Mathematics Teaching *is* Democratic Education[1]

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Abstract: It is a commonly held belief that mathematics teaching has no political effects. Astonishingly, however, the fact is that the style of argument now used in mathematics everywhere was not developed originally to do mathematics. Originally its function was to counteract the teaching by the early Greek sophists of rhetoric. Their training gave the rich and privileged such an advantage in public speaking that democracy was threatened. Making respectable a new form of argument, in which evidence and logical structure predominated, was a very radical act of enlightened democratic education. Mathematics teaching in the form of open critical dialogue between teacher and taught remains a powerful form of education in democratic attitudes. Ambitions to produce political ideas as infallible as mathematics have a modern origin. In the early part of this century, mathematics education was again becoming universal throughout Europe. In the same period the belief arose that mathematics could eventually be completed as a single structure of truth. This transformed mathematics into a paradigm of democracy in which unorthodoxy must necessarily be eliminated. Communicated to people everywhere by universal education, this belief increased respect for similar political ideas. Gödel's proof that mathematics can never be completed came too late to correct these political effects, but modern teachers can again use mathematics as a proof of the value and success of democratic attitudes and ideas. Whilst mathematics itself is ethically neutral, the ethical principles which produced both democracy and mathematics and which can be conveyed in mathematics teaching are highly relevant to the modern world, and should be understood and taught by teachers everywhere.

Kurzreferat: *Mathematikunterricht* ist *demokratische Erziehung.* Nach gängiger Vorstellung besitzt der Mathematikunterricht keine politischen Auswirkungen. Erstaunlicherweise wurde jedoch der Argumentationsstil, der heute in der Mathematik üblich ist, ursprünglich nicht nur für die Mathematik entwickelt. Die Funktion dieses Argumentationsstiles war es, ein Gegengewicht gegen den Rhetorikunterricht der frühen griechischen Sophisten zu schaffen. Durch deren Training erhielten Reiche und Privilegierte einen derartigen Vorteil für ihre öffentliche Reden, daß die Demokratie gefährdet war. Einer neuen Argumentationsform Ansehen zu verschaffen, in der Beweis und Logik dominieren, war ein außergewöhnlicher Akt aufgeklärter demokratischer Erziehung. In der Form eines offenen und kritischen Dialog zwischen Lehrenden und Lernenden ist der Mathematikunterricht eine effektive Form der Erziehung hin zu demokratischen Einstellungen. Der Wunsch, politischen Ideen den gleichen Grad von Sicherheit zu verleihen, den mathematische Ideen haben, ist modernen Ursprungs. Als am Anfang dieses Jahrhunderts die mathematische Erziehung in Europa wieder allgemein üblich wurde, entwickelte sich gleichzeitig auch die Vorstellung, daß Mathematik als geschlossenes System unangreifbarer Wahrheiten vollendet werden könnte. Diese Vorstellung verwandelte die Mathematik in ein Paradigma einer Staatsform, in der unorthodoxes Denken notwendigerweise eliminiert werden mußte. Wenn dies aber dem durchschnittlichen Bürger überall als Allgemeinbildung vermittelt wurde, dann verstärkte es das Ansehen ähnlicher politischer Ideen. Der Beweis Gödels, daß Mathematik gerade nicht vervollständigt werden kann, kam zu spät, um diese politischen Auswirkungen zu korrigieren. Mit modernen Lehrmethoden aber kann heute der Unterricht der Mathematik den Nachweis für den Wert und Erfolg der demokratischen Einstellungen und Ideen führen. Obwohl Mathematik ethisch neutral ist, sind die gemeinsamen ethischen Prinzipien, die sowohl Demokratie als auch Mathematik schufen, für die moderne Welt äußerst relevant und sollten überall von Lehrern verstanden und unterrichtet werden.

