# Analytic $K$-theory of rigid spaces

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Let $K$ be a complete discretely valued field and $R$ be the ring of integers with a prime element $\pi$. Let $\mathcal{X}$ be a (formal) scheme over $R$ and write $\mathcal{X}_n = X \otimes_R R/(\pi^{n+1})$ for $n \geq 0$. The *continuous* $K$-groups of $\mathcal{X}$ are defined as

$$K_i^{cont}(\mathcal{X}) := \varprojlim_n K_i(\mathcal{X}_n) \quad (i \in \mathbb{Z}),$$

where $K_i(\mathcal{X}_n)$ are the higher algebraic $K$-groups of $\mathcal{X}_n$ defined by Quillen and others. By studying $K_i^{cont}(\mathcal{X})$ for a projective smooth scheme over $R = k[[t]]$ with $k$ a field of characteristic zero, Bloch-Esnault-Kerz and Morrow succeeded in proving the deformational part of Grothendieck's variational Hodge conjecture.

The purpose of this talk is to explain a newly developed theory of analytic $K$-theory $KH_i^{\mathrm{an}}(X)$ for rigid spaces $X$ over $K$, which was motivated by the above work. I will explain its construction and relation to the continuous $K$-theory $K_i^{cont}(\mathcal{X})$ of a formal model $\mathcal{X}$ of $X$ over $R$. I will also explain a natural isomorphism

$$KH_0^{\mathrm{an}}(X) \simeq K_0(X)$$

for an affinoid $X$, which gives a new description of the Grothendieck group of the category of vector bundles over $X$. Finally I will formulate a conjecture on analytic $K$-theory which implies the Hodge conjecture for abelian varieties.

This is a joint work with Moritz Kerz and Georg Tamme.