Micro-Macro Duality in Quantum Physics*

Dedicated to Professor Takeyuki Hida

on the occasion of his 77th birthday

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1 Why & what is Micro-Macro Duality?

– Vital roles played by Macro –

In spite of their ubiquitous (but implicit) relevance to quantum theory, the importance of **macroscopic classical levels** is forgotten in current trends of microscopic quantum physics (owing to the overwhelming belief in the ultimate unification at the Planck scale?). Without those levels, however, **neither** *measurement* processes **nor** *theoretical descriptions* of microscopic quantum world would be possible! For instance, a *state* $\omega : \mathfrak{A} \to \mathbb{C}$ as one of the basic ingredients of quantum theory is nothing but a **micro-macro interface** assigning macroscopically measurable *expectation value* $\omega(A)$ to each microscopic quantum observable $A \in \mathfrak{A}$. Also physical *interpretations* of quantum phenomena are impossible without *vocabularies* (e.g., spacetime x, energy-momentum p, mass m, charge q, particle numbers n; entropy S, temperature T, etc., etc.), whose communicative powers rely on their close relationship with macroscopic classical levels of nature.

– **Universality of Macro** due to Micro-Macro duality –

Then one is interested in the question as to *why* and *how* macroscopic levels play such essential roles: the answer is found in the **universality** of "Macro" in the form of *universal connections* of a special Macro with generic Micro's. To equip this notion with a precise mathematical formulation we introduce the notion of a categorical *adjunction* $\mathcal{Q}\underset{E}{\overset{F}{\rightleftarrows}}\mathcal{C}$ which controls the mutual relations between [*unknown generic* objects $\mathcal{Q}$ (: microscopic quantum side) to be *described, classified* and *interpreted*] and [*special familiar* model $\mathcal{C}$ (: macroscopic classical side) for *describing, classifying* and *interpreting*], related by a pair of functors E(: $c \to q$) and F(: $q \to c$), mutually

*Talk at International Conference on Stochastic Analysis, Classical and Quantum held in Nagoya in November 2004 inverse **up to homotopy** $I \xrightarrow{\eta} EF$, $FE \xrightarrow{\varepsilon} I$, via a *natural isomorphism*:

$$\mathcal{Q}(\omega, E(a)) \quad \underset{E(\cdot)\eta_\omega}{\overset{\varepsilon_a F(\cdot)}{\rightleftarrows}} \quad \mathcal{C}(F(\omega), a) \ ,$$

so that

> an 'equation' $E(a) \sim \omega$ in $\mathcal{Q}$ to compare an unknown object ω with controlled ones $E(a)$ specified by known parameters a in $\mathcal{C}$ can be 'solved' to give a solution $a \sim F(\omega)$ which allows ω to be interpreted in the vocabulary a in $\mathcal{C}$ in the context and up to the accuracy specified, respectively, by (E, F) and (η, ε).

Abstract mathematical essence of "**Micro-Macro Duality**" can be seen in this notion of adjunction, whose concrete meanings are seen in the following discussion. What to be emphasized before going into details is the vast *freedom in the choices* of categories $\mathcal{Q}$, $\mathcal{C}$ and functors E, F which are *not* to be fixed but *adjusted* and *modified flexibly* so that our descriptions are adapted to each focused context of given physical situations and to the aspects to be examined. This point should be contrasted to the rigidity inherent to the ultimate "Theory of Everything". The simplest example of duality is given by the Gel'fand isomorphism,

$$CommC^*Alg(\mathfrak{A}, C_0(M)) \simeq HausSp(M, Spec(\mathfrak{A})), \tag{1}$$

between a commutative C*-algebra and a Hausdorff space defined by $[\varphi^*(x)](A) := [\varphi(A)](x)$ for $[\mathfrak{A} \xrightarrow{\varphi} C_0(M)] \rightleftarrows [M \xrightarrow{\varphi^*} Spec(\mathfrak{A}) = \{\chi : \mathfrak{A} \to \mathbb{C} \ ; \chi\text{: character s.t. } \chi(AB) = \chi(A)\chi(B)\}]$ and for $A \in \mathfrak{A}, x \in M$. Through our discussion on the Micro-Macro duality below, we will encounter various kinds of fundamental adjunctions appearing in quantum physics as follows:

1) Basic duality between algebras/ groups and states / representations

"*Micro-Macro Duality*" underlies "a unified scheme for generalized sectors based upon selection criteria" [14] proposed by myself in 2003 to control various branches of physics from a unified viewpoint. Extracted from a new general formulation of local thermal states in relativistic QFT (Buchholz, IO and Roos [2]), this scheme has played essential roles in my recent work to extend the Doplicher-Haag-Roberts superselection theory [5, 6] to recover a field algebra $\mathfrak{F}$ and its (global) gauge group G from the G-invariant observable algebra $\mathfrak{A} = \mathfrak{F}^G$ and its selected family of states, according to which its range of applicability restricted to unbroken symmetries has been extended to not only *spontaneously* but also *explicitly broken symmetries* [15].