ZDM-Classification: A30, A40

1. Introduction

"We alone," said Pericles, the leader of Athenian democracy, in 431 BC, "regard a man who takes no interest in public affairs, not as a harmless person, but as useless. Whilst few of us are original in our thinking, we are all sound judges of a policy. In our opinion, the greatest obstacle to action is not discussion, but the lack of knowledge gained by discussion before action is taken." And he added: "Our city is open to the world. We never prevent any foreigner from seeing or learning any secret that might profit an enemy if he knew it" (Bury, 1900). How amusing that was. It is likely that Pericles and his audience knew perfectly well that the greatest secret of Athens' strength was visible and audible most of the time. For all her citizens to be trained "judges of policy", being thus able to discuss and determine possible alternatives before any action was needed; if every person was then able to act in the best way co-operatively, but also if necessary alone – this was their greatest strength.

The kind of discussion that this required was intensely practical and realistic, with every person stating his opinion as simply and strongly as possible, and it was developed in Athens. The Greeks called this *techne logos:* rational debate, from which comes the word technology. From this beginning, in the need for effective *political* discussions, grew what we now call mathematics.

A radical appraisal of mathematics teaching has been taking place in Europe in the past few years. This has been accelerated by the TIMSS survey published in 1998, but a first step was the lengthy Danish investigation under Gunhild Nissen which pointed out that modern citizens are unable to understand the work of their own governments without an adequate degree of mathematics education, and that depriving people of this education is seriously to limit their participation in democracy.

We propose, however, much more than these conclusions.

Belief in democracy is wide-spread, and is growing, but it is tempered with anxiety about democracy's ability to survive in an increasingly disordered and dangerous world. One question we need to address, therefore, is how to improve people's confidence that in choosing the difficult path of supporting democracy they are making the right choice.

My own understanding of mathematics' part in implicitly supporting democracy has largely resulted from teaching mathematics for over twenty years. I also remember as a young boy listening to a science teacher carefully explaining an experiment, and realising that his explanations not only communicated information, they also meant that people's opinions matter; and that, if their opinions are to be changed, this must be achieved by persuasion, and not by force.

At an early age I was therefore beginning to understand that the basis of democracy is shared respect between people. I also understood that this respect is not to be found in equations or formulae. It is a moral code operating between the teacher and the taught, ultimately between the rulers and the ruled. I suggest principally that this morality is what mathematics really is: not its results, but the moral agreement between human beings to work together, as near as possible as if all are of equal importance.

In the entrance of Trinity College Chapel, Cambridge, there is a sculpture of Isaac Newton which is worth seeing. Amongst the mathematicians of the 17th and 18th centuries it was a common belief that they might read the language of God in mathematics, the *logos* by which God created the Universe. Isaac Newton certainly thought so. Besides discovering differential calculus and his new laws of dynamics and gravity, Newton spent much of his life attempting to read the mind of God even more directly, by decoding numerical relationships which he believed could be found in the Bible. Some modern mathematicians go even further than Newton. They have suggested that understanding fundamental physical laws would allow those who can do so actually "to read the mind of God".

My feeling is that this search for the language of God in physics or mathematics – or for a glimpse into the mind of God through any understanding of the material universe outside of man – is basically misguided. If one expects to find the language of God written somewhere in the world, I think one should look for it between people.

I want therefore to show you a relationship between people which is, in itself, quite extraordinary – all the more so because it is at least 2500 years old. It is this relationship called democracy. Democracy is so surprising because it is so unlike anything one could predict of our "selfish genes" working only to improve the chance of their own reproduction. Democracy is odd because it depends on trust and respect between people extending beyond the limits of family and tribe, to include all the members of an entire society. To be productive, what this mutual respect then needs is some kind of systematic, clear and open argument by which people can communicate and co-operate with each other intelligently. In other words, it needs the sort of argument used in mathematics.

There are therefore two interacting factors. One is this all-important style of clear and open argument. The other is the democracy that already is willed to exist, and which it will support. Each is really a co-factor of the other.

2. The evidence for mathematics as the co-factor of democracy

If mathematics has always acted as the co-factor of democracy, we should want to show that – contrary to common opinion – mathematics is invariably a strong shaper of political ideals, and that this is especially true when it is taught throughout a society. More precisely, when mathematics is taught by means of continual open dialogue between free individuals, we should be able to show that it encourages healthy democracy. The evidence for this relationship is of three kinds: pedagogical, social, and historical.

