Differences in Teaching and Opportunities for Learning in Primary Mathematics Classes

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Abstract: The widespread acceptance in the view that learning is an active constructive process requires teaching that is fundamentally different from classical pedagogy. It is generally accepted that teaching must consist of highly interactive and discursive situations. However, these differences in teaching are not well understood. In this paper, examples of teaching drawn from primary classrooms illustrate the differences in teaching and the ways these distinctions influence children's opportunities for learning.

Kurzreferat: *Unterschiedliche Unterrichtsmethoden und Lernmöglichkeiten im Mathematikunterricht der Grundschule.* Die weitverbreitete Akzeptanz der Ansicht, dass Lernen ein aktiver konstruktiver Prozess sei, erfordert Unterrichtsmethoden, die sich von denen der klassischen Pädagogik wesentlich unterscheiden. Es ist Allgemeinkonsens, dass Unterricht außerordentlich interaktive und diskursive Situationen enthalten muss. Die daraus resultierenden Unterschiede in den Unterrichtsmethoden werden jedoch nicht richtig verstanden. In diesem Beitrag werden solche Unterrichtsunterschiede und ihr Einfluss auf die Lernmöglichkeiten der Schüler durch Beispiele aus dem Grundschulunterricht veranschaulicht.

ZDM-Classification: C52, C72, D42

1. Introduction

Two aspects that characterize the current attempts to improve mathematics education are the influence of the "cognitive revolution" in psychology that redefined how individuals learn, and the attempt to change the perception of mathematics and what it means to do mathematics in school. There is widespread acceptance in the view that learning is an active constructive process. As children learn they impose their own interpretation and create theories about their experience that makes sense to them. Additionally, rather than conveying a view of mathematics as narrowly consisting of executing well-established procedures, the goal is to present it as a subject that consists of patterns and relationships that are understandable through cognitive processes that involve reasoning and logic. In light of these changes, comparable changes have been proposed for pedagogy as well. However, the view of teaching proposed is largely drawn from theoretical and epistemological tenets of learning and the shift in the view of mathematics generalized to a hypothetical view of teaching, rather than a perspective grounded in the practice of teachers. Thus, the purpose of this paper is to present empirical examples of teaching drawn from a research project in primary classes to illustrate the differences in teaching and the ways these influence children's opportunities for learning.

In order to examine teaching, it is important to take into consideration that teaching by definition is an interactive activity and teaching in school settings is an endeavor that is conducted as children are participating as members of a group (Wood 1995). Thus, it is essential to not only characterize the teaching that is observed but to also examine the ways teachers create a culture for learning in order to fully understand how pedagogy influences students' opportunities for mathematics learning. Therefore, in the paper I consider the social expectations the teacher establishes with the children for "doing mathematics" that define the nature of social interaction that occurs in the classes. Following this, I describe the ways teaching acts to enable children to deepen their understanding of mathematics.

2. Theoretical perspective

2.1 Social interaction and children's learning

In addition to the revision in the psychological theory of learning, there is also an increasing awareness and interest in the social aspects of learning mathematics (Bauersfeld 1988, 1995). The theorists Goffman (1959) and Garfinkel (1967) both claim that the social structure in everyday life consists of normative patterns of interaction and discourse. Once established, these patterns become the reliable routines found in interactive situations. As Evans-Pritchard (1954) commented:

> "It is evident that there must be uniformities and regularities in social life ... or its members could not live together. It is only because people know the kind of behaviour to expect from others ... that each and all are able to go about their affairs" (p. 19).

Sociologists are less interested in the personal construction of meaning per se than in the processes involved in the construction of common meanings among humans. For Blumer (1969) these meanings are "social products" and are "creations that are formed in and through the defining activities of people as they interact" (p. 5).

This view is further supported by Bruner's contention that children need to adapt to a social existence and to develop a system of shared meanings in order to participate as members of their culture (Bruner 1990, 1996). Therefore, in social situations young children must direct much of their attention to learning the expectations held for their behavior. In order to interact and communicate, children also need to learn the common meanings that are taken as an implicit basis of reference when members of a culture are speaking to one another. Thus, for children holding shared meanings is essential to their development of successful human interaction (Bruner 1990).