2) Adjunction as a **selection criterion** to select states of physical relevance to a specific physical situation, which ensures at the same time the physical interpretations of selected states. This is just the core of the present

approach to **Micro-Macro Duality** between *microscopic quantum* and *macroscopic classical* worlds formulated mathematically by categorical *adjunctions*:

$$\text{(generic:) } \mathbf{Micro} \underset{c-q}{\overset{q-c}{\rightleftarrows}} \mathbf{Macro} \text{ (: special model space with } \mathbf{universality}\text{)},$$

where $c \to q$ ($q \to c$) means a $c \to q$ ($q \to c$) channel to transform classical states into quantum ones (vice versa).

3) **Symmetry breaking patterns** constituting such a hierarchy as *unbroken / spontaneously broken / explicitly broken* symmetries: the adjunction relevant here describes and controls the relation between [broken $\rightleftarrows$ unbroken], playing essential roles in formulating the criterion for symmetry breakings in terms of *order parameters*. Through a Galois extension, an *augmented algebra* can be defined as a composite system consising of the object physical system and of its macroscopic environments including externalized breaking terms, where broken symmetries are "recovered" and the couplings with external fields responsible for symmetry breaking are naturally described.

4) If we succeed in extrapolating this line of thoughts to attain an adjunction between [irreversible historical process] $\overset{\text{homotopical dilation}}{\rightleftarrows}$ [stabilized hierarchical domains with reversible dynamics] through enough controls over mutual connections among different physical theories describing different domains of nature, we would be able to envisage a perspective towards a theoretical framework to describe the historical process of the cosmic evolution.

2 Basic scheme for Micro/Macro correspondence

2.1 Definition of sectors and order parameters

In the absence of an intrinsic length scale to separate quantum and classical domains, the distinctions between Micro and Macro and between quantum and classical are to some extent 'independent' of each other, admitting such interesting phenomena as "macroscopic quantum effects". Since this kind of "mixtures" can be taken as 'exceptional', however, we put in parallel micro//quantum//non-commutative and macro//classical//commutative, respectively, in *generic* situations. The essence of Micro/Macro correspondence is then seen in the fundamental duality between non-commutative algebras of quantum observables and their states, where the latter transmit the microscopic data encoded in the former at invisible quantum levels into the visible macroscopic form. While the relevance of *duality* is evident from such prevailing *opposite directions* as between maps $\varphi : \mathfrak{A}_1 \to \mathfrak{A}_2$ of algebras and their *dual* maps of states, $\varphi^* : E_{\mathfrak{A}_2} \ni \omega \longmapsto \varphi^*(\omega) = \omega \circ \varphi \in E_{\mathfrak{A}_1}$, their relation cannot, however, be expressed in such a simple clear-cut form as the Gel'fand isomorphism Eq.(1) valid for commutative algebras, because of the difficulty in recovering algebras on the micro side from the macro data of states. The essence of the following discussion consists, in a sense, in the efforts of circumventing this obstacle for recovering Micro from Macro.

Starting from a given C*-algebra $\mathfrak{A}$ of observables describing a Micro quantum system, we find, as a useful mediator between algebras and states, the category $Rep_{\mathfrak{A}}$ of representations $\pi = (\pi, \mathfrak{H}_\pi)$ of $\mathfrak{A}$ with intertwiners T, $T\pi_1(A) = \pi_2(A)T$ $(\forall A \in \mathfrak{A})$, as arrows $\in Rep_{\mathfrak{A}}(\pi_1, \pi_2)$, which is nicely connected with the state space $E_{\mathfrak{A}}$ of $\mathfrak{A}$ via the *GNS construction*: $\omega \in E_{\mathfrak{A}} \underset{\text{unitary equiv.}}{\overset{\text{1:1 up to}}{\longleftrightarrow}} (\pi_\omega, \mathfrak{H}_\omega) \in Rep_{\mathfrak{A}}$ with $\Omega_\omega \in \mathfrak{H}_\omega$ s.t. $\omega(A) = \langle \Omega_\omega \mid \pi_\omega(A)\Omega_\omega \rangle$ $(\forall A \in \mathfrak{A})$ and $\overline{\pi_\omega(\mathfrak{A})\Omega_\omega} = \mathfrak{H}_\omega$. Two representations π_1, π_2 without (non-zero) connecting arrows are said to be *disjoint* and denoted by $\pi_1 \overset{\shortmid}{\circ} \pi_2$, i.e., $\pi_1 \overset{\shortmid}{\circ} \pi_2 \overset{\text{def}}{\Longleftrightarrow} Rep_{\mathfrak{A}}(\pi_1, \pi_2) = \{0\}$. The opposite situation to disjointness can be found in the definition of *quasi-equivalence*, $\pi_1 \approx \pi_2$, which can be simplified into

$$\begin{aligned}&\pi_1 \approx \pi_2 \text{ (: unitary equivalence up to multiplicity)}\\ &\quad \Longleftrightarrow \pi_1(\mathfrak{A})'' \simeq \pi_2(\mathfrak{A})'' \Longleftrightarrow c(\pi_1) = c(\pi_2) \Longleftrightarrow W^*(\pi_1)_* = W^*(\pi_2)_*.\end{aligned}$$

