

## A Characterization of Hering's Plane of Order 27

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**Abstract.** Hering's translation plane of order 27 has been characterized by its order and the fact that it admits  $SL(2, 13)$  in its translation complement (see [1]). We show that, aside from the Desarguian plane and a Generalized André plane, it is the only plane of order 27 which admits a subgroup of  $SL(2, 13)$  of order  $13 \times 12$ .

### 1. Introduction

In section 2 we review the relevant subgroups of  $SL(2, 13)$ ,  $GL(3, 3)$  and  $GL(6, 3)$ . We show in proposition 1 that  $GL(6, 3)$  contains a unique conjugacy class of subgroups of order  $13 \times 12$  isomorphic to a subgroup of  $SL(2, 13)$  of the same order.

In section 3 we introduce three spread sets which admit a given group of order  $13 \times 12$ . We prove in proposition 2 that there are no more such spread sets. The next lemma characterizes the planes arising from these spread sets and the following theorem is deduced:

**Theorem.** *The only translation planes of order 27 which admit a subgroup of order  $13 \times 12$ , isomorphic to that of  $SL(2, 13)$ , in their translation complement are the Desarguian plane, a Generalized André plane and Hering's translation plane of order 27.*

Our notation is standard and we follow [2].

### 2. Subgroups of $SL(2, 13)$ , $GL(3, 3)$ and $GL(6, 3)$

Since the order of the normalizer of a Sylow 13-subgroup of  $SL(2, 13)$  is  $13 \times 12$ , it follows that  $SL(2, 13)$  has a unique conjugacy class of subgroups of order



$13 \times 12$ . One representative of this class is generated by the concrete matrices

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix}.$$

As an abstract group it can be presented as

$$S = \langle h, c \mid h^{13} = c^{12} = 1, c^{-1}hc = h^{10} \rangle.$$

The Sylow 13-subgroup of  $GL(3, 3)$  is of order 13. From the factorization

$$X^{13} - 1 = (X - 1)(X^3 - 1)(X^3 - X - 1)(X^3 + X^2 + X - 1) \\ (X^3 + X^2 - 1)(X^3 - X^2 - X - 1)$$

in irreducible factors over  $GF(3)$ , we see that there are four conjugacy classes of elements of order 13 in  $GL(3, 3)$ . We can choose  $h$  with minimal polynomial  $X^3 - X - 1$  to generate the Sylow 13-subgroup of  $GL(3, 3)$ . In rational canonical form

$$h = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

and if

$$x = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

then  $|x| = 3$  and  $x^{-1}hx = h^3$ . We have that

$$N_{GL(3,3)}(\langle h \rangle) = \langle -I \rangle \times \langle h \rangle \rtimes \langle x \rangle.$$

The conjugacy classes of elements of order 13 are represented in  $\langle h \rangle$  by  $h, h^2, h^4, h^8$ .

A Sylow 13-subgroup of  $GL(6, 3)$  is of order  $13^2$  and,

$$T = \left\{ \begin{pmatrix} h^i & 0 \\ 0 & h^j \end{pmatrix} \mid i, j = 0, \dots, 12 \right\}$$

being one them, it follows that every element of order 13 of  $GL(6, 3)$  is conjugate to an element of the form  $\begin{pmatrix} h^i & 0 \\ 0 & h^j \end{pmatrix}$ . It can be deduced that  $GL(6, 3)$  also has four conjugacy classes of elements of order 13 which are represented in  $T$  by

$$\begin{pmatrix} I & 0 \\ 0 & h \end{pmatrix}, \begin{pmatrix} h & 0 \\ 0 & h \end{pmatrix}, \begin{pmatrix} h & 0 \\ 0 & h^2 \end{pmatrix}, \begin{pmatrix} h & 0 \\ 0 & h^4 \end{pmatrix}.$$

If we let  $C = \begin{pmatrix} 0 & -I \\ x^2 & 0 \end{pmatrix}$  then  $C$  is of order 12. Moreover if  $H = \begin{pmatrix} h & 0 \\ 0 & h^4 \end{pmatrix}$  then  $C^{-1}HC = H^{10}$  and the group  $G = \langle H, C \rangle$  is a subgroup of  $GL(6, 3)$  isomorphic to  $S$ .

