A Property of Spaces whose Strong Dual is a Schwartz Space*

ERNESTO BRUNO COSSI

THEOREM. *Let E be a semireflexive locally convex Hausdorff space and let its strong dual E' be a Schwartz space. Then E is a semi-Montel space.*

PROOF. Let B be any nonempty bounded subset of E whose topology is τ. Then B^0, the polar of B in the duality between E and E', is a neighborhood of 0 in E' and B^{0*}, the polar of B^0 in the duality between E' and the bidual E'' of E, is an equicontinuous subset of E''. By the dual characterization of Schwartz spaces, there exists a neighborhood V of 0 in E' such that B^{0*} is relatively compact in the semi-normed space E''_{V^*} generated by V^* whose semi-norm q_{V^*} is the gauge of V^*. Let D be a balanced, convex, closed and bounded subset of E such that $B \subset D$ and $D^0 \subset V$. Then D is weakly closed and $D^{00} = D$. The topology induced by the semi-norm $q_{D^{0*}}$ of $E''_{D^{0*}}$ in E''_{V^*} is coarser than the topology of the semi-norm q_{V^*}, therefore B^{0*} is relatively compact in $E''_{D^{0*}}$. Let ϕ be the canonical imbedding of E onto E'' because E is semi-reflexive; it follows then that $\phi(D^{00}) = D^{0*}$ and

$$\phi \,|\, E_D : E_D \to E''_{D^{0*}}$$

is an isomorphism, where E_D and $E''_{D^{0*}}$ are equipped respectively with the topologies of the semi-norms q_D and q_{D^*}; hence $\phi^{-1}(B^{0*}) \cap E_D$ is relatively compact in E_D. But $\phi^{-1}(B^{0*}) = B^{00}$ and $B \subset B^{00} \cap E_D$, therefore B is relatively compact in E_D. The topology induced by τ in E_D is coarser than the topology of the semi-norm q_D, hence B is relatively compact in E. Then E is a semi-Montel space.

COROLLARY. *Let E be a reflexive locally convex Hausdorff space and let its strong dual E' be a Schwartz space. Then E is a Montel space.*

*Recebido pela SBM em 15 de dezembro de 1972.

REFERENCE

HORVÁTH, J., Topological Vector Spaces and Distributions, Vol. 1, Addison-Wesley Publ. Comp., 1966.

Instituto de Matemática
Universidade Federal do Rio Grande do Sul
Porto Alegre