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Renormalization using Domain Wall Regularization

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Abstract

We formulate the renormalization procedure using the domain wall regularization which is based on the heat-kernel method. The quantum effects of both fermions and bosons (gauge fields) are taken into account. The background field method is quite naturally introduced. As for the treatment about the loop-momentum integrals, an interesting contrast between the fermion-determinant part and others is revealed. These things are explained taking some examples. The Weyl anomalies for the 2D QED and 4D QED are correctly obtained. It is found that the "chiral solution" produces $(1/2)^{d/2}$ $\times$ "correct values", where d is the space dimension. Taking the model of 2D QED, both Weyl and chiral anomalies are directly obtained from the effective action. The mass and wave function renormalization are explicitly performed in 4D QED. We confirm the multiplicative renormalization (not additive one), which shows the advantage of no fine-tuning.

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1 Introduction

In the lattice world, the new era looks to begin [1, 2]. The so-called "doubling problem" becomes clarified and the chirality of fermions can be controlled more efficiently than before. We may say the new regularization has been established in lattice. In accordance with this great development , the counterpart in the continuum is progressing [3, 4, 5, 6, 7]. The merit of such approach is firstly that we can clearly understand the essential part of the new regularization, which is often hidden in the complexity of the discretized model. Secondly we can compare it with the ordinary regularizations used so far and apply it to various (continuum) field theories. The continuum version, at least, should explain the qualitative features of the lattice domain wall.

In the domain wall approach so far, at least in the continuum approach, the gauge field in the fermion determinant is mostly treated as the external field. (As for the lattice model analysis, the gauge field quantum effect was examined and the renormalizability was checked at 1-loop in [8].) Clearly the situation is not satisfactory in that the gauge field is not quantumly treated and that the general perspective about its use in the (perturbative) quantum field theory is not presented. Indeed the fermion determinant could be the most important part among other parts, but we must formulate it within the general setting in order to regard it as a new regularization in the field theory. This motivates us to do the present work.

Recently a new treatment has appeared in the continuum approach[9, 10]. The main idea is the following. The (regularization) ambiguity and the divergences of the fermion determinant can be resolved by introducing "direction" in the system. It is based on the analogy to the well-defined determinant of the elliptic operator, which can be expressed as the form of the heat equation solution. Here we recall the facts: 1) the heat propagates in a fixed direction, that is, from the high temperature to the low temperture (the second law of the thermodynamics) and, generally, the *heat equation* prescribes such behaviour; 2) The famous procedure to introduce the heat equation into the general quantum system is the heat-kernel method[11]. The fermion determinant has been so often examined in this method. (The anomaly is formulated in this standpoint in [12].) We have shown the above idea works well if we look at the 4 dim theory from the 1+4 dim space-time.

We summarize this new treatment in Sec.2. New results about 2D and 4D QED Weyl anomalies are also presented. In Sec.3, the renormalization procedure is introduced and the (renormalized) effective action itself is used to derive anomalies. At this stage, we still keep the gauge field external in order to make explanation simple. In Sec.4 we introduce the background field method[13, 14] and take into account the quantum effect of the gauge field besides the fermion. We do the renormalization of the fermion self energy explicitly. In Sec.5, we present the complete formulation combining the results of Sec.3 and 4. Finally we point out that the domain wall regularization distinctly treats the fermion determinant part from other parts.

2 Domain Wall Regularization

For a fermion system described by the quadratic form $\mathcal{L} = \bar{\psi}\hat{D}\psi$ where the operator $\hat{D}$ satisfies

$$\gamma_5\hat{D} + \hat{D}\gamma_5 = 0 \quad , \quad (\gamma_5)^2 = 1 \quad , \tag{1}$$

the fermion determinant (the effective action) can be expressed as

$$\begin{aligned} \ln\, Z[A_\mu] &= -\int_0^\infty \frac{dt}{t}\mathrm{Tr}\,\left[\frac{1}{2}(1+i\gamma_5)\exp\{it\gamma_5\hat{D}\} + \frac{1}{2}(1-i\gamma_5)\exp\{-it\gamma_5\hat{D}\}\right] \\ &= -\lim_{M\to 0}\int_0^\infty \frac{dt}{t}\frac{1}{2}(1-i\frac{\partial}{\partial(tM)})\mathrm{Tr}\,(G_+^{5M}(x,y;t) + G_-^{5M}(x,y;t)) \ , \end{aligned} \tag{2}$$

where t plays the role of the inverse temperature, M is introduced here as a *regularization* mass parameter. A_μ stands for the background gauge field appearing in $\hat{D}$. We take $M > 0$ for simplicity. G_+^{5M} and G_-^{5M} are defined as

$$\begin{aligned} G_+^{5M}(x,y;t) &=< x|\exp\{+it\gamma_5(\hat{D}+iM)\}|y> \quad , \\ G_-^{5M}(x,y;t) &=< x|\exp\{-it\gamma_5(\hat{D}+iM)\}|y> \quad , \end{aligned} \tag{3}$$

and they satisfy the heat equations with the *first* derivative operator $\mp i\gamma_5(\hat{D}+iM)$. We call G^{5M}_+ and G^{5M}_-, "(+)-domain" and "(-)-domain" respectively. The key observation is that the heat equations turn out to be the 1+4 dim Minkowski *Dirac equation* after appropriate *Wick rotations* for t. For the system of 4 dim Euclidean QED, $\hat{D} = i\gamma_\mu(\partial_\mu + ieA_\mu)$, $(\mu = 1,2,3,4)$, they are

$$
\begin{aligned}
(i\partial\!\!\!/ - M)G^{5M}_+ &= ie A\!\!\!/ G^{5M}_+ \quad , \quad (X^a) = (-it, x^\mu) \quad , \\
(i\partial\!\!\!/ - M)G^{5M}_- &= ie A\!\!\!/ G^{5M}_- \quad , \quad (X^a) = (+it, x^\mu) \quad ,
\end{aligned}
\tag{4}
$$

where $A\!\!\!/ \equiv \gamma_\mu A_\mu(x)$, $\partial\!\!\!/ \equiv \Gamma^a \frac{\partial}{\partial X^a}$, $(a = 0,1,2,3,4)$. Now we can fix the above solution based on the another key observation that the system should have a *fixed direction.* Generally the solution of (4) is given by two ingredients (see a standard textbook, for example, [15]): 1) free solution $G_0(X,Y)$, and 2) propagator $S(X,Y)$ as the following form.

