On the Role of Creative Thinking in Problem Posing

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Abstract: This paper addresses the relationship between creative thinking and problem posing as well as problem posing tasks in mathematics domains. Empirical studies were conducted to investigate on relationships and on tasks. Results of a study on arithmetic problem posing and its replication suggested that fluency is general in verbal creativity and problem posing, but flexibility is specific in problem posing. Further investigations into general mathematical problem posing were also carried out, having each of ninety-six elementary school children of Taiwan completing 18 problem posing test items in the Test on General Problem Posing. Results suggested that a general, rather than specific, problem posing competence exists in children and can be measured by the test.

Kurzreferat: *Über die Rolle des kreativen Denkens beim Aufgabenerfinden.* In dem Beitrag geht es um die Beziehung zwischen kreativem Denken und dem Aufgabenerfinden sowie Aufgaben zum Erfinden von Problemen in Mathematik. Dazu gibt es empirische Untersuchungen. Die Ergebnisse einer Untersuchung über das Erfinden arithmetischer Probleme und ihre Reproduktion legen nahe, daß Gewandtheit eine allgemeine, und Flexibilität eine spezifische Fähigkeit ist. Weitere Untersuchungen zum Erfinden mathematischer Probleme mit je 96 Grundschulkindern aus Taiwan, die 18 Testaufgaben des "Tests zum Erfinden allgemeiner Probleme" zu bearbeiten hatten, wurden durchgeführt. Den Ergebnissen nach zu schließen scheinen Kinder eher eine allgemeine statt eine spezifische Fähigkeit zum Erfinden von Problemen zu besitzen, die durch den Test gemessen werden kann.

ZDM-Classification: D50

1. Problem posing and creative thinking

Problem posing has been identified as a multi-facet construct under separate covers (Silver, 1994). To be stringent, and according to mathematicians, problem posing is real only when the problem has not been solved by anyone before. To a great extent, and according to numerous studies, problem posing means the formation of novel problems, with solutions unknown to at least the one who formulates it (e.g. Mamona-Downs, 1993; Van den Heuvel-Panhuizen, Middleton & Streefland, 1995). Subsequently, problem posing also refers to the act of converting a given problem into a different representation. For example, when solving a problem, the problem solver may need to re-present the given problem in order to understand the problem (Cohen & Stover, 1981). In this case, the solution to the problem is still unknown to the one who poses the problem again. There are also cases when the person who poses the problem may already know how to solve the problem. One example is the posing of problem isomorphs; where cognition is inherent in the act of posing isomorphs for human or computer problem solving. In that case, posing a problem isomorph would change the level of difficulty in solving (Simon, 1989). Another example can be observed in any ordinary classroom. Teachers often pose problems whose solutions are known to them. For example, teachers may need to pose textbooks problems differently in order to meet their instructional objectives (Leung, 1991). Furthermore, problem posing may also mean the construction of test items and the revision of test items after administration (Leung, 1996). Despite its varied characteristics and heterogeneity, problem posing essentially means creating a problem with solutions unknown to the target problem solver for whom the problem has been created.

Creativity is similar to problem posing in its multiplicity in nature. Psychologists identified it as a special construct other than intelligence. For a long period in history, creativity meant a form of divergent thinking (e.g. work of Guilford). Torrance followed Guilford's work and developed tests to measure creativity by fluency, flexibility, and originality (Torrance, 1966). Until today, there is no consensus on what creativity is. In Latin, the word create is *creare,* which means "bring into being". However, to account for creativity, the entity that comes into being must be of cultural value (Ochse, 1990). Therefore, in mathematical creativity, a piece of valuable mathematics is brought into being. This "mathematics in the making" is a hallmark in Polya's *How To Solve It,* an important book in problem solving (Polya, 1945). The mathematics that is brought into being can be a solution to a problem that no one has solved before; a novel solution; or a mathematics problem that is either novel, original or of some importance in the field.

Given the "creating a problem" characteristic of problem posing and the "bring into being" nature of creativity one might see problem posing as a kind of creativity. In fact, problem finding has sometimes been considered as a creative process in itself (Dillon, 1988; Voss & Means, 1989). Even in psychological investigations into problem solving by machines, attention was given to the part of having machines find problems (e.g. Langley, Simon, Bradshaw, & Zytknow, 1987). Studies on mathematical creativity were reviewed (Haylock, 1987) and one may see problem posing ability as a creative ability. In a recent comprehensive review paper on problem posing, however, Silver extended the discussion and commented that a general relationship between problem posing and creativity was still unknown (Silver, 1994).

