Fifth International Conference on
Geometry, Integrability and Quantization
June 5–12, 2003, Varna, Bulgaria
Ivaïlo M. Mladenov and Allen C. Hirshfeld, Editors
SOFTEX, Sofia 2004, pp 178–188

THE ZEROS OF POLYNOMIALS ORTHOGONAL WITH RESPECT TO q-INTEGRAL ON SEVERAL INTERVALS IN THE COMPLEX PLANE

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Abstract. We construct the sequence of orthogonal polynomials with respect to an inner product defined in the sense of q-integration over several intervals in the complex plane. For such introduced polynomials we prove that all zeros lie in the smallest convex hull over the intervals in the complex plane. The results are stated precisely in some special cases, as some symmetric cases of equal lengths, angles and weights.

1. Introduction

We will start with well-known facts from q-calculus [1], [2], where q is a real number from the interval $(0, 1)$. The basic number $[x]_q$ is given by

$$[x]_q = \frac{1 - q^x}{1 - q} \quad (x \in \mathbb{R})$$

and factorial of q-natural numbers

$$[0]_q! = 1, \quad [n]_q! = [n]_q[n-1]_q! = [n]_q[n-1]_q \cdots [1]_q, \quad n \in \mathbb{N}.$$

We define q-shifted factorials by

$$(a; q)_0 = 1, \quad (a; q)_n = \prod_{k=1}^{n}(1 - aq^{k-1}), \quad (a; q)_\infty = \prod_{k=1}^{\infty}(1 - aq^{k-1})$$