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My research is based in probability theory, with a particular focus on random walks in random environments. Work in this area, which has been a major focus of probability over the last four decades, is motivated by the goal of understanding the interplay between the geometry of a space and the stochastic processes that live upon it.

In this report, I will discuss a recent project on the Mott random walk. This model is a classical example of a random walk in a random environment, originally introduced as a model for electron transport in a disordered medium at low temperatures. In particular, the basic space in which an electron is moving consists a countable number of localization sites, which are typically modelled using a random collection of points in $\mathbb{R}^d$. Moreover, electrons hop as a random walk between localisation sites, with the probability of jumping from a localisation site at x to a localisation site at y given by

$$p(x,y) \propto e^{-|x-y|},$$

where the proportionality is over the variable y. In dimensions ≥ 2, if the localisation sites are modelled by a Poisson process, then it is known that the random walk has a usual diffusive scaling limit [4]. On the other hand, in one-dimension (which one might think of as modelling a wire), the same model shows an interesting phase transition, whereby, as the density of the Poisson process is decreased, the process starts to exhibit sub-diffusive behaviour (i.e. the mean-square displacement grows sub-linearly) [1]. Together with Ryoki Fukushima (University of Tsukuba) and Stefan Junk (Tohoku University), as reported in this venue two years ago, we derived an anomalous, sub-diffusive scaling limit in this regime [2]. As described there, '[c]orresponding to intervals of low conductance in the discrete model, the discontinuities in the scale function [of the limiting process] act as barriers off which the limiting process reflects for some time before crossing'. More recently, in [3], we have studied a regime that exhibits even more severe blocking (and sub-polynomial scaling). For this, we demonstrate that, for any fixed time, the appropriately-rescaled Mott random walk is situated between two environment-measurable barriers, the locations of which are shown to have an extremal scaling limit. Moreover, we give an asymptotic description of the distribution of the Mott random walk between the barriers that contain it.

Whilst the previous discussion is quite specific to the Mott random walk, I highlight that the techniques used to study it involve a recently developed theory that relates the convergence of processes to that of associated resistance metric measure spaces. In this approach, one transfers questions about a random walk to those about an associated electrical network, which are often easier to study. Such a viewpoint has proved useful for other examples of random walks in random environments, and it is my goal to explore further applications of it. Moreover, in my research, I seek to develop further general tools that provide insights concerning other models arising in physics and elsewhere.

References

[1] P. Caputo and A. Faggionato, *Diffusivity in one-dimensional generalized Mott variable-range hopping models*, Ann. Appl. Probab. **19** (2009), no. 4, 1459–1494.

[2] D. A. Croydon, R. Fukushima, and S. Junk, *Anomalous scaling regime for one-dimensional Mott variable-range hopping*, to appear in Ann. Appl. Probab., 2020.

[3] ———, *Extremal regime for one-dimensional Mott variable-range hopping*, preprint appears at arXiv:2208.12102, 2022.

[4] A. Faggionato, H. Schulz-Baldes, and D. Spehner, *Mott law as lower bound for a random walk in a random environment*, Comm. Math. Phys. **263** (2006), no. 1, 21–64.