2.1 The pedagogical evidence

The rules which we use in mathematics today were familiar to the Greeks in their discussions over two thousand years ago. It was a time of bitter conflict for democracy in Greece between its champions, like Pericles, and its critics, of whom Plato and Socrates are perhaps the most famous. Socrates is quite rightly regarded as the great champion of individual inquiry and thought, the basis of Western spiritual and intellectual endeavour. According to Plato's account, however, he was never reconciled with democracy. The offences for which a democratic assembly eventually compelled him to end his own life seem trivial to us today. His peers never expected that he would choose death rather than face temporary exile. It is also very possible that suspicion against him was more serious than the charges. For decades Socrates' self-appointed business had been to be the "gad-fly" of the Athenians. By the year of his trial in 399 BC he seems to have been regarded as a constant critic of democracy – claiming, as he frequently did, that only those should rule who were born to rule. More sinister still was the suspicion – unvoiced at his trial – that he inspired intrigue against democracy itself. After their defeat by Sparta in 404 BC the Athenians had an oligarchy of some thirty Athenian aristocrats imposed on them who, it was claimed by a contemporary, killed in eight months more Athenians that Sparta had in ten years of war. A chief amongst the tyrants, incidentally, was Plato's uncle Critias, who had also been a pupil of Socrates.

In 403 BC the Athenians restored their democracy, and began – with astonishing forbearance – by forgiving everyone who had supported and benefited from the tyrants. Only Socrates, apparently, eventually irritated them too much. About some details of their history, therefore, we know a great deal. About other details we know almost nothing. We do not know, for example, if anyone taught the Athenians their rules of democracy, or if, as is more likely, they evolved, how long the process took.

We do know how they used and modified the rules in a number of contexts. Mathematics, for example, was taught according to these rules: I would like to call them moral axioms:

2.1.1 Teachers must treat their pupils, and the pupils must learn to treat each other, as intellectual equals.

2.1.2 All the teacher's arguments must be openly and completely explained.

2.1.3 The teacher's arguments are only confirmed as satisfactory by their pupils' free understanding and assent.

Each of these rules is extraordinary – all the more so in the context in which we find them. None of them are what may be called the normal rules of early Athenian society. They are ideal rules. The normal social rules were very different. The Athenians were strongly tribal; class-conscious; slave-owning; women subordinating. Much of their time was – necessarily and unnecessarily – given to war or to the preparation for war. They were intensely proud of themselves, and despised almost everyone else – except Sparta, which many admired as the perfect military society. Of course they did not use these ideal rules in all discussion; nor did they use them only in what we now call mathematics. What is important to us, however, is the very close resemblance of these rules governing the teaching of mathematics, and the rules governing their democracy. Let us look at these.

2.2 The social evidence

Democracy obeys these moral axioms:

2.2.1 Political leaders must treat their people – and they must learn to treat each other – as political equals.

2.2.2 All the leaders' policies must be openly and completely explained.

2.2.3 Their policies are only finally confirmed as satisfactory by the people's free understanding and assent.

There *is* a very close resemblance between this style of teaching of mathematics and the practice of democracy. Could it be only a coincidence? Let us look at them side by side:

The Pedagogical evidence	The Social evidence
2.1.1. Teachers must treat their pupils, and the pupils must learn to treat each other, as intellectual equals.	2.2.1. Political leaders must treat their people – and they must learn to treat each other – as political equals.
2.1.2. All the teacher's arguments must be openly and completely explained.	2.2.2. All the leader's policies must be openly and completely explained.
2.1.3. Their arguments are only finally confirmed as satisfactory by the pupils' free understanding and assent.	2.2.3. Their policies are only finally confirmed as satisfactory by the people's free understanding and assent.

It *could* be only a coincidence.

