PROGRESSIVE DISCOURSE IN MATHEMATICS CLASSES – THE TASK OF THE TEACHER

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This paper uses data from two mathematics lessons to explore the nature of progressive discourse and examine critical features of teacher actions that contribute to mathematics classrooms functioning as communities of inquiry. Features found to promote progressive discourse include a focus on the conceptual elements of the curriculum and the use of complex, challenging tasks that problematised the curriculum; the orchestration of student reporting to allow all students to contribute to progress towards the community's solution to the problem; and a focus on seeking, recognizing, and drawing attention to mathematical reasoning and justification, and using this as a basis for learning.

INTRODUCTION

Classroom discourse can be progressive in the same sense as science as a whole is progressive. Scientific progress is not one homogeneous flow; it contains innumerable local discourses that are progressive by the standard of the people participating but that, with respect to overall progress in science, may only be catching up or even may be heading in the wrong direction. The important thing is that the local discourses be progressive in the sense that understandings are being generated that are new to the local participants and that the participants recognize as superior to their previous understandings. (Bereiter, 1994, p. 9)

Our interest in classroom discourse arises, in part, from previous collaborative work based on the notion of *Communities of Inquiry*, which underpins the *Philosophy for Children* movement (see, for example, Splitter & Sharp, 1995). Key features of classrooms functioning as communities of *philosophical* inquiry are the development of skills and dispositions associated with good thinking, reasoning and dialogue; the use of subject matter which is conceptually complex and intriguing, but accessible; and a classroom environment characterized by a sense of common purpose, mutual trust and risk-taking. Our concern has been how these features can be made a part of everyday classroom practice in mathematics.

In earlier work, we have reported a high level of support among principals, teachers and mathematics educators for mathematics classrooms functioning as communities of inquiry, together with a realization that current Australian practice falls far short of this goal, partly because the cognitive demands of typical lessons are low and do not challenge children (Groves, Doig & Splitter, 2000; Doig, Groves & Splitter, 2001); and the critical role of conceptually focused, robust tasks that can be used to support the development of sophisticated mathematical thinking (Groves & Doig, 2002). In this paper, we focus on aspects of classroom discourse associated with classrooms functioning as communities of *mathematical* inquiry.

According to Bereiter (1994), classroom discourse can be progressive in the same sense as science, with the generation of new understandings requiring a commitment from the participants to working towards a common understanding, based on a growing collection of propositions that can or have been tested. In a similar vein, Cobb, Wood and Yackel (1991) contrast discussion in traditional mathematics classrooms, where the teacher decides what is sense and what is nonsense, with genuine dialogue, where participants assume that what the other says makes sense, but expect results to be supported by explanation and justification. Mercer (1995) proposed three forms of talk that can be used to aid the analysis of classroom talk and thinking: *disputational talk,* featuring disagreement and individualized decision making, with few attempts at synthesis; *cumulative talk*, in which speakers build positively, but uncritically, on previous speakers' utterances; and *exploratory talk,* where critical, but constructive, use is made of another's ideas, challenges are justified, and alternative explanations offered. It is this last category of *exploratory talk* that resonates with good thinking, reasoning and dialogue in *Communities of Inquiry.*

This paper uses data from two, apparently quite different, mathematics lessons to explore the nature of progressive discourse and examine critical features of teacher actions that contribute to mathematics classrooms functioning as communities of inquiry.

A YEAR 1 LESSON ON ADDITION IN JAPAN

This lesson, observed by both authors late last year in Japan, was taught by an "expert teacher", Hiroshi Nakano, to a Year 1 class of 40 children. The lesson was part of a sequence of lessons on addition.

The lesson commenced with children being presented with a series of flashcards with shaded and unshaded dots arranged in two rows of five, and children being asked to show how many more shaded dots were needed to "make 10". This was followed by a similar task where the flashcards showed single numerals instead of dots.

