The Mathematics of War

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Abstract: During the Desert Storm, the Gulf war, it was possible to read in the newspapers words such as: “In mathematical terms, war is becoming more and more electronically controlled and, as a result, it is moving away from the battlefield. Then, when war comes down to earth, it becomes bloody, it loses its mathematical asceticism.” Reading the newspapers in those days, one had the impression that modern warfare is based on mathematics, as if it were not men but computers that decided where to carry out “surgical operations”. By contrast, the volume published a few years before the Gulf war conceived as a didactic unit to be used in schools with a guide for the teacher with the title *La matematica della guerra* (The Mathematics of War) published by Gruppo Abele in Turin begins with the words: “Mathematics, like any other discipline, lends itself to building several paths towards education for peace.” The volume, written by a group of teachers belonging to an anti-violence organisation forming part of the “education for peace” project, highlights the power or ambiguitiy of mathematical models used to simulate war or conflict situations and demonstrates that in some cases the use of mathematics leads to a better understanding of the situation, but in other cases, the mathematical model itself can lead to conclusions which are either wrong or morally unacceptable.

Kurzreferat: *Mathematik des Krieges.* Während des Golfkriegs konnte man folgendes in den Zeitungen lesen: “Kriege werden – mathematisch gesprochen – mehr und mehr elektronisch kontrolliert. Das bedeutet, sie werden vom Schlachtfeld entfernt. Wenn der Krieg dann aber wieder ‘zur Erde zurückkehrt’, wird er blutig, er verliert seine mathematische Reinheit”. Die Zeitungen zu jener Zeit erweckten den Eindruck, modernes Kriegswesen sei auf Mathematik gegründet, als ob nicht Menschen, sondern Computer für die Ausführung “chirurgischer Operationen” verantwortlich seien. Ein anderes Bild hingegen vermittelt eine Lehrerhandreichung *La matematica della guerra* (Mathematik des Krieges), die einige Jahre vor dem Golfkrieg von der Turiner Gruppe Abele entwickelt wurde. Sie beginnt mit den Worten: “Wie jede andere Disziplin kann auch die Mathematik mehrere Wege zu einer Erziehung zum Frieden bauen”. Der Band beleuchtet die Macht bzw. manchmal auch die Zweifelhaftigkeit mathematischer Modelle zur Simulation von Kriegs- und Konfliktsituationen und zeigt, daß der Gebrauch der Mathematik in manchen Fällen zu einem besseren Verständnis der Situation beitragen, in anderen Fällen jedoch das mathematische Modell selbst zu falschen oder moralisch nicht vertretbaren Schlüssen führen kann.

ZDM-Classification: A40, M90

“This American war is taking place according to a pact of non-interference between political and military powers. The Pentagon has taken over the visual images and the supply of information remembering how important these factors were during the Vietnam war. And it releases the material drop by drop. In mathematical terms, war is becoming more and more electronically controlled and, as a result, it is moving away from the battlefield – in other words, it keeps troops, photographers, TV operators and journalists at a distance from the enemy. Then, when war comes down to earth, it becomes bloody, it loses its mathematical asceticism, and the feasibility of live broadcasting becomes impracticable for those involved.” These are the words of Bernardo Valli in *La Repubblica* newspaper on February 2^{nd}, 1991. Similar articles were written by many other journalists at the time of the Gulf war. “Bombing with surgical precision, following the fine ray of a laser, with sophisticated technology, with the circumspection and precision of science,” wrote Lidia Ravera in *L'Unità* on January 25^{th}, 1991. The idea is that war is based on high technology, that war is aseptic. This is the most terrible kind of war because everything seems to be based on a high-tech game, almost mathematical in its precision: a mathematical war.

These words in Italian newspapers made me think of the final paragraphs of Oriana Fallaci’s novel *Insciallah* [2] which I had read in August 1990 when Iraq invaded Kuwait. My curiosity was kindled by the fact that several reviewers had suggested mathematics as a possible key to understanding the book. I was interested in trying to understand this popular writer’s attitude towards mathematics.

In particular, I remembered *Insciallah* because mathematics in the book becomes a sort of evil *Deus ex machina* in the account of Italian soldiers sent to Lebanon a few years ago as a peace-keeping force. Even though the author emphasises the fact that mathematics alone cannot solve any of life’s problems, the theme of the book evolves around a framework involving a mathematical equation that expresses the eternal struggle between Life and Death. “The plot is interwoven with destiny which reason denies and which is confirmed by external forces beyond our control, beyond our freewill. The many people involved include the key person who seeks Life’s formula in order to fight against Death whose formula has been revealed. The dilemma, never mentioned but always present, finally explodes with the question: is Chaos really destructive – Chaos which according to the equation consumes Life; does Death really defeat Life?”

From the book’s epilogue: a motorboat packed with explosive material sets out to sink an Italian ship laden with troops. We don’t know what happens because the ending is left in suspense. One can’t say that mathematics makes a good impression in the book: it doesn’t answer any of the major questions posed by the author; it’s an unreliable science in which the truth of a statement and its opposite are equivalent; its only purpose seems to be for doing calculations that lead to destruction and death. Reading the newspapers at the time of the Gulf war, one found the same points of view put forward in different forms and with more precision.