Mathematicians have noticed a part of this similarity before. In 1988, for example, André Lichnérowicz, one of France's most distinguished algebrists, wrote, "Mathematics was created for us in ancient Greece by men who conceived a type of argument which would be without misunderstanding or ambiguity, an argument capable of persuading any kind of person, citizen or slave, Greek, metic or barbarian; an argument capable of achieving agreement because its very form forbids disagreement."[2]

Classical scholars have also long proposed the reason why this style of argument was developed: as a reaction against the vagaries and uncertainties of professional rhetoric, and of oracular magic: both of which, apparently, could prove anything, or nothing, often depending on the size of the fee that was paid. Here for example is the explanation of a distinguished classicist, Geoffrey Lloyd, summarising the factors which brought about the development of scepticism and critical analysis in Greek thought. He has already dealt with the vagaries of magic. Here he is concerned with political debate.

"The new professionalism in the art of speaking, provoked hostile reactions from such writers are Aristophanes and Plato. ... The citizens of Athens had ample opportunity to exercise their judgement of skilful argument: but by the end of the fifth century they were also being frequently warned, by different speakers and in different contexts, not just against those who set out to make the worse appear the better cause, but also more generally against rhetoric itself. ... The institutions of the city-state called for new qualities of leadership, put a premium on skill in speaking and produced a public who appreciated the exercise of that skill. Claims to particular wisdom and knowledge in other fields besides the political were similarly liable to scrutiny, and in the competition between the many and varied new claimants to such knowledge those who deployed evidence and argument were at an advantage compared with those who did not."[3]

This, therefore, is how distinguished scholars in the two very different fields have noted the appearance of this style of argument, and, each with a different perspective, has carefully explained its purpose. But no-one, apparently, has asked precisely who benefited most from this new style of argument. In form, at least, it was a very simple style of argument. It was also very powerful. Everyone could learn it far more easily than one could become proficient in rhetoric, and at almost no cost. Once learnt – as Lichnérowicz noted, because it convinced because of its form alone – it could be applied to many different kinds of problems.

This was truly revolutionary. Almost contemptuously it swept aside political and legal advantages that the rich had enjoyed in hiring private tutors, speech-writers, and clever lawyers who could turn even bad cases into good. It reduced the advantages of the richer businessmen and traders as well, who could also afford such assistance. Instead this invention gave everyone the means to argue in a successful but simple way, and increasingly rhetoric came to be less well regarded, whilst this form of argument – techne logos – came to be respected.

And so the answer to the question who benefited most, is that the poor and low-born and less well-educated benefited most. But democracy was strengthened for everyone – and ultimately this also benefited everyone. Most astonishing of all is that the rich and powerful ultimately paid for this development and saw to it that this new power was transferred to all the people freely through public schools and demonstrations. It was the greatest act of enlightened social engineering in history. Certainly it was the most momentous. For the first time, the weaker social classes were invited to contribute their ideas to government – and the strongest gave them the means to do so.

In Europe, in Germany, the Württemberger Friedrich List was suggesting this route in the early 1800s, proposing universal free education to transform the German working classes. Ironically it was List's proposals – not those of his countryman Karl Marx – which caused Metternich, to call List "the most dangerous man in Europe". Eventually, frustrated, exhausted and ill, List took his own life in 1846. He was 57.

Some scholars have called Chancellor Bismarck List's true disciple, and certainly he followed many of List's ideas. Ending internal tariffs, universal suffrage and universal education – with a great emphasis on technology – opened the path for Germany to become a modern industrial state and world power.[4] Far more sadly, the British government's rejection of List's earlier plan, to combine the British and German economies, the London Economist wrote, "greatly contributed to draw Germany into the field of manufacturing industry by rejecting her agricultural produce; or, at least, by refusing to admit that agricultural produce at uniform and regular rates of duty. It is too late. Germany, wrongly or rightly, is now determined to be a manufacturing and a commercial power, and she will not recede from her resolution."[5]

