THE ROLE OF GESTURES IN CONCEPTUALISATION: AN EXPLORATORY STUDY ON THE INTEGRAL FUNCTION

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The paper reports on a case study from a teaching experiment on the construction of meaning of integral at secondary school level, starting from the cognitive root of area. In the considered activity, students are faced with some graphs of functions and are asked to work in group to trace the corresponding integral functions. The analysis considers the gesture-speech relationship and is carried out integrating cognitive and semiotic perspectives. The aim is to study how gestures enter in the process of conceptualisation, in a context of social construction of knowledge.

INTRODUCTION AND THEORETICAL FRAMEWORK

Despite their simple appearance, graphs of functions are very complex mathematical objects. Far from being transparent with respect to their meaning, they carry in a holistic and compressed way a big amount of information. To cope with graphs and effectively use them to interpret, model, or build new relations involves making use (often in an unconscious way) of different resources, among which language and gestures play important roles. The focus of this paper is to analyse how gestures intervene in the construction of meaning for the integral function, as a case of conceptualisation process in a functional context.

The relation between gestures, language and conceptual processing is still debated. Different hypothesis have emerged in psycholinguistics, among which the Lexical Retrivial Hypothesis and the Information Packaging Hypothesis (Kita, 2000). Whereas the former restricts the role of gesture in giving lexical access to generate sentences, the latter recognizes it as having an essential role not only in the process of *speaking*, but also in that of *thinking*: it states that "gesture is involved in the *conceptual planning* of the message to be verbalised, in that it helps speakers to "package" spatial information into units appropriate for verbalisation" (Alibali et al., 2000, italics in the original). Studying the relationships between gesture and speech, some researchers have identified the *gesture-speech match*, when a gesture expresses the same information conveyed by the verbal utterance, and the *gesture-speech mismatch*, when a gesture contains information not expressed in the uttered speech (Alibali et al., 2000; Goldin-Meadow, 2000). Kita (2000) argues: "[gesture-speech] discordance appears because spatio-motoric thinking explores organizings of information that analytic thinking cannot readily reach". *Spatio-motoric thinking* and *analytic thinking* constitute two different yet complementary modes of thinking: the former "organizes information with action schemas and their modulation according to the features of the environment" (ibid.), whereas the latter "organizes information by hierarchically structuring decontextualized conceptual templates" (ibid.). McNeill

(1992), on a different position, maintains that for the speaker gesture and speech form complementary yet intricately interwoven parts of a *single integrated system*, and that they together allow constructing multiple representations of a single task.

The link between gesture and language seems to have neural basis, thus lying in the very nature of human body. Rizzolatti and his colleagues in their neurophysiological studies on monkeys have found the existence of some single neurons, dubbed as *mirror neurons*, that are active both when the subject makes particular gestures of reaching and when it watches a person making the same gesture (Rizzolatti & Arbib, 1998). By representing the observed action in terms of internal motor encoding, the mirror neurons construct a link between self-actions and observed actions, thus providing a mechanism for the sharing of meaning. These cells are in an area (homologous to Broca's area in the human brain) that is critically involved in the programming of human speech. According to Rizzolatti, this supports the hypothesis of a gestural origin of language in human evolution: the impulse to imitate the chains of actions performed by our fellows would have brought about, through the inhibition of the physical action, the transfer of the imitation to a symbolic level.

In science education, the role of gesture has been taken into account for instance by Roth (2002). Analysing videotaped science classes of all levels, he has found that independently of the age, individuals draw a lot in gesture, especially when, in laboratory context, they have to explain facts and things they are not yet familiar with. In his perspective, "gestures are not only a co-expression of meaning in a different modality but also constitute an important stepping stone in the evolution of discourse" (ibid.).

With respect to mathematics, recent studies have begun to analyse the gesture component in the learning context, as a means of gaining information about the cognitive processes of a subject (see Edwards, 2003; Arzarello & Robutti, in print). Developing from the work of McNeill (1992), a classification of different kinds of gestures has been pointed out, identifying: *deictic* gestures (pointing to an object), *metaphoric* gestures (the content represents an abstract idea that has no physical form), and *iconic* gestures (bearing a relation of resemblance to the semantic content). The iconic gesture has been further analysed: Edwards (2003) refers to *iconic-physical* gesture in the case of concrete or physical referent, and to *iconic-symbolic* gesture if it relates to "written symbolic or graphical inscriptions, and/or to the procedures associated with these inscriptions"; Arzarello and Robutti (in print) speak of *iconic-representational* gesture when it refers specifically to graphs or other graphic representations of mathematical concepts.

