FROM SINGLE BELIEFS TO BELIEF SYSTEMS: A NEW OBSERVATIONAL TOOL

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Two of the greatest problems of research on affective factors, and in particular, research on beliefs, is what *and* how *we observe. The first difficulty is due to the lack of a clear terminology; but even once it has been clearly decided what to observe, it is not easy to put this into practice. This report describes from a theoretical point of view the results obtained using a new questionnaire appositely designed to overcome some critical points of beliefs' observation.*

INTRODUCTION

The recent book on beliefs (Leder, Pehkonen & Toerner, 2002) shows a growing interest for this construct in mathematics education. The contributions underline the most important problems of research in this field: on the one hand the lack of an agreement on terminology (see Furinghetti & Pehkonen, 2002; Op't Eynde, de Corte & Verschaffel, 2002), on the other the difficulty in designing efficient observational tools (see Leder & Forgasz, 2002).
The problem of observing pupils' conceptions is common in mathematics education research; as Balacheff claims (1990, p.262):

> It is not possible to make a direct observation of pupils' conceptions related to a given mathematical concept; one can only infer them from the observation of pupils' behaviors in specific tasks, which is one of the more difficult methodological problems we have to face.

As concerns research about affective factors, and in particular the problems related to their observation, McLeod (1992) underlines the need of a multiple approach which alternates qualitative methods (such as interviews and direct observations) to quantitative ones (such as standard questionnaires like Likert scales) and Schoenfeld (1992, p.364) claims that:

> The older measurement tools and concepts found in the affective literature are simply inadequate.

The observational tools used in most research can substantially be grouped under five typologies (see McLeod D. & S., 2002): a) physiological measures, b) interviews, c) direct observation of subject, d) diaries, essays, etc. e) questionnaires.
Questionnaires have been proposed in different typologies (see Leder, 1985) and certainly are the most used instruments because they are easy to construct, administer and score. But, during the last years, the limits of questionnaires that ask students their agreement with certain opinions have been clearly highlighted.
First of all, with this method the beliefs that *the researcher* considers important (Munby, 1984; Eagly & Chaiken, 1998) are selected *a priori*: instead, *open* tests, such as essays or interviews, are much more effective in this respect. Moreover,

respondents generally answer these questionnaires in a context and with goals that are different to those experienced when they deal with mathematics. In these cases, the well-known difference (Schoenfeld, 1989) between *beliefs expoused* and *beliefs in action* is highlighted: sometimes they are contradictory beliefs which respond to different goals and appear in different contexts (see also Cobb, 1986), i.e. sets of beliefs grouped in separate non-interacting clusters. Besides, it is often arbitrarily assumed that certain beliefs elicit in all individuals the same emotion (see Di Martino & Zan, 2003): respondents to questionnaires are requested to express their agreement with a certain statement and their emotional disposition (like or dislike) toward the statement is inferred from the grade of agreement. But as Green underlines (1971, p.42):

> Whenever a person holds a certain belief, he must also take some attitude[1] towards that belief; and that attitude is always itself capable of formulation as a belief. It is a belief about belief.

Last but not least, the most widely used questionnaires simply make a list of commonly held beliefs without considering the connection among the beliefs. On the contrary it is fundamental to consider the structure of belief systems[2] (i.e. not only the content of beliefs but also the way people held it) because taking into account the psychological strength of beliefs can help both overcome the mismatch between beliefs expoused and beliefs in action, and in the attempt to change beliefs (Cooney, 1993).

If the use of questionnaires is criticizable for the reasons discussed above, it is undeniable that the administration and the analysis of questionnaires require less time than those of interviews or direct observations and, at the same time, allow the collection of data having a higher statistical relevance.

In a three-years Italian Project (involving many researchers) about the evolution of attitude towards mathematics (*Negative attitude towards mathematics: analysis of an alarming phenomenon for the culture in the new millennium)*, in addition to essays, interviews, direct observations and Likert scales, a questionnaire called Integrated Questionnaire on Beliefs (IQB) has been specifically designed. IQB attempts to take into account some of the criticisms to questionnaires while maintaining some of their positive features. Although the strength of the project lies in the possibility of analyzing jointly the results obtained with different instruments, this report focuses on the discussion of the results of IQB from a theoretical point of view.

METHOD

The analysis of two questionnaires used in a previous study (Di Martino & Zan, 2002, 2003) suggested us the idea for a new questionnaire (IQB). In particular we wanted to take in account the complexity of the relationships among beliefs (belief

[1] He identifies attitude with emotional disposition.