But, let us pause. We have still not really proven that there is a causal connection between universal education in mathematics and the improvement of democracy. To be honest, I don't think we can. One may use one's judgement as to whether it is likely or not, fall back onto the old presumption that the exact similarity of their moral structures is just an accident. To prove, scientifically, that two factors are linked causally, we should need to change one factor, and to show that other changes follow both consistently and as predicted. Clearly to do this convincingly would require an experiment on the scale of Europe. It would need to last for several generations, and it would need to produce a clear range of related and consistent results, all showing that democratic behaviour does change in parallel with mathematical training. If they changed in the same way, we might be able to believe that they are linked. This huge experiment is impossible. But we can examine instead a later historical coincidence, which in effect is precisely the experiment we envisaged. A new style of mathematics *was* taught in Europe, and political ideals *did change* in the same way. We can ignore one coincidence. But two?

2.3 The historical evidence

Democracy is a human activity. It has extremes. Single-minded democracies are entirely possible. For example, in a 17th century New England Puritan community a man's wife told the elders of their church that he talked in his sleep. When angry, she added, he threw peas about the house. He was found guilty of witchcraft, and was hanged. Such communities generally do act swiftly to remove differences and silence dissent. This is very efficient. But ultimately such communities find it difficult to change from one course of action to another, simply because other options are so rarely considered.

The other extreme must be democracies in which there are almost as many options as people. The difficulty here is to learn how to manage this variety. But a democracy which does learn will have a great reserve of options in response to change. This is slower, and seemingly less efficient, but in the long term history has shown that it is more efficient than depending on only one man – or one idea; on putting all of society's eggs in one basket.

And yet how, exactly, does mathematics teaching help produce this kind of individual and social pluralism? How can it produce variety and flexibility, respect for dissent, and, most important of all, respect for dissenters? Using mathematics as a model for society seems to be moving in the wrong direction. Many people seem to think that this must be the road to extreme authoritarianism: to dictators and tyranny, to death camps, gulags, and killing fields. And they are not wrong. These are *precisely* the lessons of history. Throughout Europe in the 19th century, and well into the 20th, mathematics was taught with increasing confidence as a one-option science. The expectation was that it would soon be the first science of mankind to be "completed". By the early 20th century it was being taught both explicitly and implicitly in every school and university, just as in previous centuries it had been taught that Christianity was the first complete religion. There was, indeed, a strong resemblance.

Mathematicians cannot be blamed entirely for this evolution of their ideas. They were the heirs of Newton, and before him, of Galileo, and before him, of Aquinas – and long, long before him, of Plato. For there is, Plato wrote, in mathematics "something which is necessary and cannot be set aside ... and, if I mistake not, of *divine* necessity; for as to the human necessities, of which the Many talk in this connection, nothing can be more ridiculous than such an application of the words"[6]. It was Aquinas who first suggested that some of God's creation might eventually be understood by human minds, and that this "natural theology" would furnish more evidence of the existence of God. It was natural to expect that it would be in one language. Galileo first insisted that it would be mathematics.

By the end of the 19th century all of the evidence seemed to show that mathematicians were on the right track. Of course they had to ignore a few individuals. There was that wicked Georg Cantor in Halle, for example, whose concept that there are ever higher levels of numbers for which the mathematics of the known set of numbers might simply not work, threatened their whole system of ideas. Poor Cantor. Now known to have been one of the most powerful thinkers of mathematics, in his lifetime the great Henri Poincaré called his work "pathological" and "a disease". To Leopold Kronecker, even more influential in Germany, he was simply a charlatan. To meet these criticisms, wrote Morris Kline more sympathetically: "Cantor invoked metaphysics, and even God" (Kline 1980). Exhausted and disturbed Cantor died in a mental sanatorium in 1918.

But at the Second International Congress of Mathematicians, in Paris in 1900, despite some misgivings that not all the foundational problems of mathematics had been solved, the mood was triumphant. "Have we at last attained absolute rigor?" Poincaré asked. "At each stage of its evolution our forerunners believed that they had obtained it. If they were deceived, are we not like them also deceived?" His conclusion was pure hubris. "One may say today that absolute rigor has been attained."[7] The Congress envisaged a programme for the completion of mathematics as a single structure of truth. This structure would contain not only all the mathematics known, but all the mathematics ever possible. Built on undeniable axioms, using immaculate logic, when complete it would be able to produce final solutions to all well-defined mathematical. They could not imagine failure. It was only a matter of time – of devoting sufficient courage, of accepting sufficient sacrifice, to see this project through.