Bauersfeld (1995) drawing on these ideas, contends that much of what happens in school interaction is for the purpose of maintaining institutional norms rather than scholarly norms necessary for intellectual development and that the "classroom processes function toward a curtailment of the negotiation" of meaning among the participants. "Too many teachers fall victim to the reality illustration: "recitation" ... and the "funnel pattern" ... in which instruction dominates over interaction" (p. 276). For Bauersfeld students' knowing *how* and *when* to participate is as important as their knowing *what* in mathematics. Knowing the expectations for participation allows the members of the class to create a sense of social order in which one may anticipate the actions of self and others in particular social situations.

Several researchers interested in mathematics education have also argued that the interaction created in classes influence not only what children learn in terms of content but also what they believe about the nature of mathematical knowledge and the ways in which one learns mathematics (e.g., Carraher, Carraher, & Schliemann 1985). Cobb, Wood, Yackel, McNeal (1992), from empirical analysis of data from their research-project class and a traditional class, found differences in the social settings created for learning. Two social contexts categorized as *school mathematics* and *inquiry mathematics* were distinguished by two discrete situations: a situation involving the acts of "challenging and justifying" and a situation involving the acts of "explaining and clarifying," respectively.

2.2 Social interaction and teaching

The theoretical perspectives found in sociology are also of specific interest in understanding teaching. Two central perspectives drawn from a sociological position influence the analysis of teaching. The first perspective is that to understand teaching one must examine it in conjunction with students' activity as a form of interaction. Therefore, teaching that is situated in school, serves different purposes, and is embedded in different activities and practices. The second perspective is that teaching has to do with the development of meaning, both individual and collective. Drawing on these two perspectives, teaching practices are thought to have structure and forms of repeatable interaction that can be identified in the analysis of empirical data from classrooms.

Krummheuer (personal communiqué, September 1998) maintains that given teaching is most readily observed in action, it is essential to look into the situation to examine the ways that pedagogy is contributing to the mathematical meaning that is developing among students. To accomplish this, it is important to examine first the nature of the interaction that evolves between teacher and students and the forms and meanings of these contexts. Second, it is important to examine the content of what is talked about – the topics and themes that emerge. Finally, it is necessary to examine the interactive processes through which the context is established to identify the norms that underlie the interaction. It should be noted that this is not a deterministic approach because the existence of individual experience, motivation and human agency is recognized as integral in the process of learning

3. Differences in teaching

Problem-centered activities created as part of an earlier research project formed the curriculum and met the content objectives for second-grade instruction in the school system. The children determined how to solve the problems, first by working together in pairs and later sharing their strategies and thinking with others and the teacher in class discussion. The goal was to come together as a class, after working together in pairs, to present and share personal ideas and thinking for the purpose of promoting individual and common meanings for mathematics.

Teachers wanted children to share their thinking by telling others how they solved the mathematical problems and they wanted children to listen to one another's ideas. In order for this to happen, the teachers negotiated explicit expectations with their students at the beginning of the school year with the intention of creating an environment that was free of risk or threat so that children would feel secure in sharing their personal solution methods. This "climate of trust" was created to enable children to feel the other members of the class respected their ideas. The teachers were aware that positive experiences in sharing ideas built the foundations for the open communication that is necessary for learning as students express themselves mathematically. Children did express their ideas without fear of being ridiculed or embarrassed even when they made mistakes. Cobb, Yackel, and Wood (1989) and Wood, Merkel, & Uerkwitz (1996) have discussed the importance of creating this climate of trust in relation to these classes in earlier work.

Aside from creating a climate of trust, other important pedagogical differences exist among the teachers that contributed in more fundamental ways to children's mathematical learning. These teaching differences were brought to the fore in the analysis of the interactive and discursive situations that occurred during the lesson event of class discussion. Differences in the interaction were reflected in the ways teaching influenced the mathematical activity and mathematical meaning that emerged during this event.

The findings from the analyses reveal three different patterns of interaction and communication that are distinguished by the specific expectations that teachers initiate and establish with the children for participation at the beginning of the school year (Wood & Turner-Vorbeck, in press). In addition, differences in teachers' questioning of children existed within the three forms of interaction that created situations in which children were required to reflect on and think in increasingly deeper ways about mathematics. The questions the teachers posed ranged from asking children to further describe their ways of solving problems to requiring them to think more profoundly about the mathematics underlying the problems and/or their solutions. Differences in the discussion were reflected in the ways teaching influenced the mathematical activity and mathematical meaning that emerged during this event. Therefore, differences in opportunities for children's learning varied based on the nature of the expectations for participation in the interaction and the kinds of thinking needed to answer the questions that were posed by the teacher.