$$
G^5_M(X,Y) = G_0(X,Y) + \int d^5Z\, S(X,Z) ie A\!\!\!/(z) G^5_M(Z,Y) \quad . \tag{5}
$$

This gives us the solution $G^5_M(X,Y)$ in the perturbative way with respect to the coupling e. As $S(X,Y)$, we should not take Feynman propagator which has both retarded and advanced components. We should take the retarded propagator for (+)-domain and the advanced one for (-)-domain. (They are symmetric with respect to the positive and negative energy parts.)

$$
\begin{aligned}
&\text{Symmetric retarded solution} \quad G^{5M}_+ : \\
&G_0(X,Y) = G^{\rm p}_0(X,Y) - G^{\rm n}_0(X,Y) \, , \\
&S(X,Y) = \theta(X^0 - Y^0)(G^{\rm p}_0(X,Y) - G^{\rm n}_0(X,Y))
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&\text{Symmetric advanced solution} \quad G^{5M}_- : \\
&G_0(X,Y) = -G^{\rm p}_0(X,Y) + G^{\rm n}_0(X,Y) \, , \\
&S(X,Y) = \theta(Y^0 - X^0)(-G^{\rm p}_0(X,Y) + G^{\rm n}_0(X,Y)) \, ,
\end{aligned}
\tag{7}
$$

where $G^{\rm p}_0(X,Y)$ and $G^{\rm n}_0(X,Y)$ are the positive and negative energy free solutions respectively:

$$
\begin{aligned}
G^p_0(X,Y) &\equiv -i\int \frac{d^4k}{(2\pi)^4}\Omega_+(k) e^{-i\tilde{K}(X-Y)} \quad , \quad \Omega_+(k) \equiv \frac{M + \tilde{K\!\!\!/}}{2E(k)} \quad , \\
G^n_0(X,Y) &\equiv -i\int \frac{d^4k}{(2\pi)^4}\Omega_-(k) e^{+i\bar{K}(X-Y)} \quad , \quad \Omega_-(k) \equiv \frac{M - \bar{K\!\!\!/}}{2E(k)} \quad ,
\end{aligned}
\tag{8}
$$

where $E(k)=\sqrt{k^2+M^2}, (\tilde{K}^a)=(E(k), K^\mu=-k^\mu), (\bar{K}^a)=(E(k), -K^\mu=k^\mu)$. k^μ is the momentum in the 4 dim Euclidean space. [2] $\tilde{K}$ and $\bar{K}$ are on-shell momenta($\tilde{K}^2=\bar{K}^2=M^2$), which correspond to the positive and negative energy states respectively. [3] The theta functions in (6) and (7) show the "directedness" of the solution. In this solution, both the positive and negative states propagate in the forward direction in (+)-domain, while in the backward direction in (-)-domain. We call (6) and (7) "symmetric solutions". The above solutions satisfy the following *boundary conditions*.

$$
\begin{aligned}
&G_+^{5M}(\text{Retarded}) \rightarrow -i\gamma_5\delta^4(x-y) \text{ as } M(X^0-Y^0)\rightarrow +0\ , \\
&G_-^{5M}(\text{Advanced}) \rightarrow +i\gamma_5\delta^4(x-y) \text{ as } M(X^0-Y^0)\rightarrow -0\ .
\end{aligned} \tag{9}
$$

In this procedure, we naturally notice the existence of the "chiral" solution [4] , which is obtained by choosing $G_0(X,Y)$ and $S(X,Y)$ in the following way.

$$
\text{Retarded solution} \quad G_+^{5M}:
$$

$$
G_0(X,Y)=G_0^p(X,Y) \quad , \quad S(X,Y)=\theta(X^0-Y^0)G_0^p(X,Y) \tag{10}
$$

$$
\text{Advanced solution} \quad G_-^{5M}:
$$

$$
G_0(X,Y)=G_0^n(X,Y) \quad , \quad S(X,Y)=\theta(Y^0-X^0)G_0^n(X,Y) \tag{11}
$$

They also express "directed" solutions, but do not satisfy (4). They satisfy its *chiral version* in the large M limit ($M/|k|\gg 1$).

$$
\begin{aligned}
&(i\partial\!\!\!/-M)G_+^{5M}=ieP_+A\!\!\!/G_+^{5M}+O(\frac{1}{M}) \quad , \quad (X^a)=(-it,x^\mu) \quad , \\
&(i\partial\!\!\!/-M)G_-^{5M}=ieP_-A\!\!\!/G_-^{5M}+O(\frac{1}{M}) \quad , \quad (X^a)=(+it,x^\mu) \quad .
\end{aligned} \tag{12}
$$

The configuration where the positive energy states propagate only in the forward direction of X^0 constitutes the (+)-domain, while the configuration where the negative energy states propagate only in the backward direction

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[2] The relation between 4 dim quantities(x^μ and k^μ) and 1+4 dim ones(X^a and K^a): $(X^a)=(X^0, X^\mu=x^\mu), (K^a)=(K^0, K^\mu=-k^\mu), K_aX^a=K_0X^0-K^\mu X^\mu=K_0X^0+k^\mu x^\mu$.

[3] Useful relations: $-i\tilde{K}X=-iE(k)X^0-ikx,\ i\bar{K}X=iE(k)X^0-ikx,\ M+\tilde{K}\!\!\!/=M+E(k)\gamma_5+i\,k\!\!\!/,\ M-\bar{K}\!\!\!/=M-E(k)\gamma_5+i\,k\!\!\!/$.