And what effect could this immense self-confidence have on other systems of thought? Clearly against its standards all other systems of thought must be compared. But if mathematics could become one single system of truth, deriving all its solutions from one set of axioms and one tightly defined system of logic, surely *other* systems of thought must try to do the same. Including political systems? By 1845 Marx had written his definitive science of history.[8] In 1848 he published the *Communist Manifesto.* In 1870, as Magnus Nieger has pointed out, the Catholic

Church asserted the completeness of its own system of thought in the doctrine of papal infallibility.[9]

A new political class was emerging in Europe with its own upper, middle, and lower orders. They were the scientifically trained managers, supervisors, and workers of the industrial society. Imprecision, uncertainty, chance, these were acceptable no longer. Even if they understood mathematics only imperfectly, they knew its essential characteristics, and so they knew what they wanted. The mathematics they had been taught had a structure of unchallengeable logic, built on axioms of perfect truth. It was an outcome of history, was evolution itself. They admired its power to control and direct with perfectly predicted results. They understood as well the meaning of rigor: the effort, the discipline, the ruthlessness with which a system's logic must be followed to achieve its final end. The cost might be great, the prize immeasurably greater. Once the system was perfected – even in their lifetimes – there would be an end to doubt.

An end to doubt: an end to dissent: an end to conflict: an end to injustice? As the millions who survived the First World War found massive unemployment and huge political confusion, and as the old orders were found guilty of stupidity and incompetence, these were very thrilling ideas, ideas of hope and promise.

They were also obviously true. Every village schoolmaster, every high-school teacher, every university lecturer, had taught and was still teaching that scientific progress would be achieved just as mathematical progress would be achieved, through applying rigorous logic to every problem without exception. If the logic was correct, if the effort was relentless, any problem must be overcome. This was how both logic and rigor could be proved themselves to be correct: by overcoming every problems, by accepting every challenge, by fighting every enemy, if need be on all fronts at once.

Few could resist the excitement. Political theorists soon found that they could attract new supporters by promising astonishing results. So long as they claimed their ideas were based on science, any science, on logic, any logic, there seemed no limit to the credulity they could arouse. Hitler was astonished by his success. But at both ends of the political spectrum theorists claimed that they, their theories, would bring an end to doubt, dissent, conflict; bring freedom, justice, peace to all. It mattered little what the theories themselves contained; nor even – when the time came – how they were applied. To achieve the goals they wished for, people were convinced already that they must accept their leaders' system of logic, and then use "absolute rigor" to overcome all obstacles. The terms were easily translated into actions.

I do not suggest that mathematics created totalitarianism in Europe. But undoubtedly mathematics helped shape the imagination that found totalitarian ideas attractive. "You will *never*," Hitler told party-workers in 1938, "be free again." And they cheered. This same ambition inspired in almost exactly the same way political systems with no common feature: not cultural, historical, national, not even religious. In Russia, the exactly same pattern emerged. Whole societies were obliterated; whole nations deported and starved. Such tyranny was supported by the belief of millions that only by eliminating all dissent and all dissenters, only through the sacrifice of their own peace and freedom, only through this ruthlessness would they achieve the perfect state.

What was the common factor? There was only one: the people's belief, even as they suffered from it, in the necessity for ruthless political action. This was the exact parallel of the belief that mathematicians were teaching. Mathematics itself would be completed by applying one set of axioms and one logic to all its problems. Human emotions could not be allowed to intervene. This ambition transformed mathematics from being a model of democracy allowing many kinds of freedom to being a model allowing only one kind of freedom: the freedom to agree. By the middle of the 20th century this idea had been communicated throughout Europe as the standard of many intellectual, moral and political programmes. Correspondingly it was believed that a new chapter of human history was about to be written. From it a new type of human being would emerge – Aryan or Soviet – which must inevitably dominate the world.