**Proposition 1.** *If  $G_0$  is a subgroup of  $GL(6, 3)$  and  $G_0$  is isomorphic to  $S$ , then  $G_0$  is conjugate to  $G$  in  $GL(6, 3)$ .*

**Proof.** We may assume by Sylow's theorem that  $G_0$  contains one of

$$\begin{pmatrix} I & 0 \\ 0 & h \end{pmatrix}, \begin{pmatrix} h & 0 \\ 0 & h \end{pmatrix}, \begin{pmatrix} h & 0 \\ 0 & h^2 \end{pmatrix} \text{ or } \begin{pmatrix} h & 0 \\ 0 & h^4 \end{pmatrix}.$$

To see that the first three options cannot occur, we need only remark that if  $ah^i = h^j a$  with any  $3 \times 3$  matrix  $a$  over  $GF(3)$  then  $i = j = 0$  or  $a = 0$  or  $a$  is invertible. In the latter case  $h^i$  is conjugate to  $h^j$ . If we had, for instance,  $\begin{pmatrix} I & 0 \\ 0 & h \end{pmatrix} \in G_0$  and if  $C_0 = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix}$  was such that

$$C_0^{-1} \begin{pmatrix} I & 0 \\ 0 & h \end{pmatrix} C_0 = \begin{pmatrix} I & 0 \\ 0 & h \end{pmatrix}^{10}$$

then  $Y(h^{10} - I) = (h - I)Z = 0$  and  $hW = Wh^{10}$  would lead to  $Y = Z = W = 0$ , a contradiction. (Note that  $-h$  generates a subgroup of  $GL(6, 3)$  which is the multiplicative group of  $GL(27)$  with the addition of matrices.) The same type of argument eliminates the other two and we may assume that  $H \in G_0$ .

Let  $C_0 \in G_0$  be such that  $C_0^{-1}HC_0 = H^{10}$ , say  $C_0 = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix}$ . Then we get  $X = W = 0$  and  $Y, Zx^{-2} \in C_{GL(3,3)}(h) = \langle -I \rangle \times \langle h \rangle$ . This shows that there are  $4 \times 13^2$  possibilities for  $C_0$  in  $GL(6, 3)$ . Since

$$\begin{pmatrix} 0 & x \\ -I & 0 \end{pmatrix} \begin{pmatrix} h^i & 0 \\ 0 & h^j \end{pmatrix} \begin{pmatrix} 0 & -I \\ x^2 & 0 \end{pmatrix} = \begin{pmatrix} h^{9j} & 0 \\ 0 & h^i \end{pmatrix},$$

we see that  $C$  normalizes  $T$ , but  $C$  fixes no element of  $T$ . Therefore in the group  $F = T \rtimes \langle C \rangle$ , since  $T$  does not normalize  $\langle C \rangle$  and no element of  $T$  centralizes  $\langle C \rangle$ , we have that  $N_F(\langle C \rangle) = \langle C \rangle$ . Thus  $\langle C \rangle$  has  $13^2$  conjugates in  $F$ . Since both  $C$  and  $-C$  conjugate  $H$  to  $H^{10}$ , this accounts for  $2 \times 13^2$  of the possibilities and the proof would be finished if  $C_0$  was one of these, as the conjugation of  $C$  to  $C_0$  occurs within  $F \subseteq N(\langle H \rangle)$ . But this is indeed the case as multiplication by  $J = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$  leaves invariant the set of all  $4 \times 13^2$  matrices  $\begin{pmatrix} 0 & Y \\ Z & 0 \end{pmatrix}$  with  $Y, Zx^{-2} \in \langle -I \rangle \times \langle h \rangle$  and, if  $\tilde{C}$  is conjugate to  $C$  then  $\tilde{C}^6 = -I$  while



$(J\tilde{C})^6 = I$ . Thus the remaining  $2 \times 13^2$  are not matrices of order 12, but are of order 6. This completes the proof.