2. Problem posing tasks in mathematics domain

Mathematical problem posing tasks varied. There were tasks that are open-ended in that pupils were free to write any problem they could think of without restriction to mathematics content or context. For example, Australian students were asked to write down a difficult problem (Ellerton, 1986), and U.S. students were asked to create word problems (Winograd, 1991). Other tasks may be

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semi-open and deal with subjects' perception of a problem (Silver, Mamona-Downs, Leung & Kenny, 1996) or mathematics structure (Balka, 1974; Getzels & Jackson, 1962). In the Billiard Ball Mathematics (BBM) task surveyed by Silver et al., the task environment described a ball shot at an angle from the lower left hand corner of a rectangular billiard table and rebounding. Silver et al. found that subjects also considered problems as questions related to Billiard Ball playing. In other studies, children's ability in making up mathematical problems was measured as creativity (e.g. Balka, 1974, Getzels & Jackson, 1962). In the Soviet's study, the given task was so restricted that only one particular problem "naturally follows" (Krutetskii, 1976). Elsewhere in Japan, tasks on posing were restricted to the posing of a problem "similar" to a given problem (Hashimoto, 1987) or to pose a problem matching a solution strategy (Ikeda, 1993); whereas in the Netherlands, grade five children were restricted to pose percents problems (Van den Heuvel-Panhuizen, Middleton & Streefland, 1995).

In the above studies, task environments differed, and so did evaluation methods. For example, in Krutetskii's study, the "correct" problem to ask was the one problem that naturally follows whereas in Hashimoto's study, the problems posed were evaluated according to the extent to which the problem was similar to the original problem. In the BBM study, problems were classified according to how they were posed (Silver et al., 1996). Finally, when problem posing was a test of creativity, Balka scored fluency, flexibility, and originality while Getzels and Jackson scored the problems posed according to the arithmetic operations and the number of steps used to solve them.

Since mathematical problem posing can be viewed as a kind of mathematical creativity, the evaluation of problem posing may also include the novelty in content, context, solution strategy, or difficulty level; depending on the extent of freedom given in a certain problem posing task environment. If alternatively, a person was asked to pose problems in multiple task environments, one can examine a person's creativity within and across task environments. In problem solving, multiple task environments such as drawn, verbal or telegraphic were explored (Moyer, Sowder, Threadgill-Sowder, & Moyer, 1984). In arithmetic problem posing, subjects were asked to pose problems in tasks containing numerical, verbal, or symbolic information (Leung, 1993; Leung, 1995). However, problem posing within and across multiple tasks environments (including non-arithmetic problem posing) was not explored.

3. A prior study and its implications

A prior study was completed in the United States and reported in an International Meeting for The Psychology of Mathematics Education (PME) research report in Tsukuba, Japan (Leung, 1993). Leung explored the relationship between general verbal creativity and arithmetic problem posing. Given the varied characteristics of both problem posing and creative thinking and the fact that evaluation of posing has to be done in relation to the specific requirements of the task, Leung narrowed the meaning of creative thinking and problem posing by referring to abilities measured by two specific instruments. She used the overall scores of the verbal part of the Torrance Test of Creative Thinking (TTCT) to represent verbal creative thinking. To measure problem posing, she developed a test of arithmetic problem posing (TAPP). In TAPP, she purposely included written instructions similar to those of TTCT such as "write as many as you can" or "think of unusual ones". Scores in TAPP were made according to multiple dimensions of complexity. For a more detailed report of the study see Leung and Silver (1997).