[4] In ref.[9, 10], we called "Feynman path solution" because they are "invented" by "dividing" the Feynman propagator,

constitutes the $(-)$-domain. As seen from their simple structure, the chiral solution has some advantages(, at least, in concrete calculations). The chiral solutions satisfy the following *boundary condition.*

$$i(G_+^{5M}(X,Y)-G_-^{5M}(X,Y))\rightarrow\gamma_5\delta^4(x-y)\ \text{as}\ M|X^0-Y^0|\rightarrow+0\quad.\qquad(13)$$

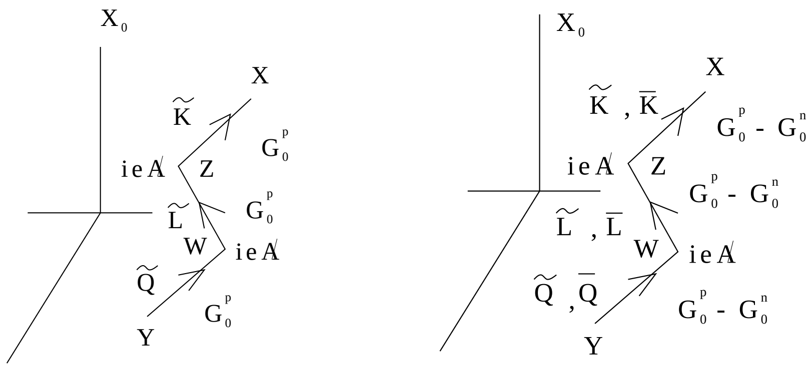
This regularization prescription finishes with taking the following "double limits":

$$\text{(i)}\quad\frac{|k^\mu|}{M}\leq 1\quad,\quad\text{(ii)}\quad Mt\ll 1\quad,$$
$$(\text{or}\quad|k^\mu|\leq M\ll\frac{1}{t})\quad.\qquad(14)$$

This relation shows the most characteristic point of this 1+4 dimensional regularization scheme. The condition (ii) comes from the usage of the regularization parameter M in (2), whereas (i) comes from the necessity of controlling the chirality without destroying the system dynamics. Note the roles of the regularization mass parameter M for the t-axis and for the x^μ-axis are different. It restricts the configuration to the *ultra-violet* region ($t\ll M^{-1}$) in the "extra space" of t, whereas to the *infra-red* (surface) region in the real 4 dim space ($|k^\mu|\leq|M|$). (This situation of configuration restriction, by (i) and (ii), will be further explained in Sect.5.) In the concrete calculation, the condition (i) is taken into account, not by doing k^μ-integral with the cut-off M but by the *analytic continuation* in order to avoid breaking the gauge invariance.

The validity of the above regularization was confirmed in Ref.[9, 10]. We found the analogous aspects to the lattice domain wall: the domain wall configuration, the overlap Dirac operator, the condition on the regularization parameter M, e.t.c.. We also confirmed the chiral anomalies (for 2D QED, 4D QED and 2D chiral gauge theory) are correctly reproduced. One of advantages of the present approach is the equal treatment of the *chiral* and *Weyl* anomalies. To see the situation, let us apply the above regularization to the Weyl anomaly calculation. We first take the simple model, 2D QED, for the later purpose. It is given, using the chiral solution (13), as[10]

$$\delta_\omega\ \ln\ J_W=2i\lim_{M|X^0-Y^0|\rightarrow+0}\text{Tr}\ \omega(x)\gamma_5(G_+^{5M}(X,Y)-G_-^{5M}(X,Y))\quad,$$
$$Tr\ \omega\gamma_5G_+^{5M}|_{AA}=\int_0^{X^0}dZ^0\int d^2Z\int_0^{Z^0}dW^0\int d^2W$$
$$\times\text{Tr}\ \omega\gamma_5G_0^p(X,Z)ie\not{A}(z)G_0^p(Z,W)ie\not{A}(w)G_0^p(W,Y)\quad.\qquad(15)$$



(i) Chiral Sol. G_+^{5M}, O(AA)
$X_0 > Z_0 > W_0 > Y_0 = 0$

(ii) Symmetric Sol. G_+^{5M}, O(AA)
$X_0 > Z_0 > W_0 > Y_0 = 0$

Fig.1 Abelian Gauge Theory, G_+^{5M}, $O(AA)$,(i) Chiral Solution and (ii) Symmetric Solution.

See Fig.1(i). Similarly for $\mathrm{Tr}\,\omega\gamma_5 G_-^{5M}$ using G_0^n. After the standard calculation explained in [10], we obtain *half* of the correct result: $\delta_\omega \ln J_W = \omega(x)\frac{1}{2}\frac{\mathrm{e}^2}{\pi}{A_\mu}^2$. When we take the symmetric solution, we evaluate

$$\delta_\omega \ln J_W = i \lim_{M|X^0-Y^0|\to+0} \mathrm{Tr}\ \omega(x)\gamma_5(G_+^{5M}(X,Y) - G_-^{5M}(X,Y)) \quad ,$$

$$\begin{aligned} Tr\,\omega\gamma_5 G_+^{5M}|_{AA} = \int_0^{X^0} dZ^0 \int d^2 Z \int_0^{Z^0} dW^0 \int d^2 W \\ \times \mathrm{Tr}\ \omega\gamma_5(G_0^p(X,Z) - G_0^n(X,Z)) ie\not{A}(z)(G_0^p(Z,W) - G_0^n(Z,W)) \\ \times ie\not{A}(w)(G_0^p(W,Y) - G_0^n(W,Y)) \quad . \end{aligned} \tag{16}$$

See Fig.1(ii). This reproduces the correct result: $\delta_\omega \ln J_W = \omega(x)\frac{\mathrm{e}^2}{\pi}{A_\mu}^2$. We have confirmed the similar situation in 4D Euclidean QED. The symmetric solution gives the correct Weyl anomaly: $\delta_\omega \ln J = \omega(x)\,\beta(e)F_{\mu\nu}F_{\mu\nu}$, $\beta(e) = \frac{\mathrm{e}^2}{12\pi^2}$, where $\beta(e)$ is the β-function in the renormalization group [5] . The chiral solution gives *one fourth* of it.

Combined with the results for the chiral anomalies in [9, 10], we conclude that the chiral solution gives $(1/2)^{d/2}$ (d: space dimension) of the correct

[5] The general anomaly calculation based on the ordinary(not using domain wall) heat-kernel is reviewed in [10].

value of the anomaly coefficients both for the chiral anomaly and for the Weyl anomaly. The use of the chiral solution reduces the degree of freedom by half for each double dimensions. This phenomenon looks contrastive with the lattice's doubling species phenomenon.

