EXPLAINING VARIABILITY IN RETRIEVAL TIMES FOR ADDITION PRODUCED BY STUDENTS WITH MATHEMATICAL LEARNING DIFFICULTIES

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Predictors of retrieval times produced by students having difficulty developing a reliance on retrieval for simple addition were discovered. The findings support the notion that separate limitations operate in working memory when retrieval occurs and call into question the use of the term' retrieval deficit' to explain difficulties experienced by these students.

INTRODUCTION

The inability to develop, strengthen, and access associations in memory that allow for the rapid and accurate retrieval of answers to problems such as 3+2, 4+5 and 8+7 is a distinguishing and persistent characteristic of a mathematical learning difficulty (Jordan & Montani, 1997; Ostad, 1997; Geary, Hamson, Hoard, 2000; Robinson, Menchetti, & Torgesen, 2002). When students with mathematical learning difficulties do retrieve answers to simple addition problems, retrieval times appear unsystematic. Factors accounting for variability in reaction times (RTs) for retrieval trials remain unidentified for children with learning difficulties (Geary, 1990; Geary and Brown, 1991).

To date, investigations of retrieval times have been examined with reference to problem-specific variables only, such as problem size. This paper outlines a study that set out to explain variability in RTs produced by students with learning difficulties as they retrieved answers to simple addition problems, using performance-specific and learner-specific variables. The results based on a 3-level hierarchical linear model indicate that retrieval time for solving an addition problem will increase if a student has previously performed the problem incorrectly or if the student has applied a different strategy to solve the problem, on a different occasion. The findings support the notion that sources of increase in RTs are two separate working memory limitations, one relating to source activation and the other relating to a response selection mechanism. These sources of limitation are the same as those identified for normally achieving children.

THEORETICAL FRAMEWORK

The 'problem-size effect' is an important aspect of research in the area of simple arithmetic given the robustness of findings supporting this phenomenon (Ashcraft, 1992). The problem-size effect refers to the finding that as the 'size' of the problem increases, the time taken between the presentation of a problem and a response (RT) also increases. The explanation of the problem-size effect in simple addition performance amongst children is straight forward: If children use a counting strategy then RTs will increase as the size of the problem increase, as more counts are required. However, the problem-size effect is also evident when answers to simple addition problems are directly retrieved by children (Geary, 1990) and adults (LeFevre, Sadesky & Bisanz, 1996). Hopkins and Lawson (2002) explain the problem-size effect in retrieval times to addition problems as follows:

> As the size of the problem increases, the time taken to retrieve the answer increases because the link associating the problem with the answer is less frequently used and is more prone to interference from links to incorrect facts associated with similar problems or with previous incorrect performance. (p.144)

Thus problem size is indirectly related to activation strength.

To identify possible performance-specific variables that could be used to predict retrieval times, theory relating to processing limitations was reviewed. Based on a cognitive science perspective, the concept of 'working memory' is commonly used to depict the limiting aspect of the processing system. However, researchers view the limited nature of working memory differently. Based on explanations of dual-task interference, at least three explanations are evident in the literature. Working memory is thought to encompass: (1) a limited mechanism that creates a bottleneck in processing when more than one response is required for selection (Pashler, 1994); (2) a source activation limitation relating to the amount of knowledge that can be activated at any one time by the information currently held in working memory (Anderson, Reder & Lebiere, 1996); and (3) a limited set of processing resources that fuel the executive component of working memory so the greater demand placed on the executive, the more its efficiency at performing certain functions will be reduced (Gathercole & Baddeley, 1993).

It is possible that the speed of retrieving an answer to a simple addition problem is influenced by a source activation limitation. Siegler's strategy choice model (Siegler & Jenkins, 1989; Siegler & Shipley, 1995) is based on the assertion that both correct and incorrect answers are stored in LTM memory each time a simple addition problem is performed. If source activation has to be divided amongst correct and incorrect answers, then the time taken to activate or discriminate the correct answer will be affected. In the present study we test the premise that retrieval times are influenced by a source activation limit.

It is also possible that the speed of retrieving an answer to a simple addition problem is influenced by a bottleneck effect as demand is placed on a limited response selection mechanism. In a review of studies comparing the processing speed of younger and older children, Chi and Gallagher (1982) found that a major limitation of children's processing speed occurred at the response selection stage. When task complexity increased, as the number of possible choices of response increased, they reported that children were particularly disadvantaged compared with adults. There are many strategies that can be applied to perform simple addition, including Count All strategies, Count On strategies and Decomposition Strategies (Hopkins & Lawson, 2002). These different strategies may compete for expression so that the problem stimulus activates not only candidate answers but also representations (schemas) for the procedural knowledge associated with the different strategies (Siegler & Jenkins, 1989). In the present study we test the premise that retrieval times are influenced by a limited response selection mechanism.