### 3. Spreads admitting the group $G$

It is easy to see that  $C_{GL(3,3)}(x) = \langle -I \rangle \times \langle x \rangle \times \langle w \rangle$  where  $w$  is an element of order 3 with minimal polynomial  $(X - 1)^2$ . If  $x$  is chosen as in 2 then we can put

$$w = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ -1 & -1 & 1 \end{pmatrix}.$$

**Proposition 2.** *Let  $V$  denote a 3-dimensional vector space over  $GF(3)$ . Then the following are the only three non-isomorphic spread sets on  $V \oplus V$  which define planes admitting the group  $G = \langle H, C \rangle$ .*

- 1)  $\Sigma_1 = \langle h \rangle x^2 \cup \langle h \rangle (-x^2)$
- 2)  $\Sigma_2 = \langle h \rangle \cup \langle h \rangle (-x)$
- 3)  $\Sigma_3 = \{(-1)^j h^i w^j x^2 h^{9i} \mid i = 0, \dots, 12; j = 1, 2\}$

**Proof.** Let  $\Sigma \subseteq GL(V)$  be an arbitrary spread set defining a spread on  $V \oplus V$  which admits the group  $G$ . By [2] the spread is of the form  $\{(V, 0), (0, V)\} \cup \{(V, VM) \mid M \in \Sigma\}$ . Since  $H$  fixes both  $(V, 0)$  and  $(0, V)$  while  $C$  interchanges them, it follows that  $G = \langle H, C \rangle$  permutes  $\{(V, VM) \mid M \in \Sigma\}$ . Since  $H$  has no other proper invariant subspaces than  $(V, 0)$  and  $(0, V)$ , it follows that  $H$  has two orbits of length 13 on  $\{(V, VM) \mid M \in \Sigma\}$ . Thus:

$$\begin{aligned} \{(V, VM) \mid M \in \Sigma\} &= \{(V, VM)H^i \mid i = 0, \dots, 12\} \cup \\ &\cup \{(V, VM')H^i \mid i = 0, \dots, 12\} \end{aligned}$$

where  $M, M'$  are chosen so that  $(V, VM)$  and  $(V, VM')$  are in the distinct orbits of  $H$ .

Since  $C$  normalizes  $\langle H \rangle$  so does  $C^4 = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}$ , which is of order 3. Thus  $C^4$  permutes the orbits of  $\langle H \rangle$  and, since it fixes  $(V, 0)$  and  $(0, V)$ , it must also fix the two other orbits. But then, being of order 3, it must fix at least one component in each. We may assume that these components are the  $(V, VM), (V, VM')$  chosen above. On a fixed component  $C^4$  fixes the origin and the point at infinity, therefore it fixes at least two of the remaining 26 points. It follows that  $C^4$  is

planar and, since  $6^2 + 6 > 27$ , the order of the fixed subplane must be 3 and  $C^4$  has no other fixed component than  $(V, 0), (0, V), (V, VM), (V, VM')$ . We conclude also that  $M = x^{-1}Mx, M' = x^{-1}M'x$  and so  $M, M' \in C_{GL(3,3)}(x)$ .

Now  $C^3$  is of order 4, it commutes with  $C^4$ , thus it permutes the fixed elements of  $C^4$ . Since it interchanges  $(V, 0)$  and  $(0, V)$ , it must fix both  $(V, VM), (V, VM')$  or interchanges them. From  $(V, VM)C^3 = (V, VM)$  it follows that  $M = -M^{-1}x$  which is a contradiction as  $M^2 = -x$  would imply that  $M$  is an element of order 12 in  $C_{GL(3,3)}(x)$ . Since there is no such elements in  $C_{GL(3,3)}(x)$  we conclude that  $M' = -M^{-1}x$ .

We have shown that the spread

$$\{(V, VM)H^i \mid i = 0, \dots, 12\} \cup \{(V, V(-M^{-1}x))H^i \mid i = 0, \dots, 12\}$$

is determined by  $M \in C_{GL(3,3)}(x)$ .

It is an easy calculation to see that for  $M = \pm w, \pm w^2, \pm wx$  or  $\pm w^2x$  we do not get a spread.

The possible spreads are then determined by the pairs  $\{M, -M^{-1}x\} = \{-I, x\}, \{I, -x\}, \{x^2, -x^2\}, \{wx^2, -w^2x^2\}$  or  $\{w^2x^2, -wx^2\}$ .