The use of the chiral solution, instead of the symmetric one, should be limitted to the simple cases such as the anomaly calculation. For the general case we should use the symmetric solution.

3 Renormalization Procedure

In this section and the next one, we develop how to use the domain wall regularization in the general field theory. We now introduce the counterterm action $\Delta\Gamma[A]$ into the effective action $\Gamma[A] \equiv -\ln Z[A]$:

$$\Gamma_{\mathrm{bare}}[A] \equiv \Gamma[A] + \Delta\Gamma[A] \quad , \tag{17}$$

in such a way that Γ_{bare} becomes finite. How to systematically define $\Delta\Gamma$ and obtain the renormalization properties is the task of this section. Let us take the simple model of 2D QED (Schwinger model) for the explanation. We consider the case $\Delta\Gamma$ is made of local counterterms. [6] From the power-counting analysis [7] , we know

$$\Delta\Gamma[A] = \int d^2x \left(c_{\mathrm{div}} \mathrm{e}^2 {A_\mu}^2 + c'_{\mathrm{div}} \mathrm{e}\, \partial_\mu A_\mu + c''_{\mathrm{div}} \mathrm{e}\, \epsilon_{\mu\nu} \partial_\mu A_\nu \right) \quad , \tag{18}$$

where $c_{\mathrm{div}}, c'_{\mathrm{div}}, c''_{\mathrm{div}}$, is some (divergent) constants to be systematically fixed. Now we take the following *renormalization condition* on $\Gamma_{\mathrm{bare}}[A]$.

$$\left. \frac{\delta^2 \Gamma_{\mathrm{bare}}[A]}{\delta A_\mu(x) \delta A_\nu(y)} \right|_{A=0} = \frac{\mathrm{e}^2}{\pi} \delta_{\mu\nu} \delta^2(x-y) \ , \quad \left. \frac{\delta \Gamma_{\mathrm{bare}}[A]}{\delta A_\mu(x)} \right|_{A=0} = 0 \quad . \tag{19}$$

The first equation defines the renormalized coupling, and the second one guarantees the stability.

---

[6] In ref.[16], the divergence structure in 2D QED is examined. As for the bosonic part of the effective action, we need not worry about the non-perturbative divergences.

[7] The mass dimensions of the gauge (photon) field A_μ and the gauge coupling (electric charge) e are 0 and 1 respectively.

In order to regularize t-integral in $\Gamma[A]$, we introduce here another regularization parameters ϵ and T $(0<\epsilon<t<T\ ,\ \epsilon\rightarrow +0\ ,\ T\rightarrow +\infty)$:

$$\Gamma[A]=-\ln\,Z[A]=$$
$$\lim_{T\rightarrow+\infty,\epsilon\rightarrow+0}\lim_{M\rightarrow 0}\int_\epsilon^T\frac{dt}{t}\frac{1}{2}(1-i\frac{\partial}{\partial(tM)})\mathrm{Tr}\,(G_+^{5M}(x,y;t)+G_-^{5M}(x,y;t))\quad .\qquad(20)$$

T is regarded as the length of the extra axis. ϵ is the "regularized point" of the origin of the extra axis. It can also be regarded as the minimum unit of length (the ultraviolet cutoff). T and ϵ regulate the infrared and ultraviolet behaviors respectively. The relevant part is the order A^2 term.

$$\begin{aligned}G_+^{5M}|_{AA}=&\int_0^{X^0}dZ^0\int_0^{Z^0}dW^0\int d^2Z\int d^2W(G_0^p(X,Z)-G_0^n(X,Z))ie\not{A}(z)\\&(G_0^p(Z,W)-G_0^n(Z,W))ie\not{A}(w)(G_0^p(W,Y)-G_0^n(W,Y))\\&=\int_0^{X^0}dZ^0\int_0^{Z^0}dW^0\int d^2z\int d^2w\int\frac{d^2k}{(2\pi)^2}\frac{d^2l}{(2\pi)^2}\frac{d^2q}{(2\pi)^2}\\&\times(-i)(\Omega_+(k)\mathrm{e}^{-iE(k)(X^0-Z^0)}-\Omega_-(k)\mathrm{e}^{iE(k)(X^0-Z^0)})ie\not{A}(z)\\&\times(-i)(\Omega_+(l)\mathrm{e}^{-iE(l)(Z^0-W^0)}-\Omega_-(L)\mathrm{e}^{iE(l)(Z^0-W^0)})ie\not{A}(w)\\&\times(-i)(\Omega_+(q)\mathrm{e}^{-iE(q)(W^0-Y^0)}-\Omega_-(q)\mathrm{e}^{iE(q)(W^0-Y^0)})\mathrm{e}^{-ik(x-z)-il(z-w)-iq(w-y)}\ .\end{aligned}\qquad(21)$$

See Fig.1(ii). Evaluating the above equation and the similar one $G_-^{5M}|_{AA}$, we obtain

$$\Gamma[A]|_{AA}=\lim_{T\rightarrow+\infty,\epsilon\rightarrow+0}\frac{\mathrm{e}^2}{\pi}\ln\frac{T}{\epsilon}\times\int d^2xA_\mu{}^2\quad .\qquad(22)$$

From the renormalization condition (19) we obtain

$$c_{\mathrm{div}}=-\frac{1}{\pi}\ln\frac{T}{\epsilon}+\frac{1}{2\pi}\quad .\qquad(23)$$

Let us check above result by calculating the Weyl and chiral anomalies *directly from the effective action.* (In eqs.(15) and (16) of this paper, and in ref.[10], anomalies have been obtained from Jacobians.) The Weyl anomaly is obtained from the scale transformation of ϵ (or T).

$$\begin{aligned}\epsilon'=\mathrm{e}^\Delta\epsilon=\epsilon+\Delta\cdot\epsilon+O(\Delta^2)\quad ,\\ \delta\Gamma_{\mathrm{bare}}=\delta\,[c_{\mathrm{div}}\mathrm{e}^2\int d^2xA_\mu{}^2]=\Delta\times\frac{\mathrm{e}^2}{\pi}\int d^2xA_\mu{}^2\quad ,\end{aligned}\qquad(24)$$

which agrees with the known result. The chiral anomaly is obtained by the variation of A_μ, $\delta\not{A} = -\frac{1}{e}\not{\partial}\alpha\cdot\gamma_5$, for $\Gamma_{\rm bare}[A]$ [8] :

$$\frac{\delta\Gamma_{\rm bare}[A]}{\delta\alpha(x)} = +\frac{i}{\pi}e\epsilon_{\mu\nu}\partial_\mu A_\nu \quad . \tag{25}$$

Both Weyl and chiral anomalies are correctly reproduced. [9] Note that the chiral anomaly derived by the Jacobian, in [9], comes from $O(A)$-part of $G^{5M}_\pm$, whereas that derived here by the effective action comes from $O(A^2)$-part. (As for the Weyl anomaly, the results by both approaches come from $O(A^2)$-part.)