[2] Green (1971) underlines three features of belief systems: quasi-logical structure (beliefs can be primary or derivative), psychologically centrality (some beliefs are more important to people than other), clusters'isolation (sets of beliefs can be protected from any relationship with other sets of beliefs).

systems) and between beliefs and emotions. The conclusions of that study were that a single belief can be linked to different beliefs in different individuals, i.e. can belong to different belief systems, and that the same belief can elicit in different individuals different emotions. We suggested the hypothesis that the emotion elicited by the given belief is not always *simply* linked to the belief itself, but to the interaction among beliefs in the cluster containing it.

We have chosen a belief (from now onwards it will be called statement A) from a list of 12 beliefs used in our previous studies:

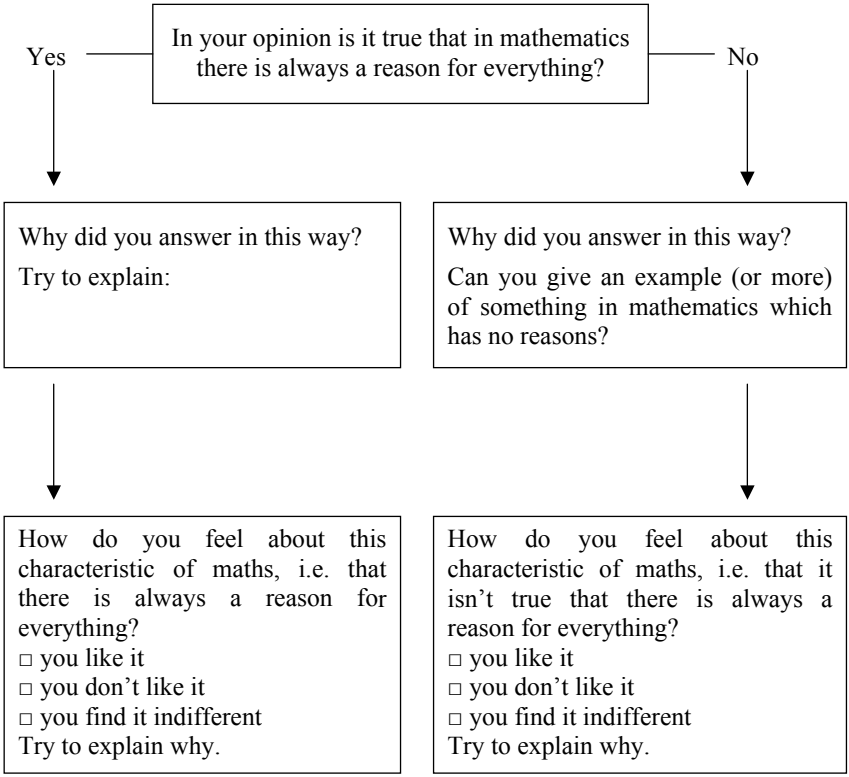
In mathematics there is always a reason for everything

The reason for the choice of statement A lies in the fact that statement A was recurrent in children's essays collected for previous research (obviously IQB's schema can be re-proposed with the choice of another belief).

Using statement A we have planned the following questionnaire:

Choose the answer (Y/N) that you most agree with.

Then follow ONLY the path signed by the arrow, and answer the subsequent questions.



IQB has been distributed in 13 classes of state middle schools (8 classes) and high schools (5 classes) and we have collected 282 questionnaires (178 from middle school and 104 from high school).

RESULTS AND DISCUSSION

The analysis of IQB has been very useful both to observe students' beliefs (together with the analysis of the results obtained with the other instruments used in the project) and for the ongoing construction of different profiles.
The numerical data are summarized in the following table:

	Yes	No
Middle school	170 (104 L-26 D-40 I)	8 (7 D-1 I)
High school	83 (46 L-12 D- 25 I)	21 (7 L-10 D-4 I)
Total	253 (150 L-38 D-65 I)	29 (7 L-17 D-5 I)

L = Like, D = Dislike, I = Indifferent

First of all we can observe that all the possible profiles (i.e. Yes-Like, Yes-Dislike, Yes-indifferent, No-Like, No-Dislike, No-Indifferent) are present, even if the percentages are considerably different. This is a further confirmation that it is arbitrary to link a fixed emotional disposition to a certain belief: for more than 40% of the students the agreement with statement A does not correspond to a positive emotional disposition and for approximately 24% of the students the disagreement with A is associated with a positive emotional disposition.
Another evidence is the high number of agreements to the statement A, but the difference in the percentages between high school and middle school students is significant: about 95.5% in middle school and about 80% in high school. Probably the greater *autonomy* of older students, who are less interested in the search for a *right* answer (i.e. an answer that they think the teacher would appreciate), can explain this difference in the percentages. A very important point for the interpretation of results is that in IQB respondents have to justify their agreement or disagreement. The analysis of the justifications shows that some answers of middle school students are influenced by commonplaces or by the belief that there is a *right* answer:[3]

6th Grade: I think so because mathematics is not an opinion[4]

7th Grade: Because mathematics is an exact science

[3] Some idiomatic expressions may be lost in the translation of the transcripts.
[4] It is an idiomatic expression in Italian meaning that mathematics is an exact science and therefore there is no room for autonomous ideas.

