IDENTITY, KNOWLEDGE AND DEPARTMENTAL PRACTICES: MATHEMATICS OF ENGINEERS AND MATHEMATICIANS

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This report explores first year undergraduate mechanical engineering and mathematics students' conceptions of the derivative and the contribution that membership of different departments may have on these conceptions. Quantitative results suggest that mechanical engineering students develop a proclivity for rate of change aspects of the derivative whilst mathematics students develop a proclivity for tangent-oriented aspects. The analysis of qualitative results further suggests that students' conceptual development of the derivative and their emerging identities are closely related and co-evolve in accordance with the departmental practices in which learning and teaching occur.

INTRODUCTION

This paper explores first year undergraduate mechanical engineering and mathematics students' differing orientations to, or conceptions of, the derivative. The mathematical education of engineers is a topic of increasing debate (see, Kent & Noss, 2002; McKenna, McMartin, Terada, Sirivedhin, & Agogino, 2001; Maull & Berry, 2000). Researchers have dealt with mathematics in the practice of engineers, developing new curricula for engineering students, students' difficulties with understanding mathematics and students' conceptual development in specific topics. Maull & Berry (2000) has commonalities with our study. They examine first and final year mechanical engineering and mathematics undergraduates alongside postgraduate students and professional engineers. They conclude that "the mathematical development of engineering students is different from that of mathematics students, particularly in the way in which they give engineering meaning to certain mathematical concepts" (ibid, p.916). They noted that both groups of students showed similar patterns of responses at the entry, by the final year, the groups' responses diverged. They did not, however, provide reasons for this emergent divergence and called for further research:

> *'There is evidence in the literature that engineering students are socialised into ways of thinking and behaving, and we may ask whether the difference found stems from socialisation, from the interactions between students and their peers, lecturers and other professional contacts, or whether there is also a second acculturation process through their discovery of what is useful in the context of their study and work' (ibid, p.916).*

Our considerations of divergence in engineers and mathematics students' conceptions have focused on issues of identity. Western research on identity has, by and large, come from a psychological perspective that conceptualises identity as an individual's sense of self (Harter, 1998). In the last five years, however, considerations of identity have been informed by social and socio-cultural theories (Wenger, 1998; Holland Lachicotte, Skinner, & Cain, 1998; Boaler, 2002; Nasir, 2002). Common to these

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perspectives is an assumption that individual growth and learning is situated in the social context/practice in which learners/practitioners live – the 'process of becoming' is practice-bound. We draw on Holland et al.'s (1998) idea of 'positional identity' in this paper. Positional identity refers to the way in which people figure out and enact their positions in the worlds in which they live. Their account of positional identity is based on their theory that identities develop in and through social and cultural practices. They argue that identities are closely related to structural features of the society and positional identities are connected to social affiliation and the way cultural systems work.

DEVELOPMENT OF THE RATIONALE OF THE STUDY

The project that this paper arises from set out to explore whether there is any difference between mechanical engineering (ME) and mathematics (M) degree students' conceptual development of the derivative concept over the first year of study and, if there are differences, to explore reasons for these differences. This study was conducted in a large university in Turkey. We do not claim that results from this study generalise beyond the confines of this university. The approach to data collection is naturalistic (Lincoln & Guba, 1985). Data were collected by a variety of means: quantitative (tests), qualitative (questionnaires and interviews) and ethnographic (classroom observations of semester 1 calculus courses and 'coffee-house' talk). In this paper we focus on two test items (see Figure 1) and provide summary data on pre-, post- and delayed post-tests and on classroom observations. The purpose of the summary data is to give the reader an appreciation of students' conceptions and their learning environments and also to inform the discussion section.