We conclude this section with listing up all regularization parameters introduced and compare them with the lattice situation. We have introduced *three* parameters M, T, and ϵ. They should satisfy the following relations with two *configuration variables*: the 4-momentum k^μ and the inverse-temperature t.

$$|k^\mu| \leq M \ll \frac{1}{t} \quad , \quad \frac{1}{T} < \frac{1}{t} < \frac{1}{\epsilon} \quad , \quad \frac{1}{T} \ll \frac{1}{\epsilon} \quad . \tag{26}$$

In the above, we clearly see an important feature of the parameter M. Among three relations above, the latter two are rather familiar ones. (Scale is cut-offed by ϵ in the UV-region, and by T in the IR-region.) The interesting one is the first. So far (before the appearance of domain wall regularization) we do not know such a regularization parameter that depends on the configuration (specified by k^μ and t in the present case) in this way. In the conclusion of this paper, we will point out another important character of the present regularization, which is obtained by the result above and that of the next section. In the lattice domain wall case, they introduce *four* parameters:

[8] The 2 dim QED, $\mathcal{L} = \bar{\psi}i(\not{\partial} + ie\not{A})\psi$, is invariant under the *local* chiral gauge transformation: $\psi' = {\rm e}^{i\alpha(x)\gamma_5}\psi$, $\not{A}' = \not{A} - \frac{1}{e}\not{\partial}\alpha\cdot\gamma_5$. ($A'_\mu = A_\mu + \frac{i}{e}\epsilon_{\mu\nu}\partial_\nu\alpha,\ \epsilon_{12} = 1.$)

[9] We notice some contrastive point in the two anomalies. The Weyl anomaly does not depend on the renormalization condition (19), whereas the chiral one depends on it. The former gives the response to the scale change, hence picks up the divergent part proportional to $\ln T/\epsilon$. On the other hand the latter gives the response to the (chiral) phase change, hence the charge normalization, which is defined by the renormalization condition (19), is crucial for it.

m_0, a, L_s, l[17]. The correpondence with the present case is

$$
\begin{array}{rcl}
\text{present paper} & & \text{Ref.[17]} \\
M & \leftrightarrow & m_0, \text{1+4 dim fermion mass} \\
\epsilon & \leftrightarrow & a, \text{lattice spacing} \\
\frac{T}{\epsilon} & \leftrightarrow & L_s, \text{the total site number along the extra axis} \\
T & \leftrightarrow & l, \text{physical extent of one direction of the 4 dim box}
\end{array}
\qquad (27)
$$

The one extra parameter in the lattice comes from the fact that the extra space is treated independently from the 4 dim space. It is adopted in the ordinary domain wall formulation, whereas the present one treats the extra space as the space of the inverse temperature which appears in expressing the determinant. In ref.[17], the 4 dim fermion mass m_f is also introduced. In such a case, we also introduce the mass parameter. (See ref.[10])

4 General Treatment of Renormalization

We have discussed only the fermion determinant for the external gauge field. In order for this approach to be regarded as an alternative new regularization for the general field theories, the quantum treatment of the gauge fields should also be described. We devote ourselves on the point in this section.

The present approach quite naturally fits in the background field method[13, 14]. We explain it taking again 4D Euclidean QED as an example. The quantum effect of both the gauge (photon) field and the fermion is taken into account. We consider the massive fermion.

$$
\mathcal{L}[\psi,\bar{\psi},A] = i\bar{\psi}\{\gamma_\mu(\partial_\mu + ieA_\mu) + im\}\psi - \frac{1}{4}{F_{\mu\nu}}^2 - \frac{1}{2}(\partial_\mu A_\mu)^2 \quad , \qquad (28)
$$

where the Feynman gauge is taken. According to the general theory of the background field method, full physical information is contained in the following background effective action.

$$
\mathrm{e}^{-\Gamma[\chi,\bar{\chi},A]} = \int \mathcal{D}a_\mu \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left[- \int d^4x \{\mathcal{L}[\chi+\psi,\bar{\chi}+\bar{\psi},A+a]\right.
$$

$$
\left. - \frac{\delta \mathcal{L}[\Phi]}{\delta \Phi^i} \phi^i - \mathcal{L}[\chi, \bar{\chi}, A] \} \right]
$$

$$
= \int \mathcal{D}a_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left[- \int d^4x \{ \mathcal{L}_2[\psi, \bar{\psi}, a] + \mathcal{L}_3[\psi, \bar{\psi}, a] \} \right]
$$

$$
\mathcal{L}_2[\psi, \bar{\psi}, a; \chi, \bar{\chi}, A] = i\bar{\psi}\{\gamma_\mu(\partial_\mu + i\mathrm{e}A_\mu) + im\}\psi - \frac{1}{2}(\partial_\mu a_\nu)^2 - \mathrm{e}(\bar{\psi}\not{a}\chi + \bar{\chi}\not{a}\psi) \quad ,
$$

$$
\mathcal{L}_3[\psi, \bar{\psi}, a] = -\mathrm{e}\bar{\psi}\not{a}\psi \quad ,(29)
$$

