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SECOND ORDER REDUCTIONS OF N-WAVE INTERACTIONS RELATED TO LOW-RANK SIMPLE LIE ALGEBRAS

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Abstract. The analysis and the classification of all reductions for the nonlinear evolution equations solvable by the inverse scattering method (ISM) is interesting and still open problem. We show how the second order reductions of the N-wave interactions related to low-rank simple Lie algebras can be embedded in the Weyl group of $\mathfrak{g}$. Some of the reduced systems find applications to nonlinear optics.

1. Introduction

It is well known that the N-wave equations [1]–[6]

$$\mathrm{i}[J,Q_t]-\mathrm{i}[I,Q_x]+[[I,Q],[J,Q]]=0, \tag{1}$$

are solvable by the inverse scattering method (ISM) [4, 5] applied to the generalized system of Zakharov–Shabat type [4, 7, 8]:

$$L(\lambda)\Psi(x,t,\lambda)=\left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}+[J,Q(x,t)]-\lambda J\right)\Psi(x,t,\lambda)=0, \quad J\in\mathfrak{h}, \tag{2}$$

$$Q(x,t)=\sum_{\alpha\in\Delta_+}\left(q_\alpha(x,t)E_\alpha+p_\alpha(x,t)E_{-\alpha}\right)\in\mathfrak{g}/\mathfrak{h}, \tag{3}$$

where $\mathfrak{h}$ is the Cartan subalgebra and E_α are the root vectors of the simple Lie algebra $\mathfrak{g}$. Indeed (1) is the compatibility condition

$$[L(\lambda),M(\lambda)]=0, \tag{4}$$