As reported in Leung and Silver (1997), a group of U.S. prospective elementary school teachers were asked to complete the test. They formulated a sequence of problems from a given situation described in story form. Results of quantitative analysis suggested no significant difference between the high and low creative thinking group, in the production of problems categorized according to predefined dimensions of complexity. However, results of quantitative analyses using two subscores of TTCT (fluency, flexibility)* showed interesting relationships between creativity and problem posing. First, subjects with a higher verbal-creativity score in fluency tended to be also fluent in problem posing ($r = 0.361$; $p < 0.01$). Second, subjects who scored high in TTCT flexibility did not necessarily score high in both comparable flexibility scores in TAPP (namely Logical/linguistic Structures and Semantic Structures). Third, scores in two comparable flexibility scores in TAPP correlated significantly ($r = 0.265$; $p < 0.05$). Finally, qualitative analysis suggested that high creative group (in TTCT) outperformed the low group in providing contexts to arithmetic word problems.

From the first result, fluency is general in both verbal creativity and problem posing but according to the second, flexibility is not. Finally, the third result suggested that flexibility is specific within arithmetic problem posing. The third result was also obtained in a replicated study in Taiwan ($r = 0.286$; $P < 0.01$; Leung, 1995).

4. About this study

The study reported here is an extended empirical study conducted in Taiwan, using Leung (1993) and Leung (1995) as the backdrop. In this study, the investigator further explored by extending work on creativity and including more problem posing tasks. The investigator included tasks that were not necessarily arithmetic, and multiple task environments to discuss problem posing ability within or across task environment.

4.1 Method

Subjects were 96 grade five elementary school children of Taiwan. The task to be completed by them was a test on general problem posing (TGPP). It consists of 18 test items, each with information which is either a text, a picture, or an answer. The items were related to topics randomly selected from four strands (Number, Quantity, Space, Statistics) documented in the curriculum standards of elementary school mathematics in the country

* No analysis was done on originality, the third subscore on TTCT, because it was hard to find a comparable originality score on problem posing when only 49 subjects were involved.

(Education Department, 1993). In each TGPP item, subjects wrote one problem below the given information and did not solve the problem. They were told to specify the changes [if any] they made to the item as they wrote down the problem they posed. In the test, 9 were pictorial and 9 textual. Three of the textual items contained answers to the problems children were asked to pose. The administration was done in two consecutive days: Day One (item 1 through 10); Day Two (item 11 through 18). Each class period was 40 minutes.

Data analysis included correlation analysis (for ability across task) and a qualitative analysis (for ability within task) on children's originality on posed problems in 3 selected items (one Text, one Answer and one Picture); scores of each having the respective highest correlation with the total score in TGPP. Two research assistants completed data-coding and reliability was attended. The first two aspects of data coding of Leung and Silver (1997) were used*. The first aspect is the Problem Type which includes Content: Math/Non-Math; Feasibility of initial state: Plausible/Implausible; and, Data required in solving: Sufficient/Insufficient/Extraneous. The second aspect is the Logical/linguistic Structure of the question or problem statement which includes: Assignment or Relational propositions; and Factual or Conditional statements. In addition, an 8-point system was developed because a representative single score was needed for correlation analyses. The scores were: Not a Problem (0); Non-math Problem (1); Implausible Math Problem (2); Insufficient Plausible Math Problem (3); Extraneous Plausible Math Problem (4); Sufficient Plausible Math Problem (5); Factual/Relational Sufficient Plausible Math Problem (6); Hypothetical/Assignment Sufficient Plausible Math Problem (7); and, Hypothetical/Relational Sufficient Plausible Math Problem (8). One limitation is noted here, for the attempt to use a single score to represent problem posing ability.

4.2 General ability in test of general problem posing (TGPP)

The investigator examined problem posing ability across tasks. Correlation analysis was completed to test if TGPP measure a general problem posing ability. The correlations of children's score in each item with the total score were computed. They ranged from 0.5451 to 0.8407, suggesting that TGPP tested a general problem posing ability. The investigator also examined if there was a specific problem posing ability in each task category: Text, Answer, and Picture. The 18 test items were divided into three categories accordingly. The three subscores (Text, Answer, Picture) were used in correlation tests, and the correlations were all statistically significant (Text-Answer: $r = 0.7512$; Text-Picture: $r = 0.7795$; Picture-Answer: $r = 0.7550$). Following the implication of the prior result, this result implied that there was a general problem posing competence across three task environments. That is to say, no proficiency in any of these three task environments is specific and not general to the other two task environments.