The first two pairs produce isomorphic spreads, as can be seen using [2; 26], as the spread set given by  $\{-I, x\}$  becomes that given by  $\{I, x\}$  under the action of  $\sigma = \begin{pmatrix} h & 0 \\ 0 & -I \end{pmatrix}$ . By the same result, applying  $\lambda = \begin{pmatrix} w & 0 \\ 0 & z \end{pmatrix}$  to the spread defined by  $\{wx^2, -w^2x^2\}$  and applying  $\mu = \begin{pmatrix} w & 0 \\ 0 & -z \end{pmatrix}$  to that given by  $\{w^2x^2, -wx^2\}$ , the spreads coincide and the last two pairs produce isomorphic planes.

The spread  $\Sigma_1$  is defined by  $\{x^2, -x^2\}$ ,  $\Sigma_2$  is defined by  $\{I, -x\}$  and  $\Sigma_3$  is defined by  $\{w^2x^2, -wx^2\}$ . The proof of the proposition will be completed once we establish that these are non-isomorphic. Both this and the proof of the theorem follow from the next lemma.

**Lemma 3.** *If  $\Pi_i$  denotes the plane defined by the spread set  $\Sigma_i$  then  $\Pi_1$  is Desarguanian,  $\Pi_2$  is a generalized André plane and  $\Pi_3$  is Hering's plane of order 27.*



**Proof.** Since

$$(V, Vh^i x^2) \begin{pmatrix} h & 0 \\ 0 & h^9 \end{pmatrix} = (V, Vh^i x^2)$$

and

$$(V, Vh^i(-x^2)) \begin{pmatrix} h & 0 \\ 0 & h^9 \end{pmatrix} = (V, Vh^i(-x^2)),$$

( $i = 0, \dots, 12$ ), it follows that the kernel of the plane defined by  $\Sigma_1$  contains a group isomorphic to  $\langle h \rangle$ . Since it also contains  $-I$  and since  $\langle h \rangle \times \langle -I \rangle \cup \{0\}$  is isomorphic to  $\text{GF}(27)$  as a field, it follows that  $\Pi_1$  is Desarguanian [2].

The plane defined by  $\Sigma_2$  does not admit  $(x, y) \rightarrow (y, x)$  as  $(V, V(-h^j x)) \rightarrow (V, V(-h^k x^2))$  and  $-h^k x^2 \notin \Sigma_2$ . If

$$\sigma = \begin{pmatrix} I & 0 \\ 0 & h \end{pmatrix} \quad \text{and} \quad \tau = \begin{pmatrix} h & 0 \\ 0 & I \end{pmatrix}$$

then a straightforward calculation shows that  $\sigma$  is an  $((\infty), (V, 0))$ -homology of order 13,  $\tau$  is a  $((0), (0, V))$ -homology of order 13 and each component is in an orbit of length at most 13 under  $\langle \sigma \rangle \times \langle \tau \rangle$ . Since 13 is a primitive divisor of  $3^3 - 1$ , the plane defined by  $\Sigma_2$  is a generalized André plane by [3; 5.2.1].

Finally, the spread defined by  $\Sigma_3$  is invariant under

$$W = - \begin{pmatrix} xwu & x^2 w^2 u \\ x^2 w^2 u & -wu \end{pmatrix}$$

where  $u$  is an involution inverting  $x$  and commuting with  $w$ . If  $x, w$  are chosen as before, one can choose

$$u = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix}.$$

But then since  $\langle W, H, C \rangle \cong \text{SL}(2, 13)$  as can be seen in [1], it follows from the same reference that the plane defined by  $\Sigma_3$  is Hering's plane of order 27. This completes the proof of the lemma, the proposition and the theorem.