where $(\Phi_i) = (\chi, \bar{\chi}, A_\mu)$ are the background fields and $(\phi_i) = (\psi, \bar{\psi}, a_\mu)$ are the quantum ones. We notice the term $\mathcal{L}_3 = -\mathrm{e}\bar{\psi}\not{a}\psi$ is the cubic power of quantum fields and contribute to the 2-loop and higher-loop orders. Terms in $\mathcal{L}_2$ are all quadratic power and contribute only to 1-loop. Among them the two terms, $-\mathrm{e}\bar{\psi}\not{a}\chi$ and $-\mathrm{e}\bar{\chi}\not{a}\psi$, are off-diagonal with respect to the quantum fields. In order to diagonalize the 1-loop part , we redefine the quantum fields: $\psi' = \psi + \Delta\psi, \bar{\psi}' = \bar{\psi} + \Delta\bar{\psi}, a'_\mu = a_\mu + \Delta a_\mu$. Here we choose $\Delta\psi, \Delta\bar{\psi}$ and Δa_μ to be *linear* with respect to quantum fields in order to *keep the loop-order structure*. We require the 1-loop part, $\mathcal{L}_2[\psi, \bar{\psi}, a]$, equal to

$$
i\bar{\psi}'\{\gamma_\mu(\partial_\mu + i\mathrm{e}A_\mu) + im\}\psi' - \frac{1}{2}(\partial_\mu a_\nu{}')^2 \quad , \tag{30}
$$

We choose $\Delta\psi$ and $\Delta\bar{\psi}$ as

$$
\begin{aligned}
\Delta\bar{\psi}(-i\overleftarrow{\not{\partial}} - \mathrm{e}\not{A} - m) &= -\mathrm{e}\bar{\chi}\not{a} \quad , \\
(i\not{\partial} - \mathrm{e}\not{A} - m)\Delta\psi &= -\mathrm{e}\not{a}\chi \quad .
\end{aligned} \tag{31}
$$

From this choice, we know $\Delta\psi$ and $\Delta\bar{\psi}$ are proportional to the vector quantum field a_μ and begin from the order of e. Then Δa_μ satisfies

$$
\int d^4x\{\frac{1}{2}(\partial_\mu a_\nu + \partial_\mu(\Delta a_\nu))^2 - \frac{1}{2}(\partial_\mu a_\nu)^2\} = \int d^4x\{-\frac{1}{2}\mathrm{e}\,\Delta\bar{\psi}\not{a}\chi - \frac{1}{2}\mathrm{e}\,\bar{\chi}\not{a}\Delta\psi\} \tag{32}
$$

where the relations (31) are used. Then we see the solution Δa_μ is obtained as the expansion with respect to the coupling e beginning from order of e^2. (RHS of (32) begins from order of e^2.)

$$
\Delta a_\mu = \mathrm{e}^2 X_\mu^{(2)} + \mathrm{e}^3 X_\mu^{(3)} + O(\mathrm{e}^4) \quad . \tag{33}
$$

Assuming $a_\nu \partial_\mu X_\nu^{(2)}$ damps sufficiently rapidly at the boundary $|x_\mu| = \infty$, $X_\mu^{(2)}$ satisfies the following equation.

$$
\partial^2 X_\mu^{(2)} = \frac{1}{2\mathrm{e}}(\bar{\chi}\gamma_\mu\Delta\psi|_\mathrm{e} + \Delta\bar{\psi}|_\mathrm{e}\gamma_\mu\chi) \quad . \tag{34}
$$

"$|_{\rm e}$" means 1-st order of e, therefore RHS is 0-th order of e.

The eq (31) can be perturbatively solved as, requiring the natural condition that $\Delta\psi$ and $\Delta\bar{\psi}$ vanish when the quantum fields vanish $(a_\mu,\psi,\bar{\psi})=0$,

$$\Delta\psi(x)=-\int d^4y\,S^A(x-y){\rm e}\not{a}(y)\chi(y)=-\int d^4y\,S(x-y){\rm e}\not{a}(y)\chi(y)+O({\rm e}^2)\ ,$$
$$\Delta\bar{\psi}(x)=-\int d^4y\,{\rm e}\bar{\chi}(y)\not{a}(y)\bar{S}^A(x-y)=-\int d^4y\,{\rm e}\bar{\chi}(y)\not{a}(y)\bar{S}(x-y)+O({\rm e}^2)\ ,\quad(35)$$

where $S^A,\bar{S}^A,S,\bar{S}$ are defined by

$$(i\not{\partial}-{\rm e}\not{A}-m)S^A(x-y)=\delta^4(x-y)\ ,\ (i\not{\partial}-m)S(x-y)=\delta^4(x-y)\ ,$$
$$\bar{S}^A(x-y)(-i\overleftarrow{\not{\partial}}-{\rm e}\not{A}-m)_y=\delta^4(x-y)\ ,\ \bar{S}(x-y)(-i\overleftarrow{\not{\partial}}-m)_y=\delta^4(x-y)\ .\quad(36)$$

$S^A(x-y)$ and $\bar{S}^A(x-y)$ are the background dependent propagators. All these propagators are (4D) Euclidean ones, therefore there is no ambiguity in the choice of boundary conditions.

The path-integral expression (29) can be rewritten by the redefined quantum fields using the measure change:

$$\mathcal{D}a_\mu\mathcal{D}\psi\mathcal{D}\bar{\psi}=J\times\mathcal{D}a_\nu{}'\mathcal{D}\psi'\mathcal{D}\bar{\psi}'\quad,\quad J\equiv\frac{\partial(a_\mu,\psi,\bar{\psi})}{\partial(a_\nu{}',\psi',\bar{\psi}')}\quad.\qquad(37)$$

Since we keep the linear relation in the choice of the redefined quantum fields, the Jacobian, J, does *not* depend on quantum fields.

$$\begin{aligned}{\rm e}^{-\Gamma[\chi,\bar{\chi},A]}=J\times\int\mathcal{D}a_\mu{}'\mathcal{D}\psi'\mathcal{D}\bar{\psi}'\exp[-\int d^4x\{i\bar{\psi}'\gamma_\mu(\partial_\mu+i{\rm e}A_\mu+im)\psi'\\-\frac{1}{2}(\partial_\mu a_\nu{}')^2+\text{cubic terms w.r.t. quantum fields}\}]\quad,\quad(38)\end{aligned}$$