The children were then presented with the problem for the day — finding the answer to 8 + 6 and explaining the reasons for their answers. Children worked individually for 5 minutes, after which the teacher wrote 8 + 6 = 14 on the blackboard and invited particular children to write their solutions on the board.
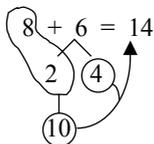
Figure 1: Girl 1's solution for 8 + 6 = 14

Girl 1's solution is shown in Figure 1. When asked, most children stated that they had used the same method. The teacher then asked the children to guess why Girl 1 had

divided the 6 into 2 and 4. Children responded that this was based on "Nishimoto-san's making 10 rule" — apparently formulated by one of the children, Nishimoto-san, in the previous lesson where the problem was to find 9 + 6.

The teacher then asked for a different solution. Boy 1's solution is shown in Figure 2.
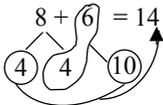
$$8 + 6 = 14$$

Figure 2: Boy 1's solution for 8 + 6 = 14

The teacher commented that this was again using "Nishimoto-san's making 10 rule", and asked for another way. Girl 2's solution, still described by the teacher as using "Nishimoto-san's making 10 rule", is shown in Figure 3. A few children said they had used this method.
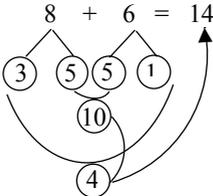
$$8 + 6 = 14$$

Figure 3: Girl 2's solution for 8 + 6 = 14

Boy 2 stated that he did not use the "making 10 rule". Children tried to guess how he found the answer — had he used a "making 5 rule"? Boy 2 said he had not and explained his reasoning as shown in Figure 4.

$$8 + 6 = 14$$

because $9 + 6 = 15$ "we did this before"

and 8 is one less than 9 . So, "if 9 becomes 8, the answer is one less".

9 + 6 = 15

1 less   1 less
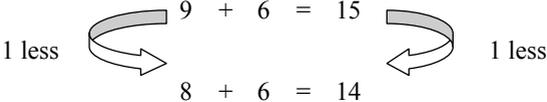
8 + 6 = 14

Figure 4: Boy 2's solution for 8 + 6 = 14

Many children clapped in response to this solution and a girl commented that this used their former knowledge of addition.

The teacher suggested that they move on to looking at 7 + 6 using the same method.

Surprisingly, rather than starting with 8 + 6 = 14, Boy 2 again started with 9 + 6 = 15 as shown in Figure 5.

$$9 \quad + \quad 6 \quad = \quad 15$$
$$8 \quad + \quad 6 \quad = \quad 14$$
$$7 \quad + \quad 6 \quad = \quad 13$$
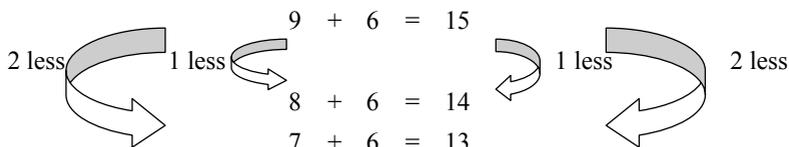
Figure 5: Boy 2's solution for 8 + 6 = 14

The teacher asked everyone to "check the hypothesis" that the answer is 13. Several children demonstrated their solutions using similar methods to those shown in Figures 1 to 3 — i.e. using the "making 10 rule".

Now that children had confirmed that 7 + 6 = 13. the teacher asked them to complete Figure 6, using Boy 2's method and confirming their answers as before.

$$9 \quad + \quad 6 \quad = \quad 15$$
$$8 \quad + \quad 6 \quad = \quad 14$$
$$7 \quad + \quad 6 \quad = \quad 13$$
$$6 \quad + \quad 6 \quad =$$
$$5 \quad + \quad 6 \quad =$$
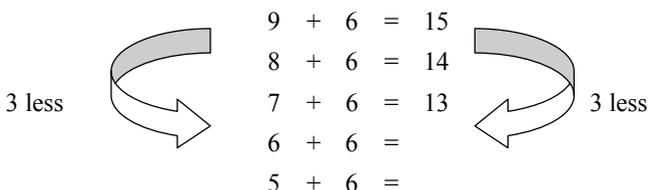
Figure 6: Using Boy 2's solution for 6 + 6 and 5 + 6

One boy continued the list to 0 + 6 and then even further to 10 + 6, 11 + 6, …, 16 + 6.

