MAKING THE CONNECTION: PROCEDURAL AND CONCEPTUAL STUDENTS' USE OF LINKING WORDS IN SOLVING PROBLEMS

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The Malaysian educational system tends to take a clear instructionally-focused approach to the teaching of mathematics. This means that many students gain a good procedural command of areas of mathematics. This paper explores one outcome of a teaching experiment in which one area of mathematics (differentiation) was taught in a radically different way which emphasized key concepts rather than procedures. The paper examines how students appear to access their mathematical knowledge in solving problems in a non-mathematical context and demonstrates that there is a strong relationship between an aspect of their language use (transitional words of consequence or contrast) and the form of their mathematical understanding in this area. We discuss the relevance of this finding to the possible structure students' mental spaces.

INTRODUCTION

If you teach procedures, they will learn procedures.

The Malaysian educational system tends to focus on ensuring that students have a good command of a wide range of different procedures (Tall and Razali, 1993). Where mathematics is being taught as support for adult students learning another topic, there may again be a tendency for the teachers to focus on the procedures they deem important for the topic they are teaching (Benn, 1997). The environment for this research was a course on differentiation for adult students studying industrial chemistry at Universiti Teknologi MARA (UiTM) in Malaysia.

The study followed 16 students on the industrial chemistry degree, focusing on their participation in a mathematics course and a physical chemistry course. We were particularly interested in examining how the students used their mathematics in solving problems from their chemistry course (in this case, a course covering topics such as rates of reaction, rate laws and half lives, which implicitly require the use of mathematics covered in their differentiation course).

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PROCEDURAL AND CONCEPTUAL DEVELOPMENT

The distinction between being able to apply a relatively well determined set of instructions to a mathematical problem (with varying degrees of fluency) and being able to explain and use links between different structural aspects of mathematics is a common one in the mathematics education literature. Skemp's (1976) distinction between instrumental (knowing how) and relational (knowing how and why) understanding is mirrored in Hiebert and Lefevre's (1986) procedural /conceptual dichotomy

Question 1	**Question 2**
Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ where i) $y = e^{-3x} + 2\sin(x+4)$ ii) $y = (\cos(4x))^2$ iii) $y = \frac{2x^3 + 8x}{x}$	Differentiate the following equation explicitly, finding y as a function of x: $y^5 + 2y - \ln y = 5x^3 + 1$

Figure 1: Example procedural questions

Earlier studies had shown that, given the teaching styles they had encountered, we should not expect our population to contain many students with a conceptual understanding of the mathematics. Indeed, a pilot study questionnaire – with students from the preceding year to the main study – using what we felt were relatively simple conceptual questions (such as that described below), found that all of the students appeared to have a very poor conceptual grasp of the topic of differentiation.

It was proposed that we attempt a teaching experiment to examine whether an alternative style of encouraging students to view the material might lead to better results. We adapted some of the ideas of Tall (1991), in particular, using active graphical approaches to working with analytic concepts such as continuity and differentiability. The mathematics module, which covered the same material as the existing course, was a hybrid of traditional teaching of procedures (required by the faculty and requested by the students) and computer-based sessions involving the investigation of particular functions.

At the end of the course, students took two tests: the first was the standard UiTM examination for the module and consisted of 'apply the procedure' questions such as those shown in figure 1.

The second test was an adaptation of the pilot study questionnaire, designed to elicit an indication of students' conceptual understanding of the material. Fundamentally, this consisted of questions of a type the students would not be familiar with and which, in our opinion and the view of two independent, experienced teachers, could not be solved with the direct application of taught procedures or simple recall. For example, the question in figure 2 might rely on students being able to link together symbolism for the derivative, the sign of the function, the derivative and the second derivate, together with meanings for the derivative and second derivative and their graphical interpretation.

The two tests were scored according to marking schemes drawn up with independent colleagues. The results (given as a scatter graph in figure 3) tallied well with our own experience of the students' competence with mathematical procedures and understanding of key underlying concepts.

State which graph(s) representing function f, which has all the three following properties:

$f(c) > 0, f'(c) < 0$ and $f''(c) > 0$.