I don’t believe that mathematics needs to be defended. However, it seems worth recalling another little book that I had bought in the same month of August 1990 while on holiday by the sea. It wasn’t, strictly speaking, a book but rather a slim volume conceived as a didactic unit to be used in schools with a guide for the teacher. The title is *La matematica della guerra* (The Mathematics of War, [3]) published by Gruppo Abele in Turin and written by a group of teachers[1] belonging to an anti-violence organisa-

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tion forming part of the "education for peace" project (see appendix A). Reading the newspapers in those days, one had the impression that modern warfare is based on mathematics, as if it were not men but computers that decided where to carry out "surgical operations". By contrast, the volume in question begins with the words: "Mathematics, like any other discipline, lends itself to building several paths towards education for peace." Four themes are dealt with, forming part of the wider issue of problems of war and peace: the arms race, nuclear deterrents, re-conversion of military budgets, and the statistics of war. The mathematical instruments used in the booklet are extremely simple; however, the choice of themes has enabled the authors "to highlight the power, in certain cases the ambiguity, of the mathematical models used to simulate war and conflict situations. In certain cases, the instrument and the mathematical model help towards a better understanding of the situation; in other cases, the model itself can lead to conclusions that are either wrong or unacceptable from the ethical point of view."

Of course, the framework of reference for the conflicts in the booklet's year of publication (1987) is the tension between the super-powers (so many years seem to have gone by!), and the authors did not attempt to show all the possible applications of mathematics to war situations. Some of the issues dealt with have to some extent been overcome by events that took place in Europe (in first place, the fall of the Berlin wall and the breaking-up of the Soviet Union), but nevertheless they retain their usefulness as laboratories of peace. The history of mankind teaches us that peace is never long-lasting and that it has to be sought day by day. If the initial aim of the volume was "to enlarge the horizons of traditional courses on mathematics" (it was written in 1987), in the light of what happened during the Gulf War several of the examples used are still interesting from many points of view. Game theory is used to illustrate the question of nuclear deterrents, in particular the "prisoner's dilemma" and the "chicken game" (see appendix B). In a subsequent chapter, the authors present a model of the arms race in which they examine the atmosphere of mutual distrust between the two opposing sides, leading to the secrecy that surrounds every improvement in technology. The case of anti-ballistic missiles (ABM) is quoted. One of the examples describes a military power that wants to protect its urban centres from missile attack with a level of efficiency of at least 50%. This ABM defence system, once it has been constructed, can easily be moved elsewhere. Therefore, even though the first nation's project was simply to protect its towns and cities, the other nation in the conflict situation doesn't trust this aim because it knows that the system can be transferred. This creates a situation of instability, and the second nation reacts by developing its own ABM system and an offensive system to counter the efficiency of the enemy's system by responding in the most elementary way: increasing the number of missile warheads. It is possible to obtain the analysis of such a model with simple first order linear equations and inequalities with two parameters. Amongst the factors considered are Cruise missiles. The conclusions of the chapter are as follows: "Two interesting results can be obtained from these models: first, the increasing sophistication of offensive and defensive weapons causes instability and accelerates the arms race; secondly, reducing the secrecy that surrounds the development of these new weapons will guarantee improved stability. It should be noted that the development of new weapons, even though they may never be used, is in itself destabilizing because the enemy power will not wait to see these weapons used; it reacts immediately by building ever-more sophisticated armaments. Because the problem of the arms race was very real in 1987, the model used is not a simple mathematical exercise; it can help to understand that more information about national security can bring more awareness to the decision making process."

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Anatol Rapoport is the author of *La teoria matematica della guerra di L. F. Richardson* (The Mathematical Theory of War by L. F. Richardson, [6]) and of *Strategia e Coscienza* (Strategy and Conscience, [5]), two books that served as sources for the didactic unit. Rapoport has written: "What mathematics can do and what common sense reasoning cannot, is to consider the overall causes and effects, sometimes interwoven in complex patterns, in order to extract the final effects. Often these models are deliberately made simple with full awareness that they do not represent real life situations. Their principal merit is that they produce interesting results which can then be compared with real life observations. More often than not, they do not agree with these observations but the nature and entity of the discrepancies often indicate possible directions for further research."

Amongst the interesting data provided, there are figures relating to World War I: 9.8 million casualties of whom 95% were military and 5% civilians. By contrast, in the Second World War, there were 52 million victims of whom 52% were military and 48% civilians. The figures are sharply inverted in the Korean War: 84% civilian casualties, 16% military. An Italian journalist, Fieschi, is right when he states that "mathematics is often surrounded by a halo of abstract purity; but we have to realise that there is no scientific discipline that can avoid being used in the military field." It is also true that there exists not only the supposed mathematical abstraction of technological warfare, but that the mathematical instrument is useful for understanding, and principally for preventing on some occasions, the effects that our non-mathematical decisions can produce.