7th Grade: Since mathematics is thinking and logics there is a reason for everything

6th Grade: I think it is the most correct answer

8th Grade: I answered this way because I have heard that it is true

Other answers are based on personal school experiences and the focus is on mathematics as a subject matter instead of as a science:

6th Grade: I answered yes because I have always met problems with explained results. As in the case of the infinite number of numbers, this is possible because one can always add one to any number

11th Grade: I answered no because we have been taught many things without a clear explanation, they are that way and that is it

9th Grade: I answered no because sometimes we use methods which we have been taught since elementary school without an explanation of why things are so

6th Grade: I answered yes because the problems I have faced up until now have always had an answer to them

The justifications to disagreement are particularly interesting above all for the request to produce a mathematical example that supports their disagreement with statement A. We can note students' doubts and curiosities and personal epistemologies:

12th Grade: Why is $\infty \bullet 0$ not 0? Since elementary school we have been taught that any quantity multiplied 0 is 0. Now we discover that it is not true, why?

9th Grade: I do not understand why a number times zero is equal to zero and not to the number itself. As a matter of fact if I have a thing and I multiply it by nothing (zero) I still have that thing, it does not disappear!

6th Grade: For example: why is 2+2 equal to 4 and not 3 or something else

8th Grade: Why are numbers infinite?

11th Grade: An example is the association between numbers and names or figures. Why do we associate the word 'three' to the concept of three and why do we represent this concept with the symbol 3 and not 7?

Also in the case of agreement with statement A the analysis of the justification is meaningful because it can highlight belief systems linked to the agreement with statement A:

6th Grade: I answered yes because mathematics is difficult to understand so there must be an answer to all questions

11th Grade: Mathematics is the science of certainties: even situations which apparently are unreal have a proof

9th Grade: Because mathematics is based on explanations (although they are useless since I do not understand anything)

Moreover, the request to justify both the agreement with statement A and, using Green's terminology, the attitude about it can allow to single out some different interpretations of the statement: these *misunderstandings* are not rare and it is important to recognize them in order to interpret the results and to improve the instrument.

In the case of statement A it seems to us that some students have expressed their agreement/disagreement towards another statement: *In mathematics there are many*

open questions without an answer (obviously this misunderstanding depends on the language in which the questionnaires are written: in the case of IQB the language is Italian):

6th Grade: I answered yes because mathematics still has many secrets

Some students have expressed their disagreement with examples out of the mathematics context:

6th Grade: For example mathematics does not have answer to: 'does space have an ending?'

In the case of the emotional disposition associated with the agreement/disagreement on statement A some students have simply described their emotional disposition towards mathematics: for example *I like mathematics*. Probably this can be due to a careless reading of the request, nevertheless it is important, as before, to recognize the possibility that someone answers to a different question.

But the most important characteristic of IQB is the request to justify the emotional disposition associated with the agreement/disagreement to the statement A. As we discussed earlier, previous studies (see Zan & Di Martino 2003) suggested that the emotional disposition associated to a certain belief is not directly linked to that single belief but to a belief system containing it. In the design of IQB we hypothesized that asking respondents to justify the emotional disposition could give further information about their beliefs linked to the original belief (i.e. the belief that it is/is not true statement A) or about their personal epistemology:

8th Grade (YES-L): I like it because if we follow the rules we cannot fail

12th Grade (YES-I): It is indifferent to me because if my solutions to the exercises are correct I do not care if there is another explanation

8th Grade (YES-D): I do not like it because I like mysteries and unsolvable enigmas. In mathematics everything has an explanation, it is a boring world for boring people

9th Grade (NO-L): I like the fact that in mathematics not everything has an explanation because it makes it more intriguing. Besides this stimulates the desire to discover new things

12th Grade (NO-I): It is indifferent to me because I am not a mathematician

9th Grade (NO-D): I do not like it because it is difficult to learn rules by heart which we do not understand or which we cannot find with reasoning

The request to justify the attitude highlights that there are deeply different motivations for answering *indifferent,* these typologies are not so evident using other instruments as semantic differential scales or Likert scales.

In fact the answer *indifferent* can derive from:

a) A like or a dislike not too marked (this is the case typically considered with semantic differential scales):

9th Grade: It is indifferent to me, not really, I mean I like the fact of getting lots of answer but I am not so enthusiastic about it

b) An alternation between like and dislike depending on the contexts:

8th Grade: It is indifferent to me because I like finding answers, but not to everything (…) I do not like it when the teacher asks me to explain everything I have said

c) The belief that a personal opinion, whether right or wrong, cannot change anything:

11th Grade: It is indifferent to me because mathematics is that way, and even if statement A were false, it would be that way the same

d) The difficulty in recognizing and expressing the personal attitude towards a certain belief:

8th Grade: I have never thought about it enough and out of the blue I do not know if I like or not

CONCLUSIONS

Many researchers have underlined the importance of a discussion about instruments to observe beliefs. Obviously the development of beliefs'theory contributes to this debate: in particular the increasing relevance given to belief systems rather than to single belief *forces* researchers to construct adequate instruments.