A YEAR 7 LESSON ON THE AREA OF A TRIANGLE IN AUSTRALIA

This double lesson, taught by Gaye Williams to a class of approximately 24 Year 7 girls in Australia, was videotaped as an "exemplary problem solving lesson" for teaching purposes at Deakin University. The lesson was part of a sequence of lessons on the topic of the area of a triangle. Video extracts will be shown in the presentation to supplement this necessarily brief description of the lesson.

Girls worked in groups of four, trying to find a rule for determining the area of a triangle. One group already knew the rule and was trying to find a rule for the area of a trapezium. The teacher introduced the problem by saying:

> You can draw as many triangles as you like …. What you want to do is to try and find the amount of space inside them; see if you can find any patterns; think about whether those patterns always happen; and try some more if you think you need to try more, until you think you know how to tell someone how to find the amount of space inside a triangle. I mean there might not even be a rule — except these people [the group working on the area of a trapezium] think there is.

The girls were given 10 minutes to make as much progress as they could, before one person from each group was asked to report on what their group was thinking about. Initially, some groups struggled with the difference between area and perimeter and tried to use irrelevant information such as the angle sum of a triangle.

As the groups worked, the teacher moved around the room, asking questions and observing students working, very much in the manner of the Japanese *kikan-shido* "between desk walking" or "purposeful scanning" (see, for example, Kepner, p. 7). As well as using this as an opportunity for selecting the order of reporting, the teacher also sometimes suggested specific aspects she wanted the group to report. While each group could choose who would report, there was an understanding that each member would report at some stage during the investigation.

During the initial reporting, the teacher reminded the girls that they were not allowed to contradict but only to ask for further explanations.

After some considerable time, at least one group came up with the standard rule for the area of a triangle of "base times height divided by two". Commenting on this group's report, the teacher said:

> We have a couple of interesting things here. I had a question to ask, but I didn't need to ask it. I was going to ask "Can they really say they have a pattern when they have only worked with one triangle?" And then Kathryn went on and said they'd worked with heaps of triangles! That's OK. It looks like they really have a pattern. But I hope they looked at some really unusual triangles to make sure it seemed to be happening all the time. But then I *loved* Sarah's question because when you have found a pattern that's the beginning not the end — that's when you have to think "well if it really is so, *why* is it so?"

Before discussing these lessons further, it should be made clear that neither of these teachers is "typical". Nevertheless, the Year 1 lesson shares many features with almost every Japanese lesson observed by us, although the same could certainly not be said about the Australian lesson. Nakano is a well-known teacher whose lessons have been the basis for many Lesson Studies, including one of the video exemplars used in a US-Japan Workshop (see Kepner, 2002; Nakano, 2002), while Williams is the author of a book containing a detailed theoretical and practical approach to learning through investigations (Williams, 1996).

THE TASK OF THE TEACHER

> A good discussion occurs … when the net result … is discerned as marking a definite progress as contrasted with the conditions that existed when the episode began. Perhaps it is a progress in understanding; perhaps it is progress in arriving at some kind of consensus; perhaps it is progress only in the sense of formulating the problem — but in any case, there is a sense of forward movement having taken place. Something has been accomplished; a group product has been achieved.
> (Lipman, Sharp & Oscanyan, 1980, p. 111)

We would argue that in both of these lessons there is progressive classroom discourse in the sense of Bereiter (1994). Moreover, the three key aspects of classrooms functioning as *Communities of Inquiry* could also be observed. We will now discuss what we believe are some critical features common to the two teachers' actions.

Problematising the curriculum

> [Students'] understanding increases significantly with their discovery of concepts they have built out of their own prior mathematical knowledge.   (Williams, 1996, p. 2)

In both lessons, the teachers took what is usually regarded, at least in Australia, as a standard piece of mathematics, to be taught by either exposition or what Simon (2003) refers to as *empirical activity*, and transformed it into challenging and problematic, yet accessible content. As stated earlier, the importance of the development of conceptually focused, robust tasks to support the development of sophisticated mathematical thinking should not be underestimated.