To explain the *central support* $c(\pi)$ of a representation π, we introduce the universal enveloping W*-algebra $\mathfrak{A}^{**} \simeq \pi_u(\mathfrak{A})'' := W^*(\mathfrak{A})$ of C*-algebra $\mathfrak{A}$ which contains all (cyclic) representations of $\mathfrak{A}$ as W*-subalgebras $W^*(\pi) := \pi(\mathfrak{A})'' \subset W^*(\mathfrak{A})$. In the universal Hilbert space $\mathfrak{H}_u := \oplus_{\omega \in E_{\mathfrak{A}}} \mathfrak{H}_\omega$, $W^*(\mathfrak{A})$ and $W^*(\pi)$ are realized, respectively, by the universal representation $(\pi_u, \mathfrak{H}_u)$, $\pi_u := \oplus_{\omega \in E_{\mathfrak{A}}} \pi_\omega$, and by its subrepresentations $\pi(A) := P(\pi)\pi_u(A) \restriction_{P(\pi)}$ $(\forall A \in \mathfrak{A})$ in $\mathfrak{H}_\pi = P(\pi)\mathfrak{H}_u$ with $P(\pi) \in W^*(\mathfrak{A})'$. $W^*(\mathfrak{A})$ is characterized by universality via adjunction,

$$W^*Alg(W^*(\mathfrak{A}), \mathcal{M}) \simeq C^*Alg(\mathfrak{A}, E(\mathcal{M})),$$

between categories C^*Alg, W^*Alg of C*- and W*-algebras (with forgetful functor E to treat $\mathcal{M}$ as C*-algebra $E(\mathcal{M})$ forgetting its W*-structure due to the predual $\mathcal{M}_*$) with a canonical embedding map $\mathfrak{A} \overset{\eta_{\mathfrak{A}}}{\hookrightarrow} E(W^*(\mathfrak{A}))$, so that any C*-homomorphism $\forall \varphi : \mathfrak{A} \to E(\mathcal{M})$ is factored $\varphi = E(\psi) \circ \eta_{\mathfrak{A}}$ through $\eta_{\mathfrak{A}}$ with a uniquely existing W*-homomorphism $\psi : W^*(\mathfrak{A}) \to \mathcal{M}$:

$$\begin{tikzcd} \mathfrak{A} \arrow[d, "\eta_{\mathfrak{A}}"'] \arrow[drr, "\forall\varphi"] \arrow[dr, phantom, "\circlearrowright"] & & \\ E(W^*(\mathfrak{A})) \arrow[rr, dashed, "E(\psi)"'] & & E(\mathcal{M}) \end{tikzcd}.$$

In this situation, the central support $c(\pi)$ of the representation π is defined by the minimal central projection majorizing $P(\pi)$ in the centre $\mathfrak{Z}(W^*(\mathfrak{A})) := W^*(\mathfrak{A}) \cap W^*(\mathfrak{A})'$ of $W^*(\mathfrak{A})$.

i) Basic scheme for **Micro-Macro correspondence** in terms of **sectors** and **order parameters**: The Gel'fand spectrum $Spec(\mathfrak{Z}(W^*(\mathfrak{A})))$ of the centre $\mathfrak{Z}(W^*(\mathfrak{A})) := W^*(\mathfrak{A}) \cap W^*(\mathfrak{A})'$ can be identified with a factor spectrum $\widehat{\mathfrak{A}}$ of $\mathfrak{A}$:

$$Spec(\mathfrak{Z}(W^*(\mathfrak{A}))) \simeq \widehat{\mathfrak{A}} := F_{\mathfrak{A}}/\approx: \textbf{factor spectrum},$$

defined by all *quasi-equivalence classes* of factor states $\omega \in F_{\mathfrak{A}}$ (with trivial centres $\mathfrak{Z}(W^*(\pi_\omega)) := W^*(\pi_\omega) \cap W^*(\pi_\omega)' = \mathbb{C}\mathbf{1}_{\mathfrak{H}_\omega}$ in the GNS representations $(\pi_\omega, \mathfrak{H}_\omega)$).

Definition 1 *A* ***sector*** *of observable algebra* $\mathfrak{A}$ *is defined by a* ***quasi-equivalence class of factor states*** *of* $\mathfrak{A}$.

In view of the commutativity of $\mathfrak{Z}(W^*(\mathfrak{A}))$ and of the role of its spectrum, we can regard

- $Spec(\mathfrak{Z}(W^*(\mathfrak{A}))) \simeq \widehat{\mathfrak{A}}$ as the **classifying space of sectors** to distinguish among different sectors, and
- $\mathfrak{Z}(W^*(\mathfrak{A}))$ as the algebra of **macroscopic order parameters** to specify sectors.

Then the map

$$\textbf{Micro:} \quad \mathfrak{A}^* \supset E_{\mathfrak{A}} \twoheadrightarrow Prob(\widehat{\mathfrak{A}}) \subset L^\infty(\widehat{\mathfrak{A}})^* \;\; : \textbf