In a semiotic-cultural perspective, Radford considers gestures as a type of signs. He has called *semiotic nodes* those "pieces of the students' semiotic activity, where action, gesture, and word work together to achieve knowledge objectification" (Radford et al., 2003). Conceptualising is seen in terms of objectifying knowledge, that is making things and relations apparent in the universe of discourse (ibid.).

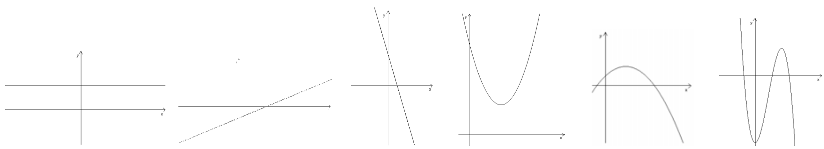
In this report, the relations between gestures, language and the process of conceptualisation are looked at through two interpretative lenses: the cognitive one, which considers how gestures enter in the thinking processes of individuals, and the semiotic one, to study how they work as body signs that join the individual to the social practice of mathematics (Radford et al., 2003).

TEACHING EXPERIMENT

The research is based on a teaching experiment aimed at studying the concept of integral starting from the cognitive root of area under a function (Tall, 2002). It was set up in a secondary school class (from an Italian "Liceo Scientifico", a scientifically oriented high school) during grades 11, 12 and 13. In grades 11 and 12, before facing calculus, students were involved in activities on area and length measurements and with approximation problems on areas under given functions, using both paper and pencil and symbolic-graphic calculators (Robutti, 2003). The notion of definite integral was thus conceptualised as the area (with sign) under a curve, whereas the traditional approach starts from indefinite integral as inverse of derivative. In the 13th grade, the class in calculus course was introduced to limits and derivative, and their meanings with respect to the graph. The integral function was then approached as the function that measures the area (with sign) under a function between two values: one fixed and the other one variable (x). The activity presented below focuses on the graphical representation of the integral function and its generation from a given graph. During the whole experiment, students were involved in small group works and in class discussions conducted by the teacher. Both group works and class discussions were video-recorded, thus providing the data for the research.

PROTOCOL ANALYSIS

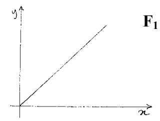
Groups are given a series of worksheets, with the following task: "Work on the graph from a qualitative point of view: for each of the following functions *f*, determine and sketch the corresponding integral function *F*". At the bottom of each page, the graph of a function is given, and the top of the page is left blank for the drawing of the corresponding integral function. Here are the six functions f:

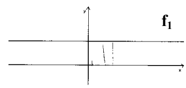


f_1 f_2 f_3 f_4 f_5 f_6

One girl and two boys compose the observed group: Erika, Francesco and Fabio. They are brilliant students, with different attitudes towards mathematics: Erika is precise and schematic in her study, Francesco and Fabio are more intuitive.

The group quickly solve the case f_1, tracing the integral function F_1 as a straight line (the figure is on the right). Then, to cope with case f_2, whose negative values raise some problems, the students come back to the previous case f_1 and reason on it:





59. Fabio: *Here you have to know* [he is putting his left forefinger on the origin of the Cartesian plane containing f_1, and with his right one is pointing to the upper straight line F_1. Then he places his right forefinger on the intersection point between the function f_1 and the y-axis] *as x increases*…[he has moved simultaneously both his forefingers from the y-axis rightwards: the left one on the x-axis, the right one pointing f_1 and covering the area under it] *how much the area increases* [keeping the left hand in the same position, he is placing his right forefinger on the straight line F_1].

60. Erika, overlapping: *How much the area is*.

61. Fabio continues: *How much the area is* [while speaking, he has repeated the previous gesture moving both his forefingers on the same Cartesian plane, and then he has set the right one to the upper straight line F_1].

62. Erika and Francesco: *Yes*.

63. Fabio moves simultaneously his forefingers on the sheet: the left one on the x-axis of the lower Cartesian plane (with f_1), from the origin rightwards, and the right one in the corresponding upper Cartesian plane, moving along the function F_1 from left to right. Concluding the movement, he says: *And here it increases more and more… because the more I go ahead, and the more the area increases*.