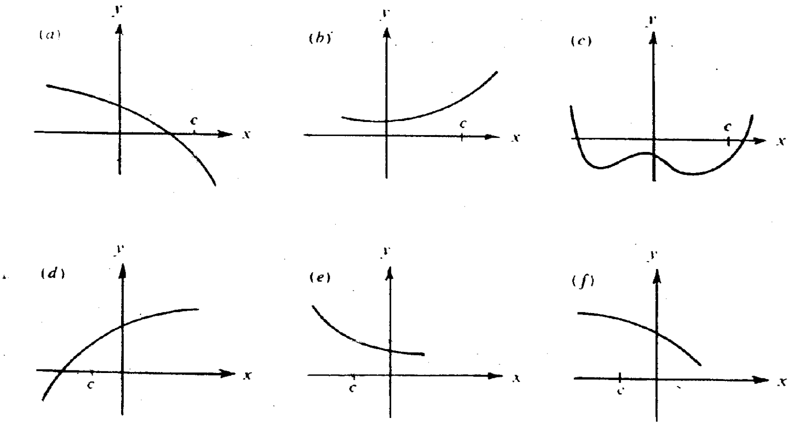


Figure 2: An example conceptual question

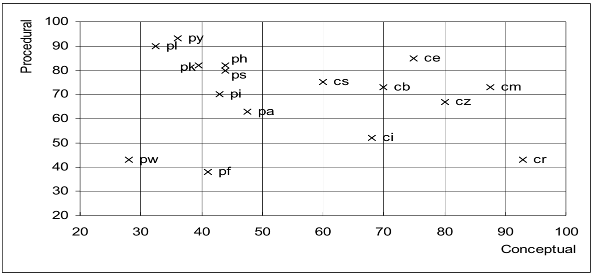


Figure 3: 'Procedural' vs 'conceptual' scores

The results show an even spread of marks across both axes, with low correlation between procedural and conceptual marks ($r^2 = 0.03$) suggesting that procedural and conceptual competence in this sample were relatively independent. Examination of the scatter diagram showed what we felt were two clear groupings based on their conceptual test scores, a cluster scoring quite poorly (marked with names indexed with the prefix P) and a spread of scores above that (marked with names indexed with the prefix C).

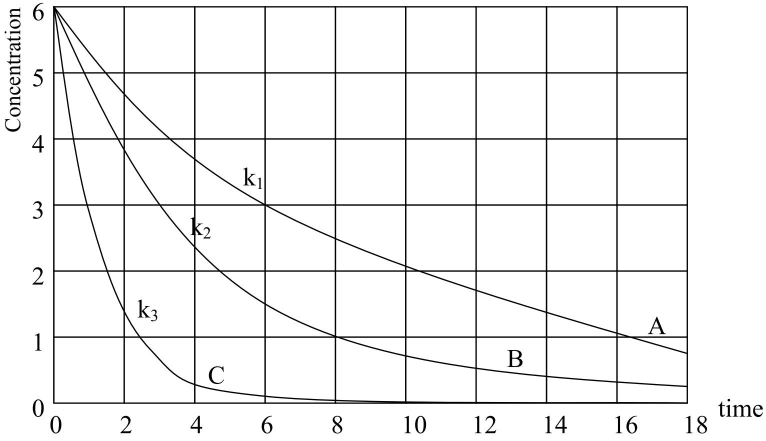
This simplistic split worked relatively well: figure 3 shows that the largest gap between conceptual scores (between candidates PA and CS) is also the point at which we chose to cleave the sample in two. For convenience, those candidates scoring above this gap (marks ≥ 60) were called conceptual (by which we meant we felt they had access to conceptual understandings of the mathematical material in the course) and those with lower scores (≤ 47.5) were called procedural (meaning those who had access only to procedural understanding).

LANGUAGE AS LINKING CONTEXTS

In addition to the two mathematical tests, the students attended interviews in which they solved questions which had an implicit mathematical content (including aspects of differentiation), but which were presented in a legitimate physical chemistry context. These questions were deemed, by their chemistry lecturer, to be questions appropriate to the chemistry course they were attending alongside the mathematics course (a typical question is shown in figure 4).

The transcripts of the students' attempts to solve these problems were subjected to many different analyses. In this paper, we will concentrate on a simple, yet surprisingly effective one which we came to call linking words analysis. Each transcript was divided into units of meaning (Kruger, 1981) and these units of meaning we sorted into two main categories: statements apparently about mathematics and statements apparently about chemistry. By asking independent raters to repeat this categorization process with a judges' manual (of the form described in Perry, 1970), we showed a good level of reliability in this categorisation. The short extract in figure 5, shows part of one student's response to the question in figure 4, with units of meaning categorized.

The diagram consists of 3 graphs of concentration versus time starting with the same initial concentration but with different rate constant k_1, k_2 and k_3.



Explain what you can about the graphs

Which graph has the highest rate of reaction at time, t = 1 and why?

Which graph has the smallest rate of reaction at t = 4 and why?

What is the order of the reaction for each of the graph?

If each of the graphs is extended, will the graph touch the time-axis and why?

Which graph has the largest rate constant and why?

Which graph has the lowest rate constant and why?

Plot a graph of rate of reaction versus concentration for each of the graph.

Figure 4: A Physical Chemistry Interview Question

In addition to this categorization, however, we noted that many students used explicit linguistic links between their statements, in which transitional words of consequence or contrast (such as ‘then’, ‘but’ and ‘therefore’) apparently indicate that the student is explaining a chemistry idea in terms of a mathematical one, or vice versa. We saw these as explicit indicators of connections in Sweetser’s (1990) epistemic domain. In the extract in figure 5, these causal connectives are highlighted in bold.

