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CARTAN FORMS AND SECOND VARIATION FOR CONSTRAINED VARIATIONAL PROBLEMS

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Abstract. Using the Cartan form of first order constrained variational problems introduced earlier we define the second variation. This definition coincides in the unconstrained case with the usual one in terms of the double Lie derivative of the Lagrangian density, an expression, that in the constrained case does not work. The Hessian metric and other associated concepts introduced in this way are compared with those obtained through the Lagrange multiplier rule. The theory is illustrated with an example of isoperimetric problem.

1. Introduction

One of the most characterizing aspects of modern variational calculus on fibred manifolds has been without doubt the promotion to "basic concept" of the so called Cartan form. Essential for the intrinsic formulation of the classical Euler-Lagrange equations and a key object for Noether's theory of infinitesimal symmetries and conservation laws, it is also a fundamental element for the multi-symplectic formulation of the theory. As one can expect, it turns out to have an important role again for the second variation, where, in the unconstrained case, this formula can be obtained both from the double Lie derivative of the Lagrangian density or via the Cartan form [4, 5, 10].

In [3] the authors proposed a Cartan formulation for first order variational problems with differential constraints, where, unlike to the unconstrained case, the Cartan form lies in the second jet of the fibred manifold where the problem is given. From this object, it is possible to introduce a (third order) Euler-Lagrange operator which characterizes critical sections, a Noether theory of infinitesimal symmetries and