## References

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## 1. Introduction

In this paper we study the topology of a closed foliated manifold  $M$ . For a foliation given by a vector field  $X$  flow without fixed points. Our main result is the following:

**Theorem 2.1.** *There exist infinitely many distinct leaf closures in  $H^2(M, \mathbb{R})$  for an infinite dimensional vector space.*

A statement of denseness and openness for foliations on the sphere was obtained by Poincaré [1908]. The result above adds the following strengthening on the sphere:

**Theorem 2.2.**  *$H^2(M, \mathbb{R})$  is a separable Banach space.*

Both theorems are proved in Section 2. Some of the results of [2] were completely calculated. The [34], [40], [41], [46], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [64], [65], [66], [67], [68], [69], [70], [71], [72], [73], [74], [75], [76], [77], [78], [79], [80], [81], [82], [83], [84], [85], [86], [87], [88], [89], [90], [91], [92], [93], [94], [95], [96], [97], [98], [99], [100], [101], [102], [103], [104], [105], [106], [107], [108], [109], [110], [111], [112], [113], [114], [115], [116], [117], [118], [119], [120], [121], [122], [123], [124], [125], [126], [127], [128], [129], [130], [131], [132], [133], [134], [135], [136], [137], [138], [139], [140], [141], [142], [143], [144], [145], [146], [147], [148], [149], [150], [151], [152], [153], [154], [155], [156], [157], [158], [159], [160], [161], [162], [163], [164], [165], [166], [167], [168], [169], [170], [171], [172], [173], [174], [175], [176], [177], [178], [179], [180], [181], [182], [183], [184], [185], [186], [187], [188], [189], [190], [191], [192], [193], 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[623], [624], [625], [626], [627], [628], [629], [630], [631], [632], [633], [634], [635], [636], [637], [638], [639], [640], [641], [642], [643], [644], [645], [646], [647], [648], [649], [650], [651], [652], [653], [654], [655], [656], [657], [658], [659], [660], [661], [662], [663], [664], [665], [666], [667], [668], [669], [670], [671], [672], [673], [674], [675], [676], [677], [678], [679], [680], [681], [682], [683], [684], [685], [686], [687], [688], [689], [690], [691], [692], [693], [694], [695], [696], [697], [698], [699], [700], [701], [702], [703], [704], [705], [706], [707], [708], [709], [710], [711], [712], [713], [714], [715], [716], [717], [718], [719], [720], [721], [722], [723], [724], [725], [726], [727], [728], [729], [730], [731], [732], [733], [734], [735], [736], [737], [738], [739], [740], [741], [742], [743], [744], [745], [746], [747], [748], [749], [750], [751], [752], [753], [754], [755], [756], [757], [758], [759], [760], [761], [762], [763], [764], [765], [766], [767], [768], [769], [770], [771], [772], [773], [774], [775], [776], [777], [778], [779], [780], [781], [782], [783], [784], [785], [786], [787], [788], [789], [790], [791], [792], [793], [794], [795], [796], [797], [798], [799], [800], [801], [802], [803], [804], [805], [806], [807], [808], [809], [810], [811], [812], [813], [814], [815], [816], [817], [818], [819], [820], [821], [822], [823], [824], [825], [826], [827], [828], [829], [830], [831], [832], [833], [834], [835], [836], [837], [838], [839], [840], [841], [842], [843], [844], [845], [846], [847], [848], [849], [850], [851], [852], [853], [854], [855], [856], [857], [858], [859], [860], [861], [862], [863], [864], [865], [866], [867], [868], [869], [870], [871], [872], [873], [874], [875], [876], [877], [878], [879], [880], [881], [882], [883], [884], [885], [886], [887], [888], [889], [890], [891], [892], [893], [894], [895], [896], [897], [898], [899], [900], [901], [902], [903], [904], [905], [906], [907], [908], [909], [910], [911], [912], [913], [914], [915], [916], [917], [918], [919], [920], [921], [922], [923], [924], [925], [926], [927], [928], [929], [930], [931], [932], [933], [934], [935], [936], [937], [938], [939], [940], [941], [942], [943], [944], [945], [946], [947], [948], [949], [950], [951], [952], [953], [954], [955], [956], [957], [958], [959], [960], [961], [962], [963], [964], [965], [966], [967], [968], [969], [970], [971], [972], [973], [974], [975], [976], [977], [978], [979], [980], [981], [982], [983], [984], [985], [986], [987], [988], [989], [990], [991], [992], [993], [994], [995], [996], [997], [998], [999], [1000].

**Theorem 4.3.** *The following are equivalent on the  $L^2$  space  $L^2(M, \mathbb{R})$ :*

- a)  $\dim H^2(M, \mathbb{R}) = 1$ .
- b)  $\mathbb{R} \cdot X$  is contained in a distributional linear foliation.

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