

Some problems of contemporary university education

by

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FOREWORD

On my recent visit to the Instituto de Matematica e Estatistica of the University of São Paulo, I was asked by various people to speak about the problems confronting those who are concerned with the teaching of students at our universities. I agreed to introduce a discussion on this topic in the hope that my remarks would generate interest among my colleagues in São Paulo. I was certainly not disappointed. At the end of my remarks, Professor Newton Costa, who had acted as chairman of the session, asked me to set down my thoughts on paper. I agreed to do this too, provided it was clearly understood that this would be a discussion paper and not a definitive document. Thus I have not attempted, in what follows, to arrive at any precise diagnosis or recommendation.

Having been asked to speak about problems of university education, I did not feel I should take up the time of my audience by discussing preuniversity education. However, I do wish to set on record my conviction that many of the most intractable problems of mathematical education at the university stem from causes which have their roots in the secondary school and even further back in the primary school. Many of the bad thought-habits and bad work-habits of our students have been inculcated and established early in their education. It is virtually impossible to eradicate them by the time they reach the university. Also the inflexibility of approach and stereotyped presentation of many university teachers are due to an attitude towards their subject which was acquired early in their own experience and has become fixed and unalterable. In this paper I must content myself with this reference to what is nevertheless a core problem of mathematical education.

On the other hand, I do not think it sensible to draw too rigid a distinction between undergraduate and graduate education. Some of my remarks may be more pertinent to the undergraduate scene, others to the problems of graduate education today. In any case the level of attainment indicated by the first degree varies substantially from country to country and even within a given country. Since I would not wish my remarks to be too parochial, it is clearly best for me to take account of both phases of university education.

Since I am a mathematician, many of my remarks will be of particular, even perhaps unique relevance to the teaching of mathematics. However I believe that many of the problems which face us are not peculiar to mathematics and in these cases my remarks, insofar as they are specifically directed to the teaching of mathematics may be taken as illustrating a more general problem.

I have divided my observations into three sections. These I have called Orientation of Mathematics Courses; Student Attitudes; Teaching and Research. In the first of these sections I am very particularly concerned with the teaching of mathematics and I do not pretend to know just what relevance my remarks have to other fields of scholarship (although, of course, the problem of the job market extends far beyond mathematics). In the second section my formulations are deliberately general since the problems discussed there affect all students, but it is useful to bear in mind that the study of mathematics is under particular attack from those whose attitudes are discussed in that section. In Section 3 I make general remarks on the relation of teaching to research, but in proposing specific ways faculty members may seek to improve their teaching, I am thinking very explicitly of mathematicians. Naturally, the concerns expressed in these sections show considerable overlap, so that the subdivision is not to be regarded as absolute, and it is no surprise that certain themes are discussed in more than one section.

1. *Orientation of Mathematics Courses*

There can be no doubt that there is one particular area in which we, as teachers of mathematics, have shown considerable success; that is in reproducing our own kind. We may take it, without being smug, that we know how to train future professional mathematicians, and indeed, some of our problems stem from our very success in this area. There is now a dangerous surfeit of young mathematicians. In the United States a cadre of approximately 12,000 mathematicians is turning out approximately 1,000 new Ph. D.'s annually. It is perfectly

obvious that the academic community could not continue to absorb this huge number indefinitely. It is a matter of fact that the crisis is already upon us. However, even were it not for the exigencies of the job market, I would still maintain that it is necessary for us to diversify our training of mathematics specialists, and to give more thought to the education of those who take our mathematics courses without the intention of becoming professional mathematicians. To meet the needs of many of those in the first category and essentially all of those in the second category, we should in particular give more attention to the teaching of applications of mathematics to biology, economics, etc., to the teaching of probability and statistics, and to familiarizing our students with the role of computers. I do not think it is necessary to say much more about these recommendations, whose nature and purpose are surely self-evident, beyond the following few remarks. First, when I speak of applications I have in mind genuine application of mathematics. Too often courses announcing themselves as being concerned with applications of mathematics in reality consist of uninteresting and unnecessary applications of dull mathematics to artificial situations. We must be sure that the applications are genuine and that the mathematics plays an essential role. The sort of pattern which I would like to recommend for a course which would familiarize students with applications of mathematics would be one in which research scientists using mathematics in their work come to address informal seminars of students about that work, encouraging the students to comment on the way in which the mathematical models were set up, their relevance, their scope, their flexibility; and to consider possible modifications of the model and possible other applications of it. That is to say, the course would be concerned with the methodology of applied mathematics rather than with a specific field of application. Second, it is hardly necessary for me to underline the importance of probability and statistics to the study of the world around us. Thus, these subjects should be taught primarily with a view to explaining their basic role in scientific method. Of course, at a more sophisticated level, they would be taught as part of a mathematics curriculum in which, conceivably, the emphasis on application would diminish. Third, the student should appreciate the manifold roles of computers. Among these, there is the role of the computer in enriching the understanding of mathematics itself. It has been my experience that many students understand mathematical reasoning far better when they are exposed to the necessity of programming a problem, putting it on the machine, and studying the output. For example, the question of the existence and uniqueness of the solution of a system of linear equations

is far better understood by many students when the algorithm for solving the system of equations has been programmed and the student understands the embarrassment caused to the computer in the absence of a solution.