SUMMARY DATA

The pre-, post- and delayed-post tests were administered to 50 first year ME and 32 M students and addressed questions regarding, 'rate of change' and 'tangent' and were used to gain insight into: (a) how ME and M students' concept images of the derivative developed over the course, (b) how students dealt with rate of change and tangent concepts when questions were presented in graphic, algebraic and application forms and (c) whether there were any differences between ME and M students' performance in the different forms of these questions. The pre-test was administered to all students at the beginning of the course and there was no significant difference between ME and M students' performance. Both groups of students improved their performance in the post-test which was set at the end of the first semester. The delayed post-test was set towards the end of the second semester and both groups of students continued to improve. ME students, overall and in comparison with M students, did much better on rate of change-oriented test items regardless of whether the items were presented in algebraic, geometric or application-based forms. Similarly M students did much better than ME students on all forms of tangent-oriented questions. These trends (ME to rate of change, M to tangent) remained strong in the delayed post-test.

The calculus courses were observed and compared with students' notes to gain insights into which aspects of the derivative were 'privileged' in each department (Wertsch, 1991, p.124 uses 'privileging' in place of 'domination' to emphasise that a mediational means may be viewed as most appropriate in a particular setting). The analysis of observations and students' notes indicates (see, table 1) that ME students were taught more application (rate of change) aspects of the derivative compared to M students who were taught more theoretical or tangent-oriented aspects.

	Rate of change		**Tangent**	
	ME	M	ME	M
Duration examples	≈133 minutes (9 examples)	≈11 minutes (no examples)	≈10 minutes (no examples)	≈85 minutes (7 examples)

Table 1: The general analysis of ME and M calculus course' notes

DEVELOPMENT OF 'RATE OF CHANGE' VERSUS 'TANGENT' ITEMS

The trend (ME to rate of change, M to tangent) emerged from the post-test data. We decided to design two further items which might shed further light on the reasons behind this trend and administered these with the delayed post-test. Item 1 provided students with one rate of change-oriented question (A) and one tangent-oriented question (B) and asked them to state which one they would choose to solve if they were asked in their examination. When students chose each question, they were also asked to explain reasons behind their selection. Item 2 provided students with an imaginary situation where two students exchange ideas regarding their understanding of the derivative concept. Students were asked which view was closer to their way of thinking. These items were given to 45 ME and 32 M students.

Item 1: If the following two questions (A and B) were given in an examination and you only had to solve one of them, which one would it be? Please tick just one option and explain why you chose that one.

A.) At a certain time (t, seconds) the rate at which water flows (m^3/sec) into a water tank is given by the formula $f(t)= t^2/4 + 24t + 125$. Find:

a.) The initial amount of water in the tank and its initial rate of change?
b.) What is the rate of change of flowing water at any time, t?
c.) The time at which the rate of change is $32\ m^3/sec^2$.

B.) Find the solutions of the following questions:
a.) Verify that the gradient of the tangent to the curve $y=x^2$ at a point $(x_1, x_1^2)=2x_1$.
b.) Find the equation of the tangent to the curve $y=2x^2-x+3$ which is parallel to the line $y=3x-2$.
c.) Show that the graph of $f(x)= x^{1/3}$ has a vertical tangent line at (0,0) and find an equation for it.

Item 2: Two university students from different departments are discussing the meaning of the derivative. They are trying to make sense of the concept in accordance with their departmental studies.
Ali says that "Derivative tells us how quickly and at what rate something is changing since it is related to moving object. For example, it can be drawn on to explain the relationship between the acceleration and velocity of a moving object.
Banu, however, says that "I think the derivative is a mathematical concept and it can be described as the slope of the tangent line of a graph of y against x".
a.) Which one is closer to the way of your own derivative definition? Please explain!
b.) If you had to support just one student, which one would you support and why?

Figure 1. Two items to explore reasons for rate of change and tangent orientations

RESULTS

We first present quantitative data (frequency counts) and then a categorisation of students' reasons for their choices. Tables 2 & 3 show students' responses to items 1 & 2.

