, the gist of the laboratory studies reported in this chapter is that transfer to isomorphic problems is a complex and difficult process, which takes place only under certain experimental circumstances and after students have been involved in well-designed forms of training.

Chapter 8 is about transfer between similar problems. Whereas solving isomorphic problems requires recognising that two apparently unrelated problems can be solved in the same way by finding corresponding quantities and relations in the two problems, solving similar problems requires modifying the solution of the example problem to solve a problem that is obviously related because of the shared story context. This is the case, for instance, when someone has solved an example problem in which two workers labour for the same number of hours, and then is confronted with a test problem about workers labouring for a different number of hours. The major finding of this work is that students typically find it difficult to adapt solutions, often relying too much on the example solution without making the necessary adjustments. The chapter provides a clear overview of the laboratory studies done by the author on the conditions that facilitate or hinder the occurrence of such transfer to similar problems, as well as the instructional approaches for improving students' skill in adapting solutions.

The final case, namely that of transfer between problems that have both a different situation and a different solution, is discussed in Chapter 9. The author reviews available research on transfer that may occur at a more general level than transferring the details of a specific solution. In other words, transfer of general methods, or "representational transfer", as Reed terms it. After reviewing some studies by Schoenfeld and his associates of students perceiving and using a common solution method throughout a set of word problems, Reed focuses on one particular heuristic in problem solving, namely the use of diagrams.

We consider Part III the strongest part of the book. The overall structure and the structure of the different chapters are well thought out. There is a proper balance between the author's presentation of his own research and related work by others. Instructional implications are derived directly from the (laboratory) studies reviewed.

In the final section of the book, "Important topics in the NCTM Standards", the author's attempt to relate his book to the ongoing debate on the curriculum reform in mathematics education in the US is presented in three chapters that we found rather loosely related to each other and to the rest of the book. In Chapter 10, Reed looks at recent research on word problems wherein these problems are critically considered from the perspective of their role in promoting authentic mathematical modelling and problem solving in the mathematics classroom. He begins by briefly examining the situated cognition view on learning and teaching. We found it surprising that this theoretical framework, that has had such an impact on cognitive, developmental, and instructional psychology over the past decade, is nowhere reflected in the empirical studies, theoretical analyses, and critical reflections presented previously in the book. Having discussed the classical study of Brazilian street children by Carraher, Carraher, and Nunes, the author reviews a series of recent ascertaining studies and teaching experiments initiated by our own work about students' disposition towards non-realistic solutions of school arithmetic word problems. These studies show that students often produce stereotypic solutions that make unreasonable assumptions about the problem situation, such as that it takes ten times as much time to run a distance that is ten times as long, but also that it is possible to change students' disposition towards mathematical modelling by radically changing the culture of the mathematics classroom. This discussion is followed by a brief review of the frequently-cited Jasper Woodbury series developed by the Cognition and Technology Group at Vanderbilt to help students become more competent solvers of authentic and non-routine problems, and of the evaluation studies done by this group. Notwithstanding an expressed sympathy for the (authentic) mathematical modelling perspective advocated in these studies, Reed defends an instructional approach to learning mathematical modelling wherein traditional word problems continue to play an important role. As he did earlier in the book, he pleads for an approach that begins with training in stylised and routine problems of various types, followed by lessons wherein problems of different types are mixed together, and by problems in which the type is not obvious and by other kinds of elaborations (p. 163). It is not clear to us to what extent such a proposal differs from the traditional approach to word problem solving, except that it asks for a bit more attention to the transfer-to-the-real-world issue at the very end of instruction.

In Chapter 11, Reed looks at another recommendation of the NCTM standards, namely to pay less attention to calculating exact answers and more attention to number sense. However, rather than providing the reader with an

up-to-date view on how number sense is nowadays conceived and investigated by advocates of the reform movement, on the role of number sense in the reformed curriculum, on the recent research findings that support its valorisation, and on recently developed instructional approaches to the learning of these complex skills, he uses this chapter to present another piece of cognitive-psychological research he did in the 80s, namely a series of studies on whether students can formulate a reasonable estimate of the answers without having to solve the problem, and on students' understanding of functional relations. How these findings can be used to help solve the problems of design with which mathematics educators working at new curricula and new textbooks are confronted is unclear.