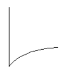
Using both his hands, Fabio (#59) performs a gesture made of three parts: first he points to the origin of the lower Cartesian plane (containing the given function) and simultaneously to the upper one (containing the traced integral function); then he moves both his fingers on the lower Cartesian plane, depicting the increasing of the abscissa and the resulting behaviour of f_1; finally he comes back to the function F_1. This complex gesture is co-ordinated with a single sentence, forming with it a unique integrated system by which the student carries out several tasks: he copes with the function f, he links f with the variation of the subtended area, and conceives it as a new function. The functions (both the given f and the integral function) are thought in terms of a *covariance view*, that is observing one variable changing respect to another (Slavit, 1997): the simultaneous movement of the forefingers on the Cartesian plane expresses very effectively the two variations and their relation, as a whole phenomenon. To give an account of the situation, the speech, that has an intrinsic linear-ordered structure, has to split it into two consecutive parts (#59: *as x increases… how much the area increases*). The gesture is not redundant with respect to the speech, since it conveys information left implicit by words, as the starting point for the area variation (that has been fixed at zero by the teacher in introducing the activity) and the positive direction of the increments. It is an *iconic-representational* gesture that, referring to the *metaphoric* structure of the Cartesian plane, represents the abstract idea of increasing of the variable x. In line #61 Fabio repeats the same gesture-word schema, and in line #63 he goes further: maintaining the established co-ordination between the two hands, he moves his finger along the graph of the integral function, by successfully correlating it with the given function f. Differently from the previous ones, the utterance is now partly self-based (#63: *the more I go head*), that is the student takes the point of view of the variable x. Roth (2002) suggests that the subject-centered perspective, requiring a lower cognitive effort, provides cognitive advantages over abstract perspective and that for this reason is often taken by pupils in early stages of conceptualisation processes. Here the self-based perspective combines with verbs of motion (#63: *I go head...it increases more and more*), describing F in dynamic terms. The conceptualisation of a curve (a static object) as a dynamic process is very frequent in mathematics and in everyday situations: the cognitive mechanism that allows it has been called *fictive motion* and consists in thinking a line as the motion of a traveller tracing that line (Nùñez et al., 1999). In the present case, Fabio has to cope with two variations (the given f and the varying area under it) expressed in a single graph. By taking the point of view of the variable x (that refers to f), he can concentrate on the behaviour of the function F, that is the focus of the conceptualisation process. The integral function rises here from the student's co-ordination between gestures and words, in a piece of the semiotic activity that can be recognizes, in Radford's terms, as a *semiotic node*.

The following excerpt regards the group tackling the case f_4:

128. Erika: *It is this area here* [with her pencil, she is pointing to the area under f_4, going from the y-axis rightwards].

129. Fabio: *Yes, yes*.

130. Francesco, reasoning by himself: *It does*... [with a single movement with his forefinger, he sketches on the desk a graph, tracing firstly quickly a vertical line segment, from the top downward, then slowing down the arc of a curve increasing rightwards from the bottom of the segment. The complete figure is represented at this side]. Fabio and Erika are not paying attention to their mate.



131. Erika traces a Cartesian plane in the upper part of the page.

136. Fabio: *It starts from zero* [he is pointing his pencil at the origin of the Cartesian plane that Erika has just finished tracing].

137. Francesco: *Yes* [with his forefinger he traces in the air in front of himself an arc of a curve similar to that represented at this side, from left to right].

138. Fabio goes on, overlapping Francesco: *Like this* [while saying *like this*, with his pencil he is miming (without writing), starting from the origin in the Cartesian plane, an arc similar to that just done by Francesco].

139. Erika: *Here* [she is covering the area under the parabola, from the y-axis to the vertex] *it does like this* [she sketches with her finger on the Cartesian plane, starting from the origin, an arc similar to those of her schoolmates].

140. Fabio: *It increases* [he is repeating his gesture of tracing an arc, as in line #138].

141. Erika: *Up to here* [she is pointing to the vertex of the parabola], *wait*....

142. Fabio: *Up to there*.

After Erika's *deictic gesture* (#128), drawing attention to the area to be represented. Francesco (#130) performs a gesture that can be identified as *iconic-representational*. Unlike the preceding iconic-representational gestures, which refer to graphs already traced on paper, the present one has not any written reference. In a semiotic perspective, it can be interpreted as a sign starting from which the graph (another sign) will later come to life, as a sort of iconic crystallized form of the gestures. It is worth observing that the student begins his representation with a vertical straight movement, which can be interpreted as tracing the y-axis: in fact he, rather than using the worksheet, is gesturing on the desk, where he has no reference to relate to. In line 141, Erika's words (#141: *here*) refer to the integral function, whereas her *deictic gesture* points to the given graph f: the *gesture-speech mismatch* allows the connection between the two functions.