IQB was created to meet this need and to preserve some of questionnaires'positive characteristics (like the easiness of administring and analyzing them).

The request of IQB is not only to express the agreement/disagreement with the statement A but also to motivate it.

But the greatest innovation of IQB is the request of expressing the reasons of one's emotional disposition toward the declared belief.

This allows the researcher to highlight other beliefs linked to the declared belief and the psychological centrality of the declared belief, thus giving information about the belief system containing it.

The results obtained when first using IQB are encouraging: analyzing the answers to IQB we found links between the agreement/disagreement with statement A and other beliefs.

The next step is to experiment IQB's schema with other statements: like those typically used in beliefs'research.

Obviously IQB can be improved and some changes have already been made. But above all, other instruments may be constructed. As a matter of fact we believe it is important to work in this direction both to improve the consistency of the instruments with the theory and, to interpret and compare the results obtained, in this field, up until now at best.

References

Balacheff N.(1990) Towards a problématique for research on mathematics teaching, *Journal for Research in Mathematics Education,* vol.21(4), 258-272.

Cobb P. (1986) Contexts, goals, beliefs, and learning mathematics, *For the Learning of Mathematics,* vol.6(2), 2-9.

Cooney T., Shealy B. & Arvold B. (1998) Conceptualizing belief structures of preservice secondary mathematics teachers, *Journal for Research in Mathematics Education,* vol.29(3), 306-333.

Di Martino P. & Zan R. (2002) An attempt to describe a 'negative' attitude toward mathematics. In Di Martino P. (Ed.) *Proceedings of the MAVI-XI European Workshop,* 22-29.

Di Martino P. & Zan R. (2003) What does 'positive' attitudes really mean?, in Pateman N., Dougherty B. & Zilliox J. (Eds.) *Proceedings of the 2003 Joint Meeting of PME and PME-NA*, vol.4, 451-458.

Eagly A. & Chaiken S. (1998): Attitude structure and function, in Gilbert D., Fiske S. & Lindzey G. (Eds.) *The handbook of Social Psychology*, Mc Graw Hill, 269-322

Furinghetti F. & Pehkonen E. (2002) Rethinking characterizations of beliefs, in Leder G., Pehkonen E. & Toerner G. (Eds.) *Beliefs: a hidden variable in mathematics education.* Kluwer Academic Publishers, 39-58.

Green (1971) *The activities of teaching*, New-York: McGraw-Hill.

Leder G. (1985) Measurement of attitude to mathematics, *For the Learning of Mathematics,* vol.5(3), 18-21.

Leder G. & Forgasz H. (2002) Measuring mathematical beliefs and their impact on the learning of mathematics: a new approach, in Leder G., Pehkonen E. & Toerner G. (Eds.) *Beliefs: a hidden variable in mathematics education.* Kluwer Academic Publishers, 95-114.

Leder G, Pehkonen E. & Toerner G. (Eds.) (2002) *Beliefs: a hidden variable in mathematics education.* Kluwer Academic Publishers.

McLeod D. (1992) Research on affect in mathematics education: a reconceptualization, in Grouws D. (Ed.) *Handbook of research on mathematics learning and teaching,* New York: MacMillan, 575-596.

McLeod D. & McLeod S. (2002) Synthesis – beliefs and mathematics education: implications for learning, teaching and research, in Leder G., Pehkonen E. & Toerner G. (Eds.) *Beliefs: a hidden variable in mathematics education.* Kluwer Academic Publishers, 115-123.

Munby H. (1984) A qualitative approach to the study of a teacher's beliefs, *Journal for Research in Science Teaching,* vol.21(1), 27-38.

Op`t Eynde P., de Corte E. & Verschaffel L. (2002) Framing students' mathematics-related beliefs. A quest for conceptual clarity and a comprehensive categorization, in Leder G., Pehkonen E. & Toerner G. (Eds.) *Beliefs: a hidden variable in mathematics education.* Kluwer Academic Publishers, 13-38.

Schoenfeld A. (1989) Explorations of students'mathematical beliefs and behavior, *Journal for Research in Mathematics Education,* vol.20(4), 338-355.

Schoenfeld A. (1992) Learning to think mathematically: problem solving, metacognition, and sense making in mathematics, in Grouws D. (Ed.) *Handbook of research on mathematics learning and teaching,* New York: MacMillan, 334-370.