The Possible and the Impossible

Martin, George E.:

Geometric Constructions

New York: Springer, 1998. – 214 p.
ISBN 0-387-98276-0

Shlomo Vinner, Beersheva (Israel)

The mathematics education community should be grateful to George Martin for his wonderful book. It should also be grateful to Springer-Verlag. It is quite rare now that a publishing house will undertake such a project as this one, namely, publishing classical stuff which is not a text book for a compulsory course. It is a topic which started with ancient Greek mathematics and kept some great mathematicians busy until the 20th century (the text includes a theorem by Gleason published in 1988). The ancient Greek undertook a challenge which in a way represents some of the most typical features of pure mathematics as an abstract discipline. It is not related to any practical need. Basically, it is an intellectual game. One principle of the purely mathematical game is the principle of minimalism. The task is to obtain as much as possible by using very minimal means. The minimal means in the case of Greek geometry is the ruler and the compass. The task was to construct various geometrical entities, as many as possible, by a ruler and a compass. This implies that measuring is not a means. This also implies that numbers become inessential. The Greek were quite ambivalent about numbers, probably because the irrational numbers undermined their mystical belief that everything in the world can be expressed as a ratio between two whole numbers (the cosmic harmony). Whether this view is true or not an astonishing geometrical edifice was erected where numbers are replaced by all kinds of ingenious geometrical ideas. The goal is to show that much can be achieved. This was already shown by the ancient Greek. Additional contributions to this domain were made by Gauss and others many centuries later. However, there were some unaccomplished tasks like trisecting an angle, duplicating a cube or squaring a circle. Mathematicians did not know how to do it by using a ruler and a compass only. Nevertheless, it took them centuries to realize that perhaps they fail to do it because it cannot be done. They fail not because they are not clever enough. Even God, who is supposed to know everything, is not able to do it. The laws of mathematics imply that this cannot be done, and God, whether being a Geometer or not cannot accomplish a mission proved by mathematics as a mission impossible. Ironically enough (to the Greek tendency to avoid the irrational numbers), in order to prove that some constructions are impossible by a ruler and a compass requires quite sophisticated tools from abstract algebra and number theory. In order to prove that the three above constructions are impossible one had “to stand on the shoulders of a giant” like Gauss. It was accomplished by Wantzel (1837) and Lindemann (1882).

The book covers many other topics in addition to the classical constructions by a ruler and a compass. It deals with the possibility of constructing geometrical entities by alternative means like the tomahawk, a compass only, a ruler only, rulers and dividers, double rulers, a ruler and a “rusty” compass, a marked ruler and more. All this is done in an interesting narrative style on one hand and, on the other hand, while keeping the traditional form of the rigorous mathematical presentation: definition, theorem and proof. There are many historical anecdotes and many inspiring quotations (starting with Walt Whitman for the preface and concluding with Lewis Carol for the index).

This is a book that every mathematics major will be happy to have on his mathematics bookshelf. It is especially recommended for mathematics teachers. Whether they decide to tell their students about the mathematics presented in the book or not, it will always remind them the unique taste of classical mathematics, something that they may forget while being busy with teaching common mathematical procedures and solving routine problems. In other words, it will remind them that mathematics is very different from the reality of common mathematics classes in which the boring drill and practice prevail.

The book is a source for many hours of pure intellectual pleasure to the mathematical mind. On page 19 of the book there are two uncommon exercises (1.4 and 1.5). The reader is asked to perform some constructions and to “note the time.” I haven’t done it. However, the time spent on reading this book is a quality time. As a matter of fact, the dimension of time disappears while reading this book. Once again, a wonderful book.

Author
Vinner, Shlomo, Prof., Ben Gurion University of the Negev, P.O. Box 653, Beersheva 84105, Israel.
E-mail: vinner@vms.huji.ac.il