A final comment

The idea that mathematics is somehow aseptic, an objective and infallible instrument, has deep roots. An analogous idea is often projected on economic models, as if the initial choices that determine the evolution of the model, and indeed, the choice of the model itself, were not actually political choices. Instruction in the modelling of phenomena involving human choices must include the instillation of an awareness of the power and ambiguity that are always a factor in mathematical models. The small book [3], even though actually conceived to serve a political situation vastly different from that existing today, is an excellent example of how to provide the tools necessary to understand how mathematical models can be used to promote a greater comprehension but, at the same time, can lead to conclusions that are unacceptable on a moral level. In the final analysis, the choices are determined by human decisions, not by the pretense of objectivity put up by a mathematical model. This is a very important message for an age in which technology seems to be an end rather than a means.

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Appendix A

From the introduction to the volume ([3]):

Mathematics, like all disciplines, lends itself to the preparation of ways for teaching about peace. The present didactic unit was organized by choosing four specific cases that permit, through mathematical modelling, the analysis of general problems of war and peace (the arms race, nuclear deterrence, reconversion of military spending and military statistics).

This choice helps shed light on the power and, in certain cases, the ambiguity, of the mathematical models used to simulate situations of conflict and war. In some cases the tools and the mathematical model itself contribute to a greater comprehension; in others the model may lead to conclusions that are wrong or morally unacceptable. Each of the proposed cases is independent of the others and may be treated separately. The necessary requisites permitting the development of each argument are also different from case to case. In the first example, the elementary knowledge of the game theory necessary to understand the example is presented in the text itself and in a brief appendix. The bibliographical references permit further study and integration whenever more research is deemed desirable.

In contrast, the second example requires a knowledge of the subjects usually covered in the first two years of high school: linear equations, first degree systems and the means for representing them on the Cartesian plane, inequalities. For the third example, appropriate for more general treatment, it is necessary to know some elements of the theory of graphs. In this case as well, the text and the appendix contain all the information and references necessary. We are dealing, then, with an exercise drawn from the experience of Emma Castelnuovo in a ninth grade class (the third year of middle school) (Castelnuovo and Barra, 1976, [1]) and therefore one that is certainly suitable for repetition at the high school level. Finally, the fourth case requires the knowledge of the concepts of logarithms and some very general points of statistics.

With this unit we did not intend to deal exhaustively with all the possible applications of mathematics in situations of war. The choice we made seemed sufficient to permit the enlargement of the cultural horizons of traditional mathematics courses, offering new stimulation. The unit's structure in itself permits the introduction of some or all of these arguments without requiring any particular changes in the program or schedule of the lessons.

For general references to the problem of the arms race, we refer you to the unit of Jannamorelli, *La corsa agli armamenti* (The Arms Race), in this same publication, while a good introduction to the general methodological criteria and to the debate on the teaching of peace is to be found in the *Guida metodologica del progetto "Scegliere la pace"* (Methodological guide to the project "Choosing Peace")(Novara and Ronda, 1986, [4]).

Appendix B

Two examples where game theory is used to illustrate the question of nuclear deterrents, the “Chicken game” and the “Prisoner’s dilemma”.

The “chicken” game

In the traditional game by this name, two children hop towards each other until they have to decide whether to bump into one another or to avoid contact. In the book *La Matematica della guerra*, this game is used as a model for a situation of attrition that might lead to open conflict. Two nations, N1 and N2, have the choice of co-operating C and not co-operating NC. The possible choices are CC (both co-operate), scores awarded in the game (3,3); one nation wants to co-operate, the other not, or vice versa, that is C NC or NC C; of course, the nation that co-operates when the other does not, has an advantage (4,2) and vice versa (2,4). Lastly, if neither of the nations co-operates, we have the maximum danger for both NC NC, score (1,1). The only reasonable possibility, if only one choice is permitted, is CC in which neither side has a significant advantage (3,3), as would occur with (4,2) or (2,4), or with (1,1) and the risk of catastrophic damage. This is the choice of the best amongst worst cases, known as the “maxmin” principle. It is quite likely that this was the strategy of the powerful nations, starting at the Yalta conference, leading to co-operation at the expense of smaller nations.

The prisoner’s dilemma

Discovered in the 1950s and formalised by A.W. Tucker, this dilemma exists in several versions. In this publication, it is used as follows: there are two nations N1 and N2 both of whom can choose between two strategies, not arm themselves A or arm themselves B. The possible combinations are AA in which both remain unarmed with the highest advantage (4,4); choices AB or BA in which one nation remains unarmed and the other not – the nation that arms itself has a significant advantage (3,1), the other a disadvantage (1,3). Lastly, both nations decide to arm themselves, score (2,2) since in this game there is a dominating strategy for both sides which leads to the best result whoever the enemy might be. It is clear that, for both sides, strategy B (arming themselves) is the dominant one; therefore, the “rational” result is (2,2). However, the AA choice is the one that provides most advantages for both – bilateral disarmament, the route taken by the two super powers after having given priority to the BB choice, the arms race.