In Japan, this is supported through the use of Lesson Study, which aims to research the feasibility or effectiveness of a lesson (see, for example, Kepner, 2002; Nakano, 2002). Moreover, a common framework for lesson planning in Japan uses a four column grid with the first showing the following steps: Posing a problem, Students' problem solving, individually or, less frequently, in small groups; Whole class discussion; and Summing up; possibly followed by Exercise/extension. Each of these is accompanied by entries under the column headings of Main learning activities; Anticipated student responses; and Remarks on teaching (Shimizu, 2002). This common lesson pattern, based on students' actual and anticipated solutions of a single problem, together with an in-depth analysis of these solutions, promotes the problematising of the mathematics curriculum.

As well as *kikan-shido*, referred to earlier, key pedagogical ideas shared by teachers and forming observational criteria include: *hatsumon* — thought-provoking questions important to mathematical development and connections; *neriage* — raising the level of whole class discussion through orchestration and probing of student solutions (Kepner, 2002); and *yamaba* — regarding a lesson as a drama structured around a climax or "yamaba" (Shimizu, 2002).

Establishing an appropriate classroom environment

> Where elegance and originality are valued; the search for the most elegant solution becomes the intrinsic motivation of the group.   (Williams, 1996, p. 2)

The classroom environment in both lessons was clearly characterized by a sense of common purpose, mutual trust and risk-taking in the sense of *Communities of Inquiry*. The common purpose was achieved through both the use of a task that was genuinely problematic, yet accessible, for students, and through the establishment of social norms that valued individual (and group) contributions to the solution process.

In the case of the Australian lesson, it was evident that a great deal of effort had been made by the teacher to establish an environment where risk-taking was both supported and simultaneously minimized — for example, as stated earlier, the teacher reminded the girls during a report they were not allowed to contradict but only to ask for further explanations. This was one of many "rules" that formed part of explicit social norms operating in her classroom (see Williams, 1996, for further details). In Japan, while such social norms still need to be established, the fact that there is a

common pattern of lessons and a shared understanding among teachers of key pedagogical ideas, means that students anticipate how a mathematics lesson will operate and do not need explicit instruction on the social norms. Moreover, Japanese teachers frequently make a point of using students' incorrect solutions as a stepping stone to the class developing their understanding. In Australia, a great deal of successful effort has gone into establishing safe classroom environments, although there is very little emphasis on establishing a common (intellectual) purpose, especially when, in many primary schools particularly, groups of students are often working on different tasks — a practice that clearly mitigates against progressive dialogue, at least in the whole-class setting.

Focusing on good thinking and dialogue

> I would like to make my class enjoyable for children's thinking. I want the class to operate so that the children's thinking can be recognized by others and also by teachers. I also like to make the class feel that they can find out about the similarities and differences of their ideas in relation to others.   (Nakano, 2002, p. 65)

In both lessons, not only were there well-established social norms relating to discussion, but also, in Yackel and Cobb's (1996) sense, well-established socio-mathematical norms for what counts as acceptable explanations and justifications. Simon (2003) describes a Year 6 lesson also on the topic of the area of triangles as constituting *empirical activity* as opposed to *logico-mathematical activity* and defines mathematical understanding as requiring a "learned anticipation of the logical necessity of a particular pattern or relationship" (p. 185). In contrast to Simon's lesson, the Australian lesson explicitly emphasized the need for this logical necessity when the teacher stated that "when you have found a pattern that's the beginning not the end — that's when you have to think well if it really is so, *why* is it so?"

CONCLUSION

While the two lessons discussed here clearly differ in many respects, there are also many similarities, with the different contexts highlighting the ways in which the teachers promoted progressive discourse. Firstly, both teachers had a clear focus on the conceptual elements of the curriculum and were able to devise and sustain the use of complex, challenging tasks, that problematised the curriculum. Secondly, progressive discourse was promoted through the orchestration of the reporting of student solutions, starting with the least mathematically sophisticated in order to allow all students reporting to progress the community's solution to the problem. This aspect requires the teacher to not only interact with students as they work on the problem, but also to anticipate potential solution strategies and select an order for student reporting. Most of all, progressive discourse was promoted through the teachers' focus on seeking, recognizing, and drawing attention to mathematical reasoning and justification, and using it as a basis for learning. Factors that appeared not to affect progressive discourse in these cases included the age of the students, the mathematical topic, nor the use of co-operative group work.

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