Furthermore, it is possible that demand placed on limited resources fuelling the executive could influence retrieval times. A retrieval model for addition presented by Campbell and Oliphant (1992) introduced the idea that a single problem can activate multiple responses (including, for example, responses relating to the operation of addition, multiplication and naming). When this occurs, the executive component is required to manage the candidate set of answers and inhibit the processing of irrelevant information. It follows then that the greater number of associations activated, the greater the demand placed on the executive, which could result in slower RTs. However, due to the difficulty in identifying when irrelevant associations are activated but inhibited, this aspect of a working memory limitation was not examined in the present study.

METHODOLOGY

Participants and procedure

Six students aged between 13yrs 9 months and 17 yrs agreed to participate in the study. All attended a secondary college (catering for students in Years 8 to 10) in suburban Adelaide and had previously been taught by the principal researcher. All students were performing poorly in mathematics, five of the six being in remedial mathematics classes. The remaining student was in Year 8 for which there was no remedial class. The students' scores on the Standard Progressive Matrices Test (Raven, 1938) were all below age means. Selection of students was also based on classroom observations that identified them as regularly using fingers to count on for simple addition.

Addition performance was assessed using a problem set consisting of 65 addition problems, written in the form m+n, where n,m>0 and n≥m. Problems included: all 45 single digit addition problems where m, n<10, 10 addition problems where m<10 and n=10, and 10 addition problems where m<10, 10<n<15.

Students individually performed 26 problems each school day, until the problem set was completed. This procedure was then repeated until the problem set was performed five times (recorded as occurring in time interval 1 to 5). Problems were shown one at a time on a computer programmed to display problems in random order, as well as record correct responses and the number of seconds taken to respond (correct to one decimal place). After each problem was solved (referred to as a trial), the student was required to describe the strategy they had used. Based on self-report plus observation (student performance was videotaped) the strategy used on each trial was coded as either a counting strategy, a decomposition strategy, retrieval (commonly reported as "I just knew it"), or as undefined. The approach is similar to that first adopted by Siegler (1987). Only RTs to correct trials where students reported using retrieval were analyzed in the present study.

Analysis

A number of assumptions are made when interpreting a traditional linear regression equation for predicting RTs. Regarding performance based on a retrieval strategy, it is assumed that the processes of encoding the problem, selecting a response and stating the answer require a constant amount of time. We refer to these collectively as the production component of processing. The activation component is dependent on a problem-specific variable relating to size and indirectly relating to activation strength. The PROD variable (the value of the product of the addends) is typically the best predictor of retrieval times (Miller, Perlmutter and Keating, 1984; Geary & Brown, 1991). The traditional equation used to predict RTs to retrieval trials for simple addition is represented by Equation (i).

$$RT = \underbrace{a}_{\text{production component}} + \underbrace{b(PROD)}_{\text{activation component}} \quad \ldots\ldots\ldots\ldots(i)$$

In the multilevel analysis framework adopted in this study, RTs to correct retrieval trials were identified as having predictors that could operate on three different levels: an occasion level, a problem level and a student level. The focus of interest in the analysis was to test whether a variable proposed to be related to a source activation limit (labeled Previous Error or PE) and a variable proposed to be related to a limited response selection mechanism (labeled Transformation or TRANS) would emerge as significant predictors of variation in student retrieval times. A description of the variables constructed at the different levels is given below.

At the occasion level, a dummy variable representing the occurrence of a previous error (PE) was tested. A trial was coded with PE=1 if a previous error was made on the same problem by the same student on any preceding trial, otherwise PE=0. At the occasion level, a dummy variable representing practice (PRAC) was also tested to account for expected variation in RTs over time, given that each problem was presented five times to students. A trial was coded with PRAC=0 if the trial occurred in either time interval 1, 2 or 3, or it was coded with PRAC=1 if it occurred in time interval 4 or 5.

At the problem level, the traditional PROD variable was tested as well as a dummy variable, labeled TRANS, representing multiple strategy use (indicative of the transformation process where a reliance on retrieval is in an early stage of development). For each student, a problem was coded with TRANS=1 if they had correctly used a strategy other than retrieval on the problem, otherwise it was coded with TRANS=0 (indicating only retrieval had been used).

At the student-level, the variable RAVEN was used to predict retrieval times, representing the raw score each student achieved on the Standard Progressive Matrices Test.