During the first weeks of the school year, teachers explicitly discussed with their children what they were expected to do to participate in the class discussion as well as in the pair work. Teachers negotiated their intended meanings for these expectations as they interacted with their children during the initial class discussions that followed these explicit discussions. It is during this time that teachers clarified what is expected not only when children talk about their solutions but also what they should do when listening to others. These interpersonal communications lead to an accomplishment of meanings that are taken-as-shared and students come to understand not only what they can expect to do themselves, but also what they can anticipate others are to do in this situation. The differences in the expectations and in the ensuing forms of interaction and communication that occurred are presented next.

4. Examples of differences in teaching

One form of interaction that occurs is one in which students explain to the other members of the class how they solved the assigned mathematical problems. In this interaction, the main goal of the teacher is to create a situation in which children report their individual strategies for solving a problem to the others. At the beginning of the year teachers establish with the children that when explaining it is expected that they would tell different ways to solve problems and provide details about their strategy if the teacher asks. In these situations, "different way" means a way that had not been shared in class discussion yet. The following episodes that occurred during whole class discussion provide examples of the differences in the pattern of interaction and in teachers' questioning.

4.1 Report ways

The problems being discussed are shown in Figure 1. Hannah gives a strategy for the problem that is third on the page (52, 33/empty box.) However, in the process of explaining she refers to the previously discussed problem 52, 30/ empty box; and the last problem 52/empty box, 33.

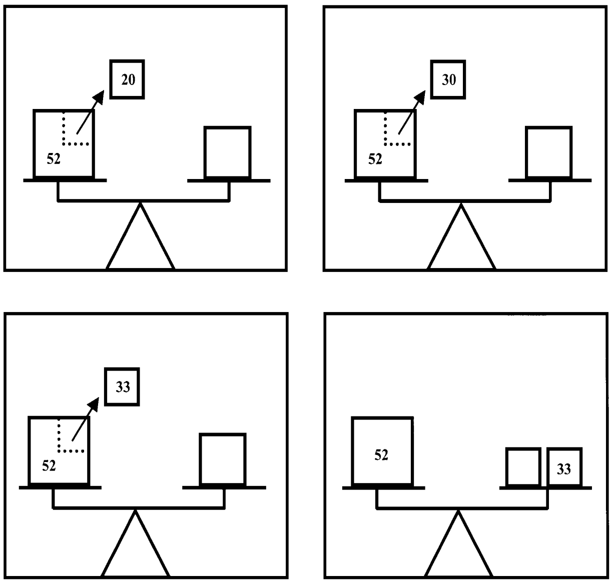

Fig. 1: Balance Problems

4.1.1 Illustrative Example

Tchr: Hannah, what was your strategy?
Hana: They're kind of like the same [referring to the problems on the sheet.] Um well, this one's the answer so, 52, so...
Tchr: What did you do? Did you use the 22 up there to help you?
Hana: No. We used (pause).
Tchr: Which one did you use to help you?
Hana: We did this one first . . . We did the last one first. (She writes 19 on the overhead)
Tchr: How did you get 19 for that one?
Hana: Well, I added up (pause) well, 33 and then with that 10 it would be 43 and (pauses as she counts up to 52 by ones on fingers) so 9.
Tchr: So 19 and 33 make 52?
Hana: (Nods yes).
Tchr: Okay. How did you do the one we are working on now?
Hana: Well there's 33 here so with that one it could be like right here (points to 33 in last problem). Like that one here so both answers are 19.
Tchr: So it is 19?
Hana: (Shakes head yes)
Tchr: Okay. Nicole did you do it a different way?

The teachers' questioning during the exchange consists of requests to the child for more information and detail about the strategy with queries such as, "How did you do it?" "What did you do?" "Where did you get 20 to add to 32?" The students' explanations consist of providing descriptions of their thinking and strategies for solving the problems. The detail of the children's descriptions varies depending on the extent to which teachers, through their questioning, demand comprehensiveness and clarity in the explanations from the children. This situation can be depicted as one of *report ways* in which students "tell how they solved the mathematics problem."

One other characteristic defines this interaction; the children listening seldom participate in the questioning. Instead, the communication is between the teacher and the child explaining, while the others listen. At the beginning of the year, each teacher established expectations for what it meant to explain as the children were giving their explanation. At the same time, the teacher also established the norms for those listening as to simply "pay attention" or, in more complex situations, the expectation was to compare their strategies to the one being given in order to know if they had a different solution to volunteer when the teacher asked.

