Information

The Role of the History of Mathematics in the Teaching and Learning of Mathematics

Discussion Document for an ICMI Study (1997–2000)

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In recent years there has been growing interest in the role of history of mathematics in improving the teaching and learning of mathematics. ICMI, the International Commission on Mathematics Instruction, has set up a Study on this topic, to report back at the next International Congress in Mathematics Education (ICME) in Japan in the year 2000. The present document sketches out some of the concerns to be addressed in the ICMI Study, in the hope that many people across the world will wish to contribute to the international discussions and the growing understandings reached in and about this area.

This discussion document will be followed by an invited conference (to be held in France in April 1998), from which a publication will be prepared to appear by 2000. The next section of the present document surveys the questions to be addressed. Your views are solicited both on the questions and on how to take the issues forward as implied in the commentary.

Some research questions

The overall intention is to study the role of history of mathematics, in its many dimensions, at all the levels of the educational system: in its relations to the teaching and the learning of mathematics as well as with regard to teacher training and in educational research. The order in which the questions are put down here carries no implication about their relative importance or significance.

1. *How does the educational level of the learner bear upon the role of history of mathematics?*
   The way history of mathematics can be used, and the rationale for its use, may vary according to the educational level of the class: children at elementary school and students at university (for example) do have different needs and possibilities. Questions arise about the ways in which history can address these differences.
2. *At what level does history of mathematics as a taught subject become relevant?*
   In analysing the role of history of mathematics, it is important to distinguish issues around using history of mathematics in a situation whose immediate purpose is the teaching of mathematics, and teaching the history of mathematics as such, in a course or a shorter session. There is also a third area, related but separate, namely the history of mathematics education, which is a rather different kind of history.
3. *What are the particular functions of a history of mathematics course or component for teachers?*
   History of mathematics may play an especially important role in the training of future teachers, and also teachers undergoing in-service training. There are a number of reasons for including a historical component in such training, including the promotion of enthusiasm for mathematics, enabling trainees to see pupils differently, to see mathematics differently, and to develop skills of reading, library use and expository writing which can be neglected in mathematics courses. A related issue is what kinds of history of mathematics is appropriate in teacher training and why: for example, it could be that the history of the foundations of mathematics and ideas of rigour and proof are especially important for future secondary and tertiary teachers.
4. *What is the relation between historians of mathematics and those whose main concern is in using history of mathematics in mathematics education?*
   This question focuses on the professional base from which practitioners emerge, and relates to the social fabric of today's mathematics education community as well as to issues about the nature of history. It is important that historians and mathematics educators work co-operatively, since historical learning and classroom experience at the appropriate level do not always co-exist in the same person.
5. *Should different parts of the curriculum involve history of mathematics in a different way?*
   Already research is taking place to investigate the particularities of the role of history in the teaching of algebra, compared with the role of history in the teaching of geometry.
   Even for the design of the curriculum historical knowledge may be valuable. A survey of recent trends in research, for example (bearing in mind that history extends into the future) could lead to suggestions for new

topics to be taught.