Hierarchical Linear Modeling (HLM) (Bryk & Raudenbush, 1992) was used to analyze retrieval times because of its appropriateness for data that have a multilevel structure. The outcome variable was the time taken to correctly retrieve an answer to a simple addition problem. The equation used in the present study to predict RTs to retrieval trials is represented by Equation (ii). Interaction variables at the same level were created by multiplying two predictor variables together (Jaccard, Turrisi & Wan, 1990).

$$\begin{aligned}\text{RT}= \; & a_1 + a_2(\text{RAVEN}) + a_3(\text{PRAC}) + a_4(\text{TRANS}) + a_5(\text{PE}) \\ & + a_6\underbrace{(\text{TRANS})(\text{PRAC}) + a_7(\text{PExPRAC})}_{\textit{production component}} \\ & + b_1(\text{PROD}) + b_2(\text{RAVEN})(\text{PROD}) + b_3(\text{PROD})(\text{PRAC}) \\ & + b_4(\text{TRANSxPROD}) + b_5(\text{PROD})(\text{PE}) \\ & + b_6\underbrace{(\text{TRANSxPROD})(\text{PRAC}) + b_7(\text{PROD})(\text{PExPRAC})}_{\textit{activation component}} \quad \text{(ii)}\end{aligned}$$

Using multilevel analysis it was possible to test the supposition that the variables TRANS and PE represent situations where different working memory limitations are operating: If TRANS is related to a limited response selection mechanism then it should influence the production component of processing (which incorporates the time taken to select a response) but not the activation component; if PE is related to a source activation limit then it should influence the activation component of processing but not the production component.

SUMMARY OF FINDINGS AND DISCUSSION

The mean percentage of trials where retrieval was correctly performed by each of the six participants was 41.9%, ranging from 20% to 62.2%. This data set (including 817 RTs) was analyzed using the HLM procedure previously described. (The steps made to construct equations at each of the three levels and a detailed description of the results is not given in this paper but is available upon request.)

An analysis of the fully unconditional model indicated that 54.2% of variance in retrieval times could be attributed to problem-level differences, 33.7% to occasion-level differences and 12.0% to student-level differences.

The analysis of the final model indicated that three factors significantly predicted RTs to correct retrieval trials. By substituting in the estimated coefficients (msecs) and including only significant predictors for the conditional model, the final equation is shown by Equation (iii).

$$\text{RT} = \underbrace{1{,}727 + 706(\text{TRANS})}_{\textit{production component}}$$

$$+ \underbrace{15(\text{PROD})(\text{PE}) - 16(\text{PROD})(\text{PExPRAC})}_{\textit{activation component}} \ldots\ldots \text{(iii)}$$

The TRANS variable was a significant predictor of retrieval times. Thus the time taken to correctly retrieve an answer to an addition problem increased if a student had previously solved the problem using a strategy other than retrieval. In our theoretical analysis the use of additional strategies is argued to impact on retrieval times due to a bottleneck effect, where a range of responses compete to be selected. The finding that the TRANS variable impacted the production component of processing and not the activation component supports this theoretical analysis.

The PROD variable explained a significant amount of variation in retrieval times for problems that had been previously performed with error (indicated by the PE variable). Thus the time taken to correctly retrieve an answer to an addition problem increased (in proportion to the size of the problem) if a student had previously given an incorrect answer to the problem (using either a counting strategy, decomposition or retrieval). The finding that the PE variable impacted the activation component of processing and not the production component was hypothesized and supports the assertion that a previous error increases the time taken to activate a correct answer because the energy source for activation is dispersed to an incorrect association. The effect the source activation limitation had on retrieval times was found to reduce with practice.

Estimated variances for the final model indicated that significance residual variance remained to be explained at the problem and student levels (particularly the student level, given that the RAVEN variable was not a significant predictor of retrieval times).

The findings in this study suggest that the retrieval times of students who have difficulty developing a reliance on retrieval for simple addition may not be as unsystematic as we have previously thought. A significant amount of variance in the retrieval times of these students can be explained by factors that are thought to underlie performance limitations of normally achieving children. This result suggests that it may be misleading to describe students with learning difficulties as having a 'retrieval deficit' (Geary, 1993; Robinson, Menchetti & Torgesen, 2002) because this label implies that limitations outside the bounds of normal variation are operating. The sources of limitation impinging on retrieval are the same for students with learning difficulties as those identified for normally achieving students. Further research is needed to test the significance of the magnitude of the limitations experienced by students with learning difficulties.

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