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タイトル TITLE	**Dissipative weak solutions of the Euler equations and vortex dynamics**		
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One of the characteristic properties of 2D turbulence is the emergence of the inertial range in the energy density spectrum of the flow corresponding to the forward cascade of enstrophy for sufficiently small viscosity[2,7,8]. It is pointed out that the emergence of the enstrophy cascade is due to the enstrophy dissipation in the inviscid limit of the incompressible flow field. However, the enstrophy dissipation never occurs for smooth solutions of the 2D Euler equations, since it conserves the enstrophy. Hence, it is necessary to consider non-smooth solutions of the 2D Euler equations to obtain the enstrophy dissipation, which is called *dissipative weak solutions*[3].

In order to investigate how non-smooth weak solutions of the 2D Euler equation can dissipate the enstrophy, we consider the 2D Euler-α equations for the point-vortex initial data. The Euler-α is originally derived by Holm et al.[4,5] as a physically relevant model of 3D turbulence. Oliver and Shkoller[9] have established the global existence of a unique weak solution of the 2D Euler-α equations for the initial vorticity data in the space of Radon measure on $\mathbb{R}^2$ in the sense of distribution. Hence, global weak solutions of the 2D Euler-α equations for the point-vortex initial data is described as the motion of many point vortices called *the α-point vortex (αPV) system*. Thus, by taking the $\alpha \to 0$ limit of the global weak solution, we are able to construct a weak solution of the 2D Euler equations in terms of the vortex dynamics.

In the presentation, we show that the three α-point vortices collapse to a point and then expand to the infinity self-similarly beyond the critical time in the $\alpha \to 0$ limit when we consider the special condition for which the self-similar triple collapse is obtained in the classical point-vortex system[1,6,10,11]. Moreover, we find that the enstrophy dissipation occurs via the self-similar triple collapse in the sense of distribution. Since the anomalous enstrophy dissipation via the triple collapse is still observed even if the special condition is perturbed, it is a robust mechanism in the sense that it is observed for a certain range of the parameter region[12].

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