Item 1	ME	M
Question A (A)	60 % (27)	22 % (7)
Question B (B)	40 % (18)	78 % (25)

Table 2. Students' responses (percentages–raw frequencies in brackets) to item 1

	ME		M	
Item 2	Item 2a	Item 2b	Item 2a	Item 2b
Ali (A)	51 % (23)	49 % (22)	19 % (6)	13 % (4)
Banu (B)	27 % (12)	49 % (22)	63 % (20)	78 % (25)
Both (A &B)	22 % (10)	2 % (1)	16 % (5)	3 % (1)
Not Attempted (NA)	0	0	3 % (1)	6 % (2)

Table 3. Students' responses (percentages–raw frequencies in brackets) to item 2

For item 1 ME students show a preference (60:40) for rate of change-oriented item over tangent-oriented one whilst M students show quite a strong preference (78:22) for tangent-oriented question. Similar preferences can be seen in item 2a. In item 2b the preference of the M students remains but the ME students are equally divided.

Tables 4 and 5 present a categorisation of students' reasons for their choices in items 1 & 2. Repeated reading of students' responses generated three emergent categories: Affiliation; Practice; and Ease. 'Affiliation' is a construct used by Nasir & Saxe (2003) to describe identification with a common cultural ancestry and distinctive cultural patterns and it appeared to us an appropriate term to apply to M and ME students who identified themselves as belonging to a particular department. 'Practice' here concerns students' calculus practices and this, as these students are novice practitioners, is related to what goes on in calculus courses. We use the term 'wider practices' in the discussion section when we attend to departmental features that are not solely concerned with calculus. 'Ease' here means what particular students reported that they found easy (not our decisions on the ease of items). We first explain how we allocated students responses to these categories and present examples of students' responses for each category.

Affiliation (ME) ME students' responses were placed in this category when they mentioned any of the following: real life; applications; rate of change; engineering.

Student 1 *'What Ali says is closer. Calculating rates of change seems to me more real. On the other hand what Banu says is not far away, even it is so close. ... But since I am going to be an engineer, Ali's idea would be just different. Because I would be the one who makes mathematics concrete'.*

Affiliation (M) M students' responses were placed in this category when they mentioned any of the following: the exact nature of the derivative; the slope of a tangent; belonging to a mathematics department; interpretation from a mathematician standpoint; the comprehensiveness of the definition.

Student 2 *'Banu interprets the derivative from a mathematician's perspective, and Ali interprets it from a physicist standpoint. At the end of the day, since I too am from mathematics department, I find Banu's explanation closer to myself. But in essence they both present the essence of mathematics'.*

Practice (ME) ME students' responses were placed in this category when they mentioned the way calculus is being covered and used in their department.

Student 3 *'We are using it in that way and learning it that way'.*

Practice (M) M students' responses were placed in this category when they mentioned not knowing much about rate of change or the way calculus is being covered in their department.

Student 4 *'We are learning in that way and I don't know much about rate of change'.*

Ease(ME & M) Students' responses were placed in this category when they mentioned the 'ease' of this way of thinking about the derivative.

Student 5 *'Because it is easier'*

Categorisation of responses	**Engineers choosing A**			**Mathematicians choosing B**		
	Item1 60% (27)	Item 2a 51% (23)	Item 2b 49% (22)	Item1 78% (25)	Item 2a 63% (20)	Item 2b 78% (25)
Affiliation	20% (9)	44% (20)	44% (20)	13% (4)	47% (15)	66% (21)
Practice	11% (5)	7% (3)	4% (2)	50% (16)	28% (9)	13 % (4)
Ease	29% (13)	0	0	16% (5)	0	0

Table 4 Responses of ME students who chose As and M students who chose Bs

'Affiliation', 'practice' and 'ease' are cited by both groups of students for item 1 and there is no clear pattern to these responses but note that 50% of M students cite 'practice'. 'Ease' is not really applicable for item 2 and all students cite either 'application' or 'practice' with 'affiliation being the dominant stated reason.