6. *Does the experience of learning and teaching mathematics in different parts of the world, or cultural groups in local contexts, make different demands on the history of mathematics?*
   A historical dimension to mathematics learning helps bring out two contrary perceptions in a dialectical way. One is that mathematical developments take place within cultural contexts. The antithesis to this is the realisation that all human cultures have given rise to mathematical developments which are now the heritage of everyone; this therefore acts against a narrow ethnocentric view within the educational system.
   The Study should explore the benefit to learners of realising both that they have a local heritage from their direct ancestors and also that every culture in the world has contributed to the knowledge and experience base made available to today's learners.
7. *What role can history of mathematics play in supporting special educational needs?*
   The experience of teachers with responsibility for a wide variety of special educational needs is that history of mathematics can empower the students and valuably support the learning process. Among such areas are experiences with mature students, with students attending numeracy classes, with students in particular apprenticeship situations, with hitherto low-attaining students, with gifted students, and with students whose special needs arise from handicaps.
8. *What are the relations between the role or roles we attribute to history and the ways of introducing or using it in education?*
   This question has been the focus of considerable attention over recent decades. Every time someone reports on a classroom experience of using history and what it achieved they have been offering a response to this question.
   The question also involves a listing of ways of introducing or incorporating a historical dimension: for example anecdotal, broad outline, content, dramatic etc. Then one would draw attention to the range of educational aims served by each mode of incorporation: the way that historical anecdotes are intended to change the image of mathematics and humanize it, for example. Or again, the way that mathematics is not, historically, a relentless surge of progress but can be a study in twists, turns, false paths and dead-ends both humanizes the subject and helps learners towards a more realistic appreciation of their own endeavours. And there are rich issues for discussion and research in, for example, the use of primary sources in mathematics classrooms at appropriate levels.
9. *What are the consequences for classroom organisation and practice?*
   The consequences of integrating history are far-reaching. In particular, there are wider opportunities for modes of assessment. Assessment can be broadened to develop different skills (such as writing and project activity), and consequences for students' interest and enjoyment have been noted. Teachers may well need practical guidance and support both in fresh areas of assessment, and in aspects of classroom organisation.
10. *How can history of mathematics be useful for the mathematics education researcher?*
    One example is the use of history of mathematics to help both teacher and learner understand and overcome epistemological breaks in the development of mathematical understanding. A constructive critical analysis of the view that "ontogeny recapitulates phylogeny" – that the development of an individual's mathematical understanding follows the historical development of mathematical ideas – may be appropriate. Another example is of research on the development of mathematical concepts. In this case the researcher applies history as possible "looking glasses" on the mechanisms that put mathematical thought into motion. Such combinations of historical and psychological perspectives deserve serious attention.
11. *What are the national experiences of incorporating history of mathematics in national curriculum documents and central political guidance?*
    This is not so much a question for discussion as a fairly straightforward empirical question, needing input from knowledgeable people in as many countries and states as possible. But of course it has policy implications too, and could lead to a sharing of experience among members of the community about how they have reached the policy-making level in their countries to influence the content or rhetoric of public documents. In some parts of the world a different relationship between history and mathematics may have been developed. For example, in Denmark and Sweden history of mathematics is regarded as an intrinsic part of the subject itself. There are also differences in styles of examination and assessment. If everyone with access to examples of such different approaches, from different countries and states, could pool their experience it would be a most valuable input to the Study.
12. *What work has been done on the area of this Study in the past?*
    The answer is: quite a lot. But it is all over the place and needs to be gathered together and referenced analytically. A major annotated critical bibliographical study of the field, which might well take up a sizable proportion of the final publication, would be an enormously valuable contribution that the ICMI Study could make. It should include a brief abstract of each paper or piece of work included, and indications of the categories to which the work relates in an analytical index. Work in progress could be made available on the WorldWideWeb.

Call for contributions

The ICMI Study on *The role of the history of mathematics in the teaching and learning of mathematics* will investigate the above questions over the next two years. The Study has three components: an invited study conference, related research activities, and a publication to appear in the ICMI Study series that will be based on contributions

to and outcomes of the conference and related research activities. The conference will be held in April 1998 in France. The major outcomes of the study will be published as an ICMI Study in 1999 and presented at the International Congress of Mathematics Education in Japan in 2000.

The International Programme Committee (IPC) for the study invites members of the educational and historical communities to propose or submit contributions on specific questions, problems or issues stimulated by this discussion document no later than 1 October 1997 (but earlier if possible). Contributions, in the form of research papers, discussion papers or shorter responses, may address questions raised above or questions that arise in response, or further issues relating to the content of the study. Contributions should be sent to the co-chairs (addresses below). Proposals for research that is on its way, or still to be carried out, are also welcome; questions should be carefully stated and a sketch of the outcome – actual or hoped-for – should be presented, if possible with reference to earlier and related studies. All such contributions will be regarded as input to the planning of the study conference.

The members of the International Programme Committee are Abraham Arcavi (Israel), Evelyne Barbin (France), Jean-Luc Dorier (France), Florence Fasanelli (USA), John Fauvel (UK, co-chair), Alejandro Garciadiego (Mexico), Ewa Lakoma (Poland), Jan van Maanen (Netherlands, co-chair), Mogens Niss (Denmark) and Man-Keung Siu (Hong Kong).

This document is a shortened version of a longer document which is available on the WorldWideWeb at

http://www.math.rug.nl/indvHPs/Maanen.html#dd

or (in French)

http://www-leibniz.imag.fr/DDM/ICMI.html

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