Fifth International Conference on
Geometry, Integrability and Quantization
June 5–12, 2003, Varna, Bulgaria
Iva"ılo M. Mladenov and Allen C. Hirshfeld, Editors
SOFTEX, Sofia 2004, pp 149–157

STRATIFIED REDUCTION OF MANY-BODY DYNAMICAL SYSTEMS

TOSHIHIRO IWAI

Department of Applied Mathematics and Physics
Kyoto University, Kyoto-606-8501, Japan

Abstract. The center-of-mass system for many bodies in $\mathbb{R}^3$ admits a natural action of the rotation group $SO(3)$. According to the orbit types for the $SO(3)$ action, the center-of-mass system M is stratified into strata. A quantum Hamiltonian system and a classical Lagrangian system are defined on $L^2(M)$ and on $T(M)$, respectively. These systems are also stratified according to the stratification of M, and then reduced by the rotational symmetry, respectively.

1. Introduction

Consider a smooth manifold M on which acts a compact Lie group G. According to the orbit types of the group action, the manifold is stratified into different strata. Mechanics will be set up on each stratum and then reduced by symmetry. We apply this idea, taking M and G as the center-of-mass system for N bodies and the rotation group $SO(3)$, respectively. The center-of-mass system M will be stratified into $M = \dot{M} \cup M_1 \cup M_0$, where $\dot{M}$ and M_1 are the set of non-singular configurations or non-linear molecules, and the set of collinear configurations or linear molecules, respectively, and M_0 is a singleton which denotes the simultaneous collision configuration. We have no need to discuss mechanics on M_0. A quantum Hamiltonian system is defined on $L^2(M)$, and stratified into those on $L^2(\dot{M})$ and $L^2(M_1)$, which are reduced to quantum systems on vector bundles over $\dot{M}/SO(3)$ and $M_1/SO(3)$, respectively. A classical Lagrangian system is defined on $T(M)$, and stratified into those on $T(\dot{M})$ and $T(M_1)$, which are reduced to classical systems on vector bundles over $\dot{M}/SO(3)$ and $M_1/SO(3)$, respectively.