Fourth, I would wish to observe that this recommendation may very well involve considerable difficulty in finding university teachers able and willing to do a good job of teaching the subjects under consideration. Consequently, the very question of the training of university teachers comes up for consideration in considering this new orientation of courses. Thus, a further recommendation must be that, in the training of the future professional mathematician, greater emphasis should be placed on his teaching skill and his attitude towards teaching. Much has been written about this, but I wish to emphasize that we will not persuade our students to regard the problems of teaching mathematics as being comparable in importance with the problems of mathematics itself, unless and until the leading figures in the world of mathematics themselves indicate that they share this attitude. I, myself, would recommend that we should adopt a far more liberal approach to what would constitute a reasonable program for a Ph. D. in mathematics. I would certainly believe that a type of thesis which involved the development of a new experimental course—for example, a course designed to teach to future biologists the sort of mathematics which they are likely to need in their professional work—may very well be far more valuable and may very well be a far more scholarly piece of work than many theses of original research of the kind currently accepted for the Ph. D. degree. I would go further and suggest that certain types of curriculum innovation should qualify the student for the Ph. D. degree even without thesis. But I recognize that this may be too radical a recommendation to have much chance of adoption at this time.

Let me face one of the difficulties which undoubtedly arise if we do endeavor to place the teaching of mathematics on a level at least comparable with that of research. Currently, we have, I believe, very fair and objective standards for judging research mathematicians. We examine their published work, we consult others who are expert in their field. The criteria of excellence in teaching mathematics are far more suspect and difficult to enunciate. Let me state categorically that I distrust the method of consulting student opinion. That is not to say that I do not believe that it is one of the methods that should be used in seeking an objective evaluation. However, I believe it to be extremely unreliable. The majority of students will favor that teacher whom they believe will best help them to pass their examinations. We must beware of regarding a university as a sort of shop, selling degrees to

those who offer examination proficiency. The ability of inspire a student or to teach a student in such a way that he obtains a genuine mastery of the subject and is able to use what he has learned later in his professional life—these abilities are difficult to evaluate contemporaneously and certainly difficult to quantize. It is one of the real difficulties in all curricular reform that we do not have reliable instant estimates of the value of work in this field. Similarly we do not have reliable instant estimates of the quality of teaching. Thus, I admit frankly that I do not know what should replace the objective criteria for evaluating research mathematicians when it comes to considering the relative and absolute merits of teachers of mathematics. I believe that this problem can be solved, I already have some ideas myself, but it is clearly a matter for further discussion.

2. Student Attitudes

The problem to which I would like to direct attention in this section is that of the non-acceptance, by many students, of traditional modes of study, traditional discipline, and traditional goals. The causes of this phenomenon are well known to all of you and need not be emphasized here. I believe that the students are well justified in asking for a further democratization of the educational process. I also believe that the university is *ipso facto* involved in society and cannot and should not seek to avoid its responsibilities to that society. However, having said this, I must add that I do not share the attitude of many of my colleagues in American universities, particularly the younger ones, when they appear to give wholehearted support to the more extreme and radical pretensions of the students for control of university affairs. In order that I should not appear to you to be shirking the issue, let me state quite categorically three views of my own which, for greater effect, I state in negative form. I do not believe that the university is primarily an instrument of social change. I do not believe that the university is a basically democratic institution. I do not believe that the university is essentially egalitarian. All these views are relevant to the problems facing the university today in the United States. I think that the faculty need to understand student attitudes, but should not necessarily feel obliged to share them. For example, I find no merit in the view that students should have a vote in the appointment and promotion of faculty.

Further, I believe that student and faculty play an essentially different role within the university and this difference should be recog-

nized in many institutional forms. With even more fervor I believe that the university plays its most essential role outside the special domain of place and time in which the university affairs happen to be currently conducted. This larger role of the university must be emphasized. In particular it needs to be stated confidently, emphatically and unequivocally, that the university does not develop the social conscience of its students simply by giving them courses for credit in social conscience. The social conscience of students emerges as a by-product of their education and of the contact and stimulus which they have received within the academic community. So too does the social conscience of the faculty. Thus, for example, while I am, myself, bitterly opposed to the Vietnam war, I would regard it as quite improper for a university to take a corporate attitude towards such an issue. I would, however, confidently hope that very many active opponents of the war would be found on the university campus; the members of a university community deriving and strengthening their social consciences through the natural interactions within that community.

I regard as a matter of supreme importance the maintenance of basic courses for undergraduates. I am highly suspicious of the various integrated curricula which are being introduced at the undergraduate level. I believe that integrated curricula are appropriate at the graduate level and even more especially at the research level. However, it is essential for any student who wishes to address himself to any of the urgent problems of the day that he acquire a taste for and an understanding of the difficulties of genuine hard work. This, I believe, can only be acquired through studying a particular subject in depth. Many students today appear to believe that by a process of purifying their emotions they prepare themselves adequately for tackling the problems of pollution and ecological imbalance which beset our society. I do not, myself, believe that such a process is adequate.