4.3 Creative problem posing

The exploration of problem posing ability went from "across task" to "within task". The investigator selected the top correlations in each category to further analyze how children posed problems; unpacking the creativity part of mathematical problem posing. The three items that were chosen are The Sandwich Problem (Item 7; $r = 0.8035$), The Speed Problem (Item 9; $r = 0.8202$) and The Temperature Problem (Item 16; $r = 0.8407$). The analysis was geared towards contents and contexts of posed problems.

In the Sandwich Problem, the given information was a text, "A piece of sandwich is in the shape of a triangle. The length of each side is 5 cm". According to empirical posed problems, children's creativity could be traced from a variety of goals such as naming a shape, finding a side, height, perimeter, area, volume, and, radius or diameter (of a circular sandwich). Other than attending to goals, they challenged the given condition and changed the triangle in the original task environment to a circle, a square, a trapezium, and a pentagon. Children attending to goals and conditions in problem posing acted similar to teachers posing problems in BBM task (Silver et al., 1996). Other than attending to contents, they also changed the context. The sandwich was changed to a loaf of meat, a hamburger, a meat bun, a piece of toast, a pencil case, a plank of wood, or, a clock. Grade five children created original problems by considering combinations of changes in contents and contexts. One original problem given was, "how can you arrange 100 sandwiches to make a figure with the greatest perimeter?"

In the Speed Problem, an answer was given for children to pose a problem. In this task environment, children had to foresee the solution of the problem they posed. In this problem, the answer is, "Answer: Speed of $A = 37$m/min; Speed of $B = 28$m/min." A child posed automatically the following problem. "The runway is 1038 m long. A finishes in 28 min. B finishes in 37 min. Find the speed of A and B". The other children attended to content and posed problems in: comparing speeds, finding average speeds, finding the catch-up time, changing speeds, walking in opposite direction, or leaving at different times. The variety of contexts included train stations, post office, hill climbing, boat racing, and money transactions. There were also instances when students posed problems whose answers were different from the answer in the item. What they did was using the answer as a basis for posing a new problem whose solution requires the information given in the answer to the item. When solving, the answers were different to the answer that was given in the task environment. In that case, the given problems were not rated as creative. They only represented instances where children failed to follow instructions.

A picture of two thermometers, instead of a text or answer, was given in the Temperature Problem (see Fig. 1). Compare or average problems were mostly given. A few of these asked fill-in-the-blank questions, or comprehension questions like "what if the day temperature?" The

* The third and fourth measures of TAPP cannot be adopted to measure non-arithmetic problems, which were also included in TGPP.

problem on finding the total temperature of the day and the night has no meaning in real and was not given merit of novelty. An example of a problem posed by attending to content was a display on sales of 3 popular local drinks in summer. The problem was to draw a pie diagram. The mathematics content switched to statistics. An original problem is given in Figure 1.

Figure included in print version only

Fig. 1: The Temperature Problem and an example of a posed problem

5. Discussion

In presenting a plenary speech at 1993 PME meeting, Silver commented that it is unknown if a clear relationship between creativity and problem posing can be found (Silver, 1994). Since then, there were initial attempts on relationships, including the part on researching in the general or specific role of creativity in problem posing. According to Leung (1993), fluency and context building is general in verbal creative thinking and in posing arithmetic word problems. However, flexibility is not general in both, suggesting that there is a specific arithmetic problem posing exists (Leung, 1993; Leung, 1995). While the two prior empirical studies required participants to pose problems in one task environment, this study required children to pose problems in 3 task categories. Findings from the extended study implied that there is a general mathematical problem posing competence. Children's competence in all 18 tasks were consistent within and across all three task environment categories: Text, Picture, and Answer. Finally, by referring to three test items, the author highlighted the creative problem posing of grade five elementary school children of Taiwan and gave examples of original problems.

The study also adds knowledge to existing literature on problem posing tasks in mathematics domains. While prior empirical studies tended to examine problem posing behaviors on one mathematics topic (e.g arithmetic in Getzels and Jackon's study, percents in Van den Heuvel-Panhuizen, Middleton & Streefland's study) or even one task (e.g. BBM task in Silver et al.'s study), this study addressed children's problem posing ability within and across 18 tasks. It was investigated by asking them to complete 18 test items in TGPP. There are instructional implications as well. The instrument is now available for use, for example as a means to foster mathematical creativity in ordinary classrooms (Pehkonen, 1996).

6. References

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