As it develops, the protocol becomes richer and richer in gestures, which tend to overlap each other and which are accompanied by simple utterances, often deictics that draw group mates' attention to concurrent action (#128, #139, #141, #142). The students neither refer to the integral function with its name, nor giving it any name, rather through the generic pronoun "*it*" (#63, #130, #136, #139, #140): it is the gesturing that clarifies any ambiguity to the referent, by making perceptually available what is required for sense making. In spite of this gestural abundance, the only descriptive word referring to the integral function is the verb "*increases*" uttered by Fabio (#140; see also #59, #63, and below, #179).

Let's look at what happens when the students face function f_5:

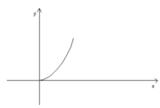
172. Erika traces the Cartesian reference in which the integral function is to be sketched.

173. Fabio points his pencil at the origin of the new Cartesian plane and immediately Erika points her pencil towards the Cartesian reference containing the function f_5.

174. While Fabio keeps his pencil pointed, Erika slowly moves her pencil rightwards along the parabola, up to the vertex.

175. Fabio: *It increases*….

178. Erika: *Here it does like this* [in the new Cartesian reference, she is tracing an arc similar to that represented at this side].



179. Fabio: *It increases*....

180. Erika: *Up to here* [she is pointing to the vertex of the parabola f_5].

Without speaking, Erika and Fabio co-ordinate their actions on the worksheet (#172-174). Fabio refers to the origin of the upper Cartesian plane, a "starting point" for the integral function, a sort of "eye" through which he looks at the graph of f and "sees" how the function F originates from it (see also #135, #138). The girl focuses attention on the behaviour of the function *f*. The resulting *iconic-representational* gesture can be compared with that previously performed by Fabio to analyse the case 1 (#63): in both cases it supports in covariational terms the link between the given graph and the new rising function. It is a sign that, put on the scene by a student, enhances reasoning not only in the subject, but in the whole group. Its features, implicitly established by the social practice, allow its use in a sort of co-operation between the students. In fact, they do not need to explain to each other what they are doing: the scene opens silently and shows very few utterances. Fabio's and Erika's words (#175-180) refer to the forthcoming integral function, describing it with in successful connection with gestures. In line #180 the girl performs a *gesture-speech mismatch*, similar to that of line #141, previously discussed.

CONCLUSIONS

The mathematical context of graphs of functions and the given task have fostered the flourishing of iconic-representational gestures. This kind of gesture appears to be performed not only with reference to represented graphs, already traced on paper (as discussed in Arzarello & Robutti, in print), but also as describers of new functional relations (i.e. the integral functions). They are expressed in graphs that come to exist only after some gestural explorations, as sort of crystallized forms of the preceding gestures. Coping with the given graphs, the students construct the integral function using a covariational view. It is supported in an effective way by the holistic character of gesturing and therefore properly described by language: in McNeill's terms, gesture can be appointed a mediating role between internal, subjective, "global-synthetic" imagery and shared, conventional, "linear-segmented linguistic structure" (McNeill, 1992).

But gesture's role goes beyond a cognitive dimension, purely internal to the subject. It is also endowed with an external dimension, coming from its semiotic nature in the social practice of mathematics. Internal and external dimensions of gestures are interwoven and mutual-affecting. Their interplay enhances individuals' thinking processes and develops a shared semiotic system in which other signs, such as graphic or symbolic representations, can emerge. In such a complex, multi-faced semiotic context, the construction of a mathematical meaning can finally be reached.

The picture portrays the scene leaving in the background two important and pervasive elements: cultural and institutional dimension. Further research is to be undertaken to widen the scope of analysis and to study how student's culture (in a broad sense) and the teaching practice affect students' gestures in the context of mathematical conceptualisation. The relationship between the cultural-social contribution and the biological-neural human basis, which is taken into account by recent research in neurology (Rizzolatti & Gallese, 1997), constitutes an intriguing issue that is still to be clarified.

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