Categorisation of responses	Engineers choosing B			Mathematicians choosing A		
	Item1 40% (18)	Item 2a 27% (12)	Item 2-b 49% (22)	Item1 22% (7)	Item 2a 19% (6)	Item 2-b 13% (4)
Affiliation	0	18% (8)	49% (22)	6% (2)	19% (6)	10 % (3)
Practice	9% (4)	9% (4)	0	0	0	0
Ease	31% (14)	0	0	13% (4)	0	0
Not categorised	0	0	0	3% (1)	0	3% (1)

Table 5 Responses of ME students who chose Bs and M students who chose As

'Ease' is the dominant cited reason in item 1 and 'affiliation' is the dominant cited reason in item 2 for students who do not follow the 'ME – rate of change, M – tangent' trend. That 49% of ME students' responses (item 2b) is with a mathematician's concept of the derivative is noteworthy.

DISCUSSION

In this section, we attend to possible reasons for ME students' tendency to rate of change and M students' tendency to tangent aspects of the derivative concept. We focus on practice and affiliation as these constructs, in the categorisation of students' responses, apply to both items 1 & 2 (see tables 4 & 5). We first discuss practice, then affiliation, then the relationship between affiliation and positional identity and end with questions for further research.

Table 1 shows that the ME students' calculus course 'privileged' rate of change examples whilst the M students' calculus course 'privileged' tangent examples. Calculus practices in each department are likely to have played an important role in the growth of the different tendencies in these two groups of students' performances In this regard, Kendal & Stacey (2000) show how students' conceptions of the derivative are strongly influenced by the aspect of the derivative privileged by their teachers. It is thus reasonable to infer that privileging of rate of change and of tangent aspects of the derivative in the two calculus courses influenced students' orientation and knowledge development.

Table 4 shows that both groups of students, but especially M students, referred to practice in explaining the reasons behind their choices in items 1 & 2, i.e. their preferences for rate of change or tangent forms of the derivative. Others, however, related their preferences for specific forms of the derivative to affiliation and ease (especially ME students). We do not believe that it is possible to separate students' feelings of affiliation and perceptions of ease from practice. We further believe that the specific calculus practices of the students, e.g. the type of examples they are most often presented with, are interrelated to the wider practices of their departments and that students' ways of participating are adapted to the constraints and affordances existing in each department (Greeno, 1998; Boaler & Greeno, 2000), i.e. these wider practices facilitate student access to some forms of the derivative but simultaneously constrain (maybe inadvertently) access to other forms.

Do the calculus practices alone explain ME students' proclivity to rate of change and M students' proclivity to tangent aspect of the derivative concept? Students responses suggest that the answer is 'no'. Affiliation (see tables 4 and 5) is the highest cited reason given by both groups of students in explaining their preferences for forms of knowledge. We interpret this as evidence for a developing personal association (being an engineer or a mathematician) towards particular conceptual forms of the derivative. The applicability of the derivative concept, for instance, was cited as a reason for their preferences for rate of change-oriented items by many ME students. In a similar manner many M students attached considerable importance to the 'exact' nature of the derivative. It is, therefore, plausible to infer that the reasons behind the students' different inclinations for the different forms of the derivative arise not only from the fact that their calculus practice privileged these forms but also because students from each department developed different affiliations towards different aspects of the derivative.

Affiliation appears to be an important construct in understanding undergraduate students' conceptions, but where does affiliation come from? We have noted that the ME to rate of change and M to tangent trend was not present in the pre-test but was present in the post-test. It appears to have developed during the first semester. We believe that Holland et al.'s (1998) notion of positional identity is relevant in explaining the genesis of affiliation. Students' developing affiliation towards particular forms of the knowledge could come into being as a result of the way that students comprehended and enacted their positions in the department to which they belong. This positioning can take many forms, mental and physical, e.g. going to the part of the mathematics section of the library that deals with applied (or pure) mathematics. In the course of their studies it is likely that students from each department began to position themselves according to their professional perspective and this positioning influenced their proclivity towards specific forms of knowledge. The concept of positional identity helps us to appreciate that what we have termed affiliation is not just a personal mental construct but, to use Holland et al.'s (1998) language, is shaped by enacting their positions. Many ME students, for example, attended meetings organised by the ME department with local professional engineers.