4.2 Inquiry

At other times, the discussion might initially begin as a reporting of ways with students telling their strategies to those listening, but move into an interaction involving inquiry. The difference that characterized inquiry was that the children explaining were also expected to clarify their meanings and to give reasons for their thinking in response to inquiry questions from the teacher and the other students. In these classes, the discussion moved back and forth from a reporting of ways exchange to an inquiry interaction. In such interactions, the listener was trying to understand and to make sense of an individual's strategy as he/she was explaining. In the inquiry interaction personal ideas were open for examination and questioning by the teacher or, in some classes, the other students, with the underlying expectation that it was important to try and make sense of the mathematics being explained. In this interaction, confusion and complexity surrounding someone's explanation was viewed by teachers as opportunities for children's learning, both the one explaining and the others listening.

The following episode that occurred during a class discussion provides an example of the pattern of interaction and teachers' questioning.

4.2.1 Illustrative example

The problem is $72 - 39 =$ ___.

Tchr: My gosh we've got three different ways already. What did you get, Sarah and how did you do it?
Sara: Um, 33. I almost did it like Kyle except I did 72 minus 30. And then I um, then I took off 10 and then I got 42. And then, I took off 10 and I got . . .Wait.
Tchr: You took 72 minus 30 and you got 42. Then you took off 10. Why did you take off 10? Do you remember why you took off 10?
Sara: Yes, because it was easier. It was close to 9.
Tchr: What do you mean it was close to 9?
Sara: Because 9 is close to 10 so then 32.
Tchr: 32? But how did you get 33?
Sara: I added 1.
Tchr: Why did you add 1?
Sara: Because you were only taking off 9 instead of 10, so I had to add a 1 back on.
Tchr: Okay. I understand now. Does anyone have another way we can do this problem?

Within an inquiry interaction, the expectation for children explaining is to not only tell how they solved the problem but also to give *reasons* for their thinking and to *clarify* their ideas to others when asked. The teacher asks the same information seeking questions illustrated in the previous interaction of report ways, but now also makes inquiries into children's meanings by asking, "What do you mean? I don't understand". Teachers also ask questions that require children to do more than give additional detail about their strategies, instead students are asked to provide reasons for their thinking, questions such as, "Why did you add 20 to 32?" This form of questioning requires children to clarify their thinking in order that others can better understand and demands deeper thinking from students in order to give reasons for their ideas. The differences seen in teaching as it occurs during an inquiry interaction is one in which students explaining are asked to "understand and reason" about mathematics.

In this form of interaction, the children listening were often seen participating in the question asking in addition to the teacher. The listeners, therefore, were also required to "understand and reason" about mathematics in order to participate in the question asking. Therefore, the expectations that the teacher established were more complex for not only the child as explainer but also as listener. The role of listeners was to understand and to make sense of what was said. If they did not, they were to ask questions in an attempt to make sense of an individual's strategy and the reasoning behind it. The expectation that listening students would also participate in questioning was important not only for their own learning but also created the possibility for public discourse about mathematical ideas. These differences in teaching create an opportunity for the private thinking of individuals to be examined and questioned for the purpose of understanding and establishing common meanings. Public understanding of personal meanings is thought to be an important process in the development of shared meanings that are fundamental to children in order to participate as members of their culture (Bruner 1990; Edwards & Mercer 1987).

4.3 Argument

An inquiry interaction frequently extends to an interaction that involves assertions and challenges being made about the validity of an answer or someone's reasoning. In an argument interaction, processes of justification and argumentation involving what might be called informal "proof" occur as students are expected to justify their reasoning if challenged by others. The movement between forms of interaction involving inquiry and argument were extremely fluid. Thus, this situation reflected the most complex forms of teaching because of the careful attention to establishing the expectations for the active mental participation of the listener as well as the explainer (c.f. Wood 1999). This was largely a result of the active role that the children listening were expected to take in making challenges and providing rationale for their disagreements. The following episode provides an example of the pattern of interaction and teachers' questioning.

4.3.1 Illustrative example

The problems being discussed are also shown in Figure 1.

Tchr: What did you get for an answer to this problem? (Points to problem 3). David?
Davd: 19.
Clas: Agree.
Tchr: (Writes 19).
Davd: Our number sentence was 52 take away 33 equals 19.
Tchr: Okay. Anyone have anything different? (to the class). Okay David. How did you solve this problem?
Davd: (Comes to the front of the room). I knew that 20 plus, (pause) no 19 plus 33 was 52, so I knew that if you took 33 it would be 19.
Tchr: David, I agree with your answer but I don't agree with how you solved it. You said you knew 19 plus 33 equals 52. How did you know that?
Davd: Because that and that (points to 3 tens and 1 ten) equals 40 and add that and that (points to 3 ones and 9 ones) equals 50 and 2, equals 33.
Tchr: Yes. But where did you get the 19? Nineteen is not given in the problem.
Davd: I already knew it because the first problem was 52 take away 20 was 32, so 52 take away 33 had to be 19.
Tchr: Ah. Okay. Now I agree with what you did.