The 1-loop part in the integrand is just the quantity that we have discussed in Sec.2 for the case $m=0$ (Sec.7 of [10] treats the case $m\neq 0$). Up to the lowest nontrivial order, J is obtained as

$$\begin{aligned}J^{-1}=\frac{\partial(a_\mu{}',\psi',\bar{\psi}')}{\partial(a_\nu,\psi,\bar{\psi})}&=\det(\delta_{\mu\nu}\delta^4(x-y)+{\rm e}^2\frac{\delta X^{(2)}_\mu(x)}{\delta a_\nu(y)}+O({\rm e}^3))\\&=\exp[{\rm e}^2{\rm Tr}\ (\frac{\delta X^{(2)}_\mu(x)}{\delta a_\nu(y)})+O({\rm e}^3)]\ ,\\ \ln J={\rm e}^2{\rm Tr}_{x=z,\mu=\nu}&\int d^4yD(x-y)\bar{\chi}(y)\gamma_\mu S(y-z)\gamma_\nu\chi(z)+O({\rm e}^3)\quad.\qquad(39)\end{aligned}$$

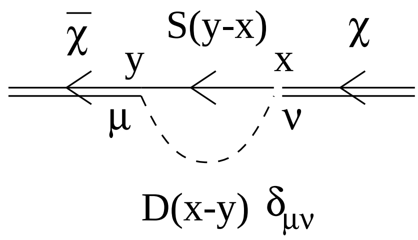


Fig.2 Fermion Self Energy.

This corresponds to the 1-loop part for the fermion self energy. See Fig.2. Because the Jacobian is *decoupled* from the main integral part, the regularization for the divergences in (39) can be taken *independently* from the regularization parameter M introduced in Sec.2. The quantity to be regularized in Sec.2 was $\det(\not\partial + ie\not A)$, while the Jacobian above is roughly $\det(1+\mathrm{e}^2\frac{1}{\partial^2}\bar{\chi}\frac{1}{\not\partial}\chi) = \det(\partial^2+\mathrm{e}^2\bar{\chi}\frac{1}{\not\partial}\chi)/\det\partial^2$. Both quantities are divergent, but the presence of "∂^2" inside of det-symbol , in the latter quantity, makes no chiral problem. (There is no ambiguity in determining the divergent quantity $\det(\partial^2+\cdots)$ because it can be expressed as the heat equation (not Dirac equation) in 1+4 dim. See Sec.2 of ref.[10].) The (momentum) integral, corresponding to "Trace" in (39), is done by the usual way appearing in the standard field theory textbook. We explain the effective-action calculation in the *coordinate* space a little more.

$$\ln J|_{\mathrm{e}^2} = \mathrm{e}^2\int d^4y\int d^4x D(x-y)\bar{\chi}(y)\gamma_\mu S(y-x)\gamma_\mu\chi(x)$$
$$= \mathrm{e}^2\int d^4y\int d^4x D(x-y)\bar{\chi}(y)\gamma_\mu S(y-x)\gamma_\mu\{\chi(y)+(x-y)^\mu\partial_\mu\chi|_y+\cdots\}$$
$$\equiv \Gamma_{\bar{\chi}\chi}+\Gamma_{\bar{\chi}\partial\chi}+\cdots \quad ,(40)$$

where $\chi(x)$ is Taylor-expanded around y, and each expanded part is defined by $\Gamma_{\bar{\chi}\chi}$,$\Gamma_{\bar{\chi}\partial\chi}$,$\cdots$. $D(x)$ comes from the photon propagation and is defined by $\partial^2 D(x)=\delta^4(x)$. The usual integral calculation gives the (ultra) divergent parts as,

$$\Gamma_{\bar{\chi}\chi} = \frac{\mathrm{e}^2}{2\pi^2}m\ln\frac{T}{\epsilon}\times\int d^4y\;\bar{\chi}(y)\chi(y) + \text{finite term} \quad ,$$
$$\Gamma_{\bar{\chi}\partial\chi} = -\frac{\mathrm{e}^2}{8\pi^2}\ln\frac{T}{\epsilon}\times\int d^4y\, i\bar{\chi}(y)\not\partial\chi(y) + \text{finite term} \quad , \qquad (41)$$

where we take the region of momentum-integral as $\frac{1}{T} < |k_\mu| < \frac{1}{\epsilon}$ where T and ϵ are introduced in Sec.3 as the infrared and ultraviolet cutoffs. Here we find the fermion part of the counter action as (the gauge part is given in (18) for 2D QED)

$$\Delta\Gamma[\chi,\bar{\chi}] = \int d^4y \left\{ \frac{\mathrm{e}^2}{2\pi^2} m \ln\frac{T}{\epsilon} \times \bar{\chi}(y)\chi(y) - \frac{\mathrm{e}^2}{8\pi^2}\ln\frac{T}{\epsilon} \times i\bar{\chi}(y)\partial\!\!\!/ \chi(y) \right\} \quad ,(42)$$

which is introduced in order to cancel the divergences of (41). $\Delta\Gamma$ is "absorbed" by the mass renormalization $m + \Delta m$ and the wave-function renormalization of the fermion $\sqrt{Z_2}$ as follows.

$$\begin{aligned} \Delta\Gamma[\chi,\bar{\chi},A] &\equiv \int d^4x \Delta\mathcal{L}[\chi,\bar{\chi},A] \quad , \\ \mathcal{L}[\chi,\bar{\chi},A;\mathrm{e},m] + \Delta\mathcal{L}[\chi,\bar{\chi},A] &= \mathcal{L}[\sqrt{Z_2}\chi,\sqrt{Z_2}\bar{\chi},\sqrt{Z_3}A;\mathrm{e}+\Delta\mathrm{e},m+\Delta m] \quad , \\ m+\Delta m = m\Big(1-\frac{3\mathrm{e}^2}{8\pi^2}\ln\frac{T}{\epsilon}\Big) \quad &, \quad \sqrt{Z_2} = 1 - \frac{\mathrm{e}^2}{16\pi^2}\ln\frac{T}{\epsilon} \quad .(43) \end{aligned}$$

Especially the mass is renormalized in the *multiplicative* way, as we expected. [10]

5 Discussion and Conclusion

Combining the results in Sec.3 and 4, we present the final form of the renormalization prescription (, taking 4D QED as an example,) with the background field gauge[14].