That mathematics teaching is not politically neutral was fully apparent to Plato over two thousand years ago. His solution was for mathematics to be taught only to the political elite. It would help them to experience the necessities of pure existence, and help them keep everyone else in the place allotted to them.[10] So far as I know, after Plato these connections between democracy and mathematics were not mentioned until 1993, when the first of my papers was published in Germany by the Arbeitskreis Mathematik und Bildung of the Gesellschaft für Didaktik der Mathematik, the working party on mathematics and education for the Association for Mathematics Teaching.[11]

Non-Germans tend to see Germany as a large, powerful, confident country at the centre of Europe. It has been its greatest industrial power since the early 1900s, and inevitably must be politically dominant. Germans know that Germany was also Europe's battle-ground for centuries. To explain their obsession with rules, order and security, one only needs to reflect on their centuries of experience of rebuilding their shattered towns, burnt houses, ruined farms.

Germany repeatedly produced spiritual and secular thinkers who have given their lives for freedom of thought and expression. It has one of the most turbulent democratic histories in Europe. But Germany has also repeatedly abandoned and destroyed these same thinkers and leaders. By 1939, for example, Hitler had killed, imprisoned, or silenced every voice in Germany which did not give him unqualified support. All dissent was weakness, and weakness must not be allowed. The agreement of millions of ordinary Germans with these ideas began in their history, but it was completed in their classrooms when their mathematics teachers taught them the same idea.

Mathematics cannot be politically neutral. Plato was accurate in his fears. Only a little training in good mathematics gives people more confidence in democracy. This is because good mathematics teaches pupils to listen, to think, and to argue more effectively; to respect others always and to accept ideas which at first they do not understand; and even to accept decisions which they do not like or respect. Democracy depends on attitudes like these.

Just as the two great totalitarian systems in Europe were moving inexorably towards war – which both, incidentally, agreed was inevitable – a young man in Vienna in 1931 proved that mathematics cannot be one system of truth. In one of the most extraordinary moments in mathematics' long and crowded history a young Austrian Jew, Kurt Gödel, published a paper which showed mathematicians that to complete mathematics is not possible. He called this the *Incompleteness Theorem.* Within a few years his argument was known to mathematicians and scientists everywhere. It was an extraordinary example of the true democracy of mathematics: that this young scholar' proof was accepted despite his youth, despite his unimportance, despite the fact that it destroyed the life-long work of many famous men. It used, incidentally, insights which can also be detected in poor Cantor's proof of the existence of transfinite numbers. His theorem – and its proof – destroyed at once the dream that mathematics, one day, could be completed as a perfect structure. Suddenly there was no support for the notion that any logical system could solve the problems of any complex reality. Mathematics ceased to be a model for one-option democracy. It reverted to being a model for multi-option democracy.

2.3.1 Democracy may allow many options or one; and mathematics may allow many options or one.

2.3.2 By the early 20th century mathematics had become strongly deterministic. One system of axioms and one logic was believe to be enough to complete system of all mathematics

2.3.3 By the middle of the 20th century Europe was dominated by two opposing political systems. Both claimed to be the complete truth and both outlawed all dissent.

2.3.4 As the realisation spread – post Gödel – that mathematics cannot be completed as a perfect system, faith began to fade in perfect political systems. Mathematics again became human. Democracy was again a system intellectuals could respect.

There are now many mathematics. It is again a plural activity, much more like a society of people, indeed, much more like the real world. In parts deeply conservative, austere, inhumanly immaculate and certain, in others untidy, adventurous, confused. Very human. More mathematicians are working and teaching in the world than ever before They produce more new mathematics than history has ever known. They communicate, they criticise, they co-operate, better than ever before. And they work democratically.

3. Conclusion

Over two thousand years ago the Athenians made a unique contribution to the evolution of the human spirit – and human society. They were the first to make mathematics intensely human. They gave it an intensely important social function. They showed the divine necessity, as we should understand it, of people learning to agree in order to co-operate, both to strengthen their society and to make it prosper.

