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HARMONIC FORMS ON COMPACT SYMPLECTIC 2-STEP NILMANIFOLDS

YUSUKE SAKANE† and TAKUMI YAMADA‡

† *Department of Pure and Applied Mathematics*
Graduate School of Information Science and Technology
Osaka University, Toyonaka, Osaka, 560-0043, Japan

‡ *Department of Mathematics*
Graduate School of Science
Osaka University, Toyonaka, Osaka, 560-0043, Japan

Abstract. In this paper we study harmonic forms on compact symplectic nilmanifolds. We consider harmonic cohomology groups of dimension 3 and of codimension 2 for 2-step nilmanifolds and give examples of compact 2-step symplectic nilmanifolds G/Γ such that the dimension of harmonic cohomology groups varies.

1. Introduction

Let $(M, \mathbf{G})$ be a Poisson manifold with a Poisson structure $\mathbf{G}$, that is, a skew-symmetric contravariant 2-tensor $\mathbf{G}$ on M satisfying $[\mathbf{G}, \mathbf{G}] = 0$, where $[\,,\,]$ denotes the Schouten-Nijenhuis bracket. For a Poisson manifold $(M, \mathbf{G})$, Koszul [5] introduced a differential operator $\mathrm{d}^*\colon \Omega^k(M) \to \Omega^{k-1}(M)$ by $\mathrm{d}^* = [\mathrm{d}, i(\mathbf{G})]$, where $\Omega^k(M)$ denotes the space of all k-forms on M. The operator d^* is called the **Koszul differential**. For a symplectic manifold (M^{2m}, ω), let $\mathbf{G}$ be the skew-symmetric bivector field dual to ω. Then $\mathbf{G}$ is a Poisson structure on M. Brylinski [1] defined the star operator $*\colon \Omega^k(M) \to \Omega^{2m-k}(M)$ for the symplectic structure ω as an analogue of the star operator for an oriented Riemannian manifold and proved that the Koszul differential d^* satisfies $\mathrm{d}^* = (-1)^k * \mathrm{d} *$ on $\Omega^k(M)$ and the identity $*^2 = \mathrm{id}$. A form α on M is called **harmonic form** if it satisfies $\mathrm{d}\alpha = \mathrm{d}^*\alpha = 0$. Let $\mathcal{H}^k_\omega(M) = \mathcal{H}^k(M)$ denote the space of all harmonic k-form on M. Brylinski [1] defined symplectic harmonic k-cohomology group $H^k_{\omega\text{-}hr}(M) = H^k_{hr}(M)$