$$\begin{aligned} (\Phi_i) = (\chi,\bar{\chi},A):\ \text{Background fields} \quad &, \quad (\phi_i) = (\psi,\bar{\psi},a):\ \text{Quantum fields} \quad , \\ \mathcal{L}[\chi,\bar{\chi},A;\mathrm{e},m] &= i\bar{\chi}\gamma_\mu(\partial_\mu + i\mathrm{e}A_\mu + im)\chi - \frac{1}{4}{F_{\mu\nu}}^2 \quad , \end{aligned}$$

[10] In the present continuum approach, this multiplicative renormalization comes from the fact that the Jacobian J decouples from the M-involved part (fermion determinant part) at (38). In the lattice, the 4D fermion mass is essentially introduced as the coupling m_1 between the walls at the two ends of the extra axis. In this case, whether the renormalized mass is proportional to m_1 or not is highly non-trivial, because M could appear additively simply from the dimensional reason. The numerical simulation supports the multiplicative renormalization. This fact is very important for no fine-tuning (good control of the small mass fermion) and for the validity of the chiral perturbation[18].

$$\mathcal{L}_{\rm gauge}[a;\xi] = -\frac{1}{2\xi}(\partial_\mu a_\mu)^2 \quad ,$$

$$\mathrm{e}^{-\Gamma_{\rm bare}[\Phi]} = \int \mathcal{D}\phi \ \exp\Big[-\int d^4x\{\mathcal{L}[\Phi+\phi] + \mathcal{L}_{\rm gauge}[a] + \Delta\mathcal{L}[\Phi+\phi] \\ -\frac{\delta}{\delta\Phi^i}(\mathcal{L}[\Phi]+\Delta\mathcal{L}[\Phi])\phi^i - \mathcal{L}[\Phi] - \Delta\mathcal{L}[\Phi]\}\Big] \quad , (44)$$

where the fermion determinant part only is regularized by the Domain Wall regularization of Sec.2, and other parts by the cutoffs for the t-axis: $\epsilon < t < T$(or $\frac{1}{T} < |k_\mu| < \frac{1}{\epsilon}$). The background field gauge is chosen here and ξ is the gauge parameter. $\Delta\mathcal{L}$ is obtained in such a way that $\Gamma_{\rm bare}$ becomes finite satisfying some proper *renormalization condition* on the following quantities.

$$\left.\frac{\delta^2\Gamma_{\rm bare}[\chi,\bar\chi,A]}{\delta A_\mu(x)\delta A_\nu(y)}\right|_{\chi=\bar\chi=A=0} \quad , \quad \left.\frac{\delta^2\Gamma_{\rm bare}[\chi,\bar\chi,A]}{\delta\chi(x)\delta\bar\chi(y)}\right|_{\chi=\bar\chi=A=0} \quad , \\ \left.\frac{\delta^3\Gamma_{\rm bare}[\chi,\bar\chi,A]}{\delta A_\mu(x)\delta\chi(y)\delta\bar\chi(z)}\right|_{\chi=\bar\chi=A=0} \quad . \qquad (45)$$

Especially the fermion mass and the gauge coupling are normalized by the second and the third condition respectively. Renormalization parameters are obtained by

$$\mathcal{L}[\chi,\bar\chi,A;\mathrm{e},m] + \Delta\mathcal{L}[\chi,\bar\chi,A] = \mathcal{L}[\sqrt{Z_2}\chi,\sqrt{Z_2}\bar\chi,\sqrt{Z_3}A;\mathrm{e}+\Delta\mathrm{e},m+\Delta m] \quad .(46)$$

Compared with the case in Sec.4, we have the relation:

$$\sqrt{Z_3} = \frac{1}{1+\frac{\Delta\mathrm{e}}{\mathrm{e}}} \quad , \qquad (47)$$

because the background gauge invariance is preserved. Some comments, in relation to the higher-loop structure, are in order[14].

1. Generally the terms in the Taylor-expansion of $\Delta\mathcal{L}$ play the role of subtracting sub-divergences in multi-loop diagrams. Its proof is largely based on the structure of the Taylor-expansion.

2. From the viewpoint of the Taylor-expansion, compare the treatment of the gauge part in this section and the previous one(Sec.4). In Sec.4, the

gauge term, $-\frac{1}{2}(\partial_\mu A_\mu)^2$, is faithfully Taylor-expanded in (29). Therefore, in this case, the subdivergence problem is manifestly solved. On the other hand, the background field gauge adopted in this section, $\mathcal{L}_{\rm gauge}$ in (44), is not Taylor-expanded. The proof of the subdivergence cancellation is solved by using the properties described in the next item. The superiority of (44) is that the effective action $\Gamma_{\rm bare}$ is guaranteed to be gauge invariant.

3. In relation to the subdivergence problem mentioned above, at 2-loop and higher orders, the gauge parameter and quantum fields suffer from the renormalization effect. (Renormalization of "internal" quantities.)

Through this analysis, the character of the domain wall regularization is revealed. First point is the condition on the regularization parameters (26) as stated in Sec.3. The second point is the treatment for the fermion loop (determinant) is different from the other types of loops which are irrelevant to the chiral problem. For the fermion loop, we do the calculation in the order: 1) the momentum (k^μ) integral, 2) taking the procedure $Mt \ll 1$, 3) the extra coordinate (t) integral. Therefore, for each t-segment, the momentum region is suppressed as $|k_\mu| \leq M_t(\ll \frac{1}{t})$, where the lower-suffix 't' indicates its t-dependence. For other types of loops, the corresponding region is suppressed as $|k_\mu| \leq \frac{1}{t}$. Two upper cut-offs have the relation $M_t \ll \frac{1}{t}$ from the requirement (26). It is expected that the different treatment does no harm to the final physical results as far as the *low energy* fermion and gauge bosons are concerned. (This situation is realised in the lattice numerical simulation.)

In the literature so far, the domain wall is discussed mainly for the determinant calculation for the external gauge field. In the present paper, we have formulated it into the general field theory framework, using the background field method, where both fermion and (gauge) boson are quantumly treated. We can calculate any term, in principle, of the effective action. We have explicitly shown the renormalization of the fermion wave function and of the fermion mass. They agree with the known results.

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