If children are taught mathematics well, it will teach them much of the freedom, skills, and of course the disciplines of expression, dissent and tolerance, that democracy needs to succeed. If, on the other hand, they are taught mathematics as if it has no room for independence; as if they must never question, doubt, or disagree; and if we therefore fail to teach them to respect and value those who have different ideas – or wrong ideas – or even no ideas at all (as Socrates insisted he had none) – then we can do more than damage their mathematics. For this kind of mathematics teaching destroys democracy. It does exactly as Plato preferred. It creates a divided society: above, dominant oligarchy; below, separated from government not by any regulated status but – far worse – by their own conviction – people who do not believe they can safely govern themselves.

In contrasting Athenian democracy with Sparta, J.B. Bury writes of the admiration felt by many Athenians for Sparta's rigid simplicity, for its "citizen absolutely submissive to the authority of the state, and not looking beyond it". This, too, was Plato's ideal; and when the name of Socrates, his intellectual mentor, is used for Europe's most far-reaching education programme, we, who are the people of Europe, are right to be wary. But Bury concludes that Socrates' directed suicide by a democratic assembly "was the protest of the spirit of the old order against the growth of individualism" (Bury 1900).

Of the old order? There is no doubt that Socrates disliked democracy. But he may still represent to us the primacy of individual thought over and against the primacy of the State and over and against what Plato called the Many. Behind his certainty in the essential importance of his own thoughts, doubts and questions, lay – as he quietly explained at his trial – the constant assurance of his daemon: of his own individual incommunicable spiritual insight.

I would like to suggest that almost everything of importance to democracy emerged from the synthesis of these two ideas: the spiritual basis of individuality and the need for rational consensus. If mathematics contains no spiritual insight, it is also true that wherever it uses its original habit of open and critical discussion, as if between equals, it demonstrates the power of democracy to reach into the heart of problems, to eliminate obscurantism, to combine people's energy and courage, and to produce the solutions that all eventually can accept. It also protects the dreamer and allows his dreams.

This is what mathematics does. And this is therefore what it is: not so much a rational as a *moral* adventure.

4. Annotations

[1] This is the fundamental historical thesis of the Comenius project "Mathematics Teaching and Democratic Education" of the European Union, directed by the Landesinstitut für Erziehung und Unterricht Stuttgart.

[2] "[La science est née pour nous dans l'ancienne Grèce d'hommes qui] conçurent le projet d'un type de discours sans quiproquo ni malentendu, un discours cohérent et contraignant pour l'autre quel qu'il soit, citoyen ou esclave, grec, métèque ou barbare, un discours capable, par sa forme même, d'interdire le refus de son contenu." The last words were underlined by the author; (in an article, "Universitalité des

mathématiques et compréhension du réel" quoted by Didier Nordon in Nordon 1993).

[3] Lloyd 1986, pp. 264–266. Lloyd adds to this sentence, "at least – to repeat our proviso once again – so far as some audiences and contexts were concerned." The text makes clear that these remarks apply especially to the political field.

[4] Biography of Friedrich List in Wendler 1996, especially concerning the thesis of Jules Domergue.

[5] ibid, quoting The Economist is of the 27th September 1845.

[6] Plato, Laws, my italics; quoted by Bertrand Russell in Philosophical Essays, Longmans and Green, 1902.

[7] Kline 1980; and also in Henri Poincaré, The Value of Science (1905).

[8] Though this – his German Ideology – was not published fully until after his death in 1883.

[9] Magnus Nieger in a forthcoming paper of the Comenius-Project.

[10] Plato, The Republic.

[11] Hannaford 1993. Sent with the recommendation of Eugen Wendler, founding director of the List Institut in Reutlingen, to Carl Friedrich von Weizsäcker, one of Germany's foremost physicists and distinguished ethical scholars, it was praised by him as "an important contribution to understanding the connection between the intellectual, moral, and political problems of our world". I shall ever be grateful for their immediate understanding and support.

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