According to the author, the book would have been incomplete without a final chapter that looks more closely at the curriculum reforms in the US that were mentioned in the introductory chapter, and, specifically, at evaluation studies of curriculum changes in line with the Standards. Accordingly, the final chapter contains a series of one-to-two page descriptions of some of the best-known curriculum reform projects in the US from the past decade, including the work done at Northwestern University by Fuson and her associates, Purdue University's "Problem-Centered Mathematics Project", and Carpenter and Fennema's "Cognitively Guided Instruction" program, followed by a couple of programs implementing curriculum changes at the high school level. Following this summary of mostly well-known reform projects in the US, the author points out two resources that have, in his opinion, been under-utilised so far in the design of curriculum reform, namely computers and cognitive research. With respect to the latter, he complains how difficult it was to find documentation of strong links between cognitively guided research and curriculum reform, except in the domain of elementary word problems, where he could find "glimpses of curriculum reform being guided by research" (p. 199). The author's call at the end of the book, therefore, is clear: "In order to provide closer links, curriculum designers need to be aware of instructionally relevant research, and researchers need to be aware of important trends in the curriculum reform movement" (p. 199) and he expresses the hope that this book will encourage a tighter link.

We applaud the aim of a mutually beneficial partnership between cognitive psychology and mathematics education, but we doubt that the approach advocated by Reed is the way to achieve this aim. Despite his recently-acquired interest in curriculum reform, his work remains within a tradition of psychological researchers using mathematics as grist to their theoretical mills that goes back at least to Thorndike of whom Kilpatrick (cited in De Corte, Greer, & Verschaffel, 1996, p. 492) remarked: "His theory was his hammer; he looked around and saw the arithmetic curriculum as something to pound".

Throughout the period since the start of the "Cognitive Revolution", which Gardner (1985) pinpointed as 1956, strong claims have been made about the contribution that cognitive science can make to mathematics education, yet, over 40 years on, the observable effects in classrooms are limited, even though Reed believes (p. 7) "that the study of specific issues in well-controlled experimental settings is providing the kind of data that will be useful for understanding mathematical reasoning, and for subsequently designing effective curriculum". However, as he acknowledges (p. 12), it is a challenge "to link the careful control of laboratory research with the messy world of the classroom".

We would suggest that the theoretical goal and related research program aimed at describing cognitive architecture through laboratory experiments and then applying that knowledge to learning and teaching mathematics has not delivered substantial results and that a more direct approach is needed to meet the challenge of linking research with the classroom. From this perspective, Reed's continuing championing of a "hard" cognitive science approach seems, to us, dated and to ignore developments in mathematics education and in cognitive science. In the last twenty years, mathematics education has become a field of study in its own right, drawing on many disciplines. During the same period, cognitive science has moved beyond the "first wave" of the Cognitive Revolution, a major characteristic of which was identified by Gardner (1985, p. 41) as "de-emphasis on affect, context, culture and history". As argued in De Corte, Greer, & Verschaffel (1996, p. 492), the first wave of the Cognitive Revolution has been followed by a second "that relocates intellectual functioning within a broader human framework and takes into account social, cultural, and affective factors". As a result, the increasingly symbiotic relationship between psychologists and mathematics educators has benefited from and contributed to major shifts – "from a focus on general to a focus on domain-specific knowledge, processes, and expertise; from a concentration on the individual to a concern for social and cultural factors; from 'cold' to 'hot' cognition; from the laboratory to the classroom as the arena for research, and from technically to humanistically grounded methodologies and interpretative approaches" (De Corte, Greer, & Verschaffel, 1996, p. 491). Thus, in our opinion, the true promise of fruitful collaboration lies in the enhancement of cognitive science by psychologists who have left their safe laboratories for the messy world of the classroom, who have enriched their narrow disciplinary theoretical frameworks with insights from other disciplines such as sociology, anthropology, and history, who have diversified their methods of inquiry and analysis well beyond the narrow range of the empirical-scientific approach, and who have engaged in new forms of collaboration with teachers as research partners.

Does this imply that we think the book has no merits? Certainly not. For people who are not familiar with Reed's work from his published journal articles, Section III and part of Section IV provide an excellent summary of his oeuvre, which makes it possible for the reader to judge for herself or himself Reed's aspirations for the applicability of his work to mathematics education. The book also contains good coverage of related work, though, as remarked at several points in this review, not as up-to-date as we would have liked. Further, the literature cited and his discussions do serve to illustrate – while not affording a deep analysis – the ongoing philosophical and ideological

debate about mathematics education in the United States and beyond.

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Authors

Greer, Brian, Dr., Center for Research in Mathematics and Science Education, San Diego State University, 6475 Alvarado Road, Suite 206, San Diego, CA 92020, USA.
E-mail: b.greer@greerb.fsnet.co.uk

Verschaffel, Lieven, Prof., Dept. of Instructional Psychology and Technology, University of Leuven, Vesaliusstraat 2, B-3000 Leuven, Belgium.
E-mail: Lieven.Verschaffel@ped.kuleuven.ac.be