Positional identity is clearly a related construct to affiliation, but what else can be said about the relationship between these two? We cannot give definitive answers as our data is inconclusive but there are interesting questions for further research. We believe the relationship is a dialectical one, i.e. that the positions enacted by students help shape departmental affiliations and that students' affiliation is a characteristic of their emerging positional identities. Further to this, and with regard to students' emerging identities and their relationship with forms of knowledge, what relation exists between students' knowledge development and their emerging positional identities?

CONCLUSION

Mechanical engineering students develop a proclivity for rate of change aspects of the derivative whilst mathematics students develop a proclivity for tangent-oriented aspects. Further to this, it appears that students' conceptual development of the derivative and the way they build relationships with its particular forms are closely related and co-evolve in accordance with departmental perspectives.

It has been argued that this difference between the conceptions of the both groups students cannot solely be attributed to the practice of the courses that the students followed. Departmental affiliation appears to have influence on cognition and play a crucial role in the emergence of this difference. The concept of positional identity has links with the concept of affiliation. An implication for further research into students' understanding of calculus at the undergraduate level is that researchers should not ignore students' departmental affiliation.

REFERENCES

Boaler, J., & Greeno, J.G. (2000). Identity, Agency, and Knowing in Mathematical Worlds. In J. Boaler (Ed.), *Multiple Perspectives on Mathematics Teaching and Learning* (pp. 171-200). Stamford, CT, Ablex.

Boaler, J. (2002). The Development of Disciplinary Relationships: Knowledge, Practice and Identity in Mathematics Classrooms [1]. *For the Learning of Mathematics* 22, 1.

Greeno, J.G. (1998). The Situativity of Knowing, Learning and Research. *American Psychologist,53*(1), 5-26.

Harter, S. (1998). *The Construction of Self.* New York: Guilford.

Holland, D., Lachicotte, W., Skinner, D. and Cain, C. (1998), *Identity and Agency in Cultural Worlds*. Harvard University Press, Cambridge, MA.

Kendal, M., & Stacey, K. (2000). Acquiring the concept of derivative: Teaching and learning with multiple representations and CAS. In T. Nakahara & T. Koyama (Eds.), *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education,* Hiroshima, Japan: PME. (Vol.3, pp.127-134).

Kent, P. & Noss, R. (2002). The mathematical Component of Engineering Expertise: The Relationship between Doing And Understanding Mathematics. http://www.ioe.ac.uk/rnoss/MCEE/Kent-Noss-EE2002-preprint.pdf

Lincoln, Y.S. & Guba, E.G (1985) *Naturalistic Inquiry*, Newbary Park, London, Sage.

Maull, W.,&Berry, J. (2000) A questionnaire to elicit the mathematical concept images of engineering students. *International Journal of Mathematical Education in Science and Technology, Vol.31, No.6, 899-917.*

McKenna, A., McMartin, F. Terada, Y., Sirivedhin, V., & Agogino, A. (2001). A Framework for Interpreting Students' Perceptions of an Integrated Curriculum. *Proceedings of the 2001 American Society for Engineering Education Annual Conference 2001, American Society for Engineering Education",* http://best.me.berkeley.edu/~aagogino/papers/ASEE01_1626.pdf

Nasir, N.S. (2002) Identity, Goals, and Learning: Mathematics in Cultural Practice. *Mathematical Thinking and Learning, 4 (2&3), 213-247.*

Nasir, N.S., & Saxe, G.B. (2003) Ethnic and Academic Identities: A Cultural Practice Perspective on Emerging Tensions and Their Management in the Lives of Minority Students. *Educational Researcher, Vol. 32, No. 5, pp. 14–18.*

Wenger, E. (1998). *Communities of Practice*. New York: Cambridge University Press.

Wertsch, J.V. (1991) *Voices of the Mind*. Cambridge, MA: Harvard University Press.