The questions I have been raising in this section may appear to be rather broad, and not to admit of solution at the classroom level. Nevertheless, I do believe that these problems have their reflection in the classroom and that the teacher in the classroom can contribute to their solution. It is today a much harder job to teach a class of students a piece of mathematics than it once was. One remains as always up against the principal difficulty that the student is unlikely to understand mathematics or to have much taste for it. But now, superimposed on that difficulty, is the extra problem that the student believes himself in many cases to be justified in questioning the validity of the requirement that he learn mathematics at all. He feels himself free to ask the teacher, "Why should I spend my time learning mathematics rather

than doing something else?" I believe we must prepare ourselves to answer such questions, but I believe it is essential we answer these questions unapologetically, with our faith in the importance in mathematics quite unimpaired. We should not compromise. We should not apologize to the students, we should not seem to share their distrustful attitudes. We should, as I say, understand why they question but, just as they are free to 'do their thing' in questioning, we must be free to 'do our thing' in confidently and with certainty stating the value and purposes of mathematical education.

The questioning attitudes to which I have referred may very well lead to serious discipline problems. These should be met in the first instance with a sympathetic approach, but if certain students persist in endeavoring to disrupt classes, I, myself, believe that we have no other recourse but to exclude them. Too often the pattern has been that the hardworking serious student has suffered because of the licence permitted to his recalcitrant neighbor to prevent the passage of ideas between teacher and student which lies at the heart of the educational process at this level.

3. *Teaching and Research*

Here the main issue is that of the roles of teaching and research in the totality of responsibilities of members of the academic profession. It seems perfectly clear to me that each of these roles is of enormous importance. What, however, has to be argued today is that, far from being incompatible, these two roles are complementary. There is no known way of ensuring that a potential faculty member will turn out to be a good teacher. However, I would claim that, in selecting for this quality, one of the best indicators is the enthusiasm of the faculty member for his subject. Here I believe my proposition has general validity, but I would be content if it would be considered simply in relation to mathematicians. There a few mathematicians, it is true, who succeed in giving inspiring and lucid lectures while not being themselves concerned very much with developments at the frontiers of knowledge. However, such individuals are rare and, generally speaking, the best teachers are active in research. It is necessary to attack and demolish the view, which is becoming increasingly prevalent, that we must choose between teaching and research and, moreover, that the community would be well-advised to choose teachers rather than research workers. The scholarly pursuit of our subject is under attack from government, bureaucracy, administration, and other ignoramuses, and also from

students. The case is stridently made out that university professors are selfishly pursuing their own interests, mathematical and financial, to the neglect of their principal responsibility which is that of educating students. I will refer only in passing to the irony that these charges are often made by students who are simultaneously claiming that there should be complete democracy within the university. Since I do not accept the latter view, I cannot dispose of the argument of these students by endorsing it!

I must simply emphasize that teaching or research is a false dichotomy. The principal function of a university is scholarly activity. The faculty and students compose the two components undertaking this activity. The main aspects of scholarly activity are communication between faculty and students, and the acquisition of knowledge by faculty and by students. Students will principally acquire their knowledge from the faculty and from reading recommended by the faculty. The faculty will principally obtain their knowledge by their own reading and by their own research.

However, it is necessary to add that although the good research man is the more likely to be the good teacher there are many serious defects within our professional community with regard to teaching. We have to take teaching desperately seriously. And, as I have emphasized earlier, this must include not only the training of the future mathematicians, but also the teaching of mathematics to intelligent people who do not themselves propose to become mathematicians. This latter aspect of our teaching job is, I believe currently the least satisfactory. We must give it the attention it deserves and treat the students with the dignity that they merit as intelligent human beings. We must understand the nature of their motives for studying mathematics, we must reinforce those motives, and we must respect them. We must not take the view that we are simply members of a secret society whose function it is to indoctrinate new acolytes. I believe that one of our difficulties in meeting the ingnorant and prejudiced charges which are being leveled against us today, is precisely that our own house is not in good order. If we could point to our success in the teaching of mathematics to the generality of our students, we could then the better answer those charges. If I were to go to the limits of tolerance, towards those people who have been attacking our profession, I would say that they have noticed certain unsatisfactory symptoms, but they have made a totally wrong diagnosis.

In emphasizing the importance of teaching as a component of the activity of the professional mathematician, I have come back again to the point which I raised earlier under the heading of the orientation

of courses. That is, that we must place greater emphasis on teaching in our training of graduate students. We cannot expect good teaching from graduate students who have been led to believe that the only important activity in their professional lives is that of producing and publishing original research. I have nowhere in my remarks intended to downgrade the importance of doing original work in mathematics. I believe that too many people are today concerned in this activity, but that is another matter. However, what needs to be said is that our mathematical community is not in any way fulfilling its responsibilities to the community at large unless it pays as serious attention to the teaching of mathematics and the training of teachers of mathematics as it currently pays to the advancement of mathematics itself.

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