In the examples given above, the differences in teaching are very apparent. However much of the time teaching is not this visible as the children as explainer and listeners come to dominate the interaction. The following example taken from a whole class discussion characterizes student interaction and the dynamic nature of the movement between the three interaction patterns (Wood & Turner-Vorbeck, in press).

4.3.2 Illustrative example

The problem is the third one shown in Figure 1.

Tchr: What did you get for an answer for this problem?
Fred: 25.
Sara: 19.
Adm: 21.
Tchr: Any other answers? Okay. Fred tell us how you got 25.
Fred: We used the unifix cubes. 52 and then we took away 33. First I took away the tens. 42, 32, 22. Then I counted back the ones, 21, 20, 19.
Karn: But you said it was 25.

Fred: I know, but now I think it is 19, because I counted it again with the cubes.
Tchr: John what do you want to say? (He has his hand raised).
John: I went back to 52 take away 30 is 22 (points to second problem on the paper). And I took away 3 more and that was 19. So I think it is 19.
Tchr: Okay. But why did you take away 3?
John: Because 52 take away 30 is 22, and 33 is 3 more than 30 so it was 19.
Tchr: How did you know that it was 19?
John: Because if 52 take 30 is 22, 52 take away 33 is 3 more than 30, so then I had to take away 3 more from 22, and that would be 19.
Tchr: That makes sense. Sarah what would you like to say?
Sara: Well if you take 30 from 50, then you would have 20. Then you would have 2 and that 3, so you could take 1 from 20, and that would be 19.
Mark: This is too confusing for me. Sarah, I don't understand why you took the 1 from 20.
Sara: Because you have 2 minus 3 and so you need 1 off the tens.
Mary: But if you took 1 from the 20, what happened to the 2 and the 3?
Sara: I took 1 from the tens and added it to the 2 to make 3. [Then] 3 minus 3 is 0. So then I had to take 1 from the tens-20 and that makes the answer 19.
Ryan: Well if you check it by adding 19 and 33, you get 52, so 19 is the answer.
Karn: I think the answer must be 19, because we did it so many different ways to figure it out. And we got 19.
Class: Agree. It is 19.

Although the teaching in this episode appears to be nominal, nonetheless it is the differences in teaching that are necessary to establish forms of interaction that require children to think more deeply about their mathematical ideas and the relationships among them.

5. Conclusion

The examples teaching presented in the instances above illustrate pedagogical practices that are fundamentally different from conventional mathematics instruction found in most elementary schools. The examples given reveal that teaching involves creating a classroom environment in which students present their mathematical thinking to others, struggle to understand and participate in the evaluation and validation of common meaning. These differences in teaching evolve from a perspective of learning as a constructive reconstructive process and, therefore, is necessarily substantially different from classical pedagogy that is derived from a view of learning as a process of absorption. Much of the difference is due to the fact that teaching from this perspective is viewed as an interactive communicative process through which meanings are shared and knowledge is constructed. The examples illustrate teaching that is comprised of the interpersonal exchange that leads to the attainment of mathematical meaning through the interactional process of negotiating meaning that contributes to the development of both individual and common knowledge.

This form of pedagogy requires that teachers understand the complex relationship between social processes that are established and the opportunities created for children's mathematical learning. There is wide spread agreement that these changes depend on elementary teachers' coming to know their students' mathematical thinking and its development and their using this knowledge in teaching. However, these differences in teaching require more of teachers than merely acquiring pedagogical content knowledge of mathematics (Ma 1999) or knowledge of children's mathematical thinking (Carpenter, Ansell & Levi, in press). Instead, the differences in teaching described in this paper involve creating interactive settings to not only bring students' individual thinking to the fore, but to also establish a context for pupils to negotiate meanings that become the common ground from which to progress in mathematical thinking. This highlights the importance of teachers' understanding children's need to adapt to a social existence and to develop a system of shared meanings. Teaching differently means that teachers understand the importance of the social interactive and discursive processes that underlie the creation of appropriate contexts for children's mathematics learning.

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