What became clear from this analysis, however, was that while within both procedural and conceptual groups of students, the ability to solve these chemistry problems varied considerably, there were radically different ways in which they solved them. In particular, we noticed that those students we had chosen to classify as ‘conceptual’ used many linking words, while those who we had classified as ‘procedural’ used very few (even in the cases of students who performed extremely well on these chemistry tasks).

Chem: These graphs show concentration of 3 different reactants versus time. k_1, k_2 and k_3 are the rate constants. Graph A, the reaction is slow.

Math: The slope, if the slope is like this, less steep (shows his hand to indicate the steepness),

Chem: **then** its rate of reaction is slow. Graph C, the reaction is very fast

Math: **because** the slope is more steep (shows his hand to indicate the steepness),

Chem: it is fast to dissolve, it is fast to ….for the reactants to form the products, it is very fast. It means that the rate of reaction is higher.

Figure 5: A coded transcript extract

In figure 6, we consider the relationship between conceptual understanding (as evidenced by the score on the second mathematical test) and the use of linking words (a simple count of number of linking words across all of the chemistry problem questions). From the figure alone is clear that there is a strong correlation between them (in fact, $r = 0.91$, significant at the $p < 0.01$ level) and it seems plausible to suggest that the conceptual understanding the students have allows them to make explicit linguistic links between the two conceptual domains.

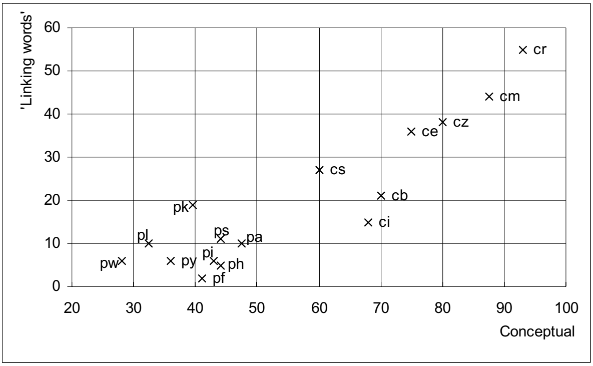


Figure 6: Correlating conceptual scores with linking words

DISCUSSION

We can consider this strong correlation in terms of Fauconnier’s (1985) notion of a mental space – “partial assemblies constructed as we think and talk, for purposes of local understanding and action.”

Assuming that people's utterances are our entrance into inferring something about the cognitive structure which underpins their thinking processes, the striking level of correlation between the number of linking words and the marks achieved on 'conceptual' problems (that is, ones that appear to require an understanding of relational aspects of the key concepts in differentiation) suggests that the conceptual students are thinking about the problems in quite different ways from the procedural.

Their use of causal connectives suggests that they are explicitly linking the two apparently disparate domains of mathematics and chemistry together. They use their knowledge of mathematics to support their knowledge of chemistry and they use their knowledge of chemistry to contextualize their knowledge of mathematics.

In Fauconnier's terms, this suggests that their sequence of utterances tend to be connected and lie within a single mental space. In contrast, the transcripts of the procedural students, which have remarkably few linking words, indicate they are developing a series of disjoint mental spaces.

The ability to construct conjoined mental spaces would appear to allow the conceptual students access to ideas from chemistry and mathematics flexibly: to use the ideas from one conceptual domain easily and naturally within the other.

References

Kruger, D. (1981). *An introduction to phenomenological psychology*. Pittsburgh, PA: Duquesne University Press.

Skemp, R.R. (1976) "Relational understanding and instrumental understanding." *Mathematics teaching*, 77, 20-26.

Perry, W.G. (1970) *Forms of intellectual and ethical development*, New York: Holt, Rinehart and Winston

Sweetser, E. (1990) From Etymology to Pragmatics: Metaphorical and cultural aspects of semantic structure, Cambridge: C.U.P

Tall, D. and Razali, Md. R. (1993), "Diagnosing Students' Difficulties in Learning Mathematics", *International Journal of Mathematics in Education, Science & Technology*, 24, 209– 202.

Benn, R. (1997) Adults Count Too, Leicester : NIACE

Hiebert, J., & Lefevre, P. (1986), "Conceptual and procedural knowledge in mathematics: an introductory analysis" in J. Hiebert (Ed.), *Conceptual and Procedural Knowledge: The case of mathematics.* Hillsdale, N. Jersey: Lawrence Erlbaum Associates.

Tall, D., (1991) "Intuition and rigour: the role of visualization in the calculus", in Zimmermann, W. & Cunningham, S. *Visualization in Mathematics*, M.A.A..

Fauconnier, G. 1985. *Mental Spaces*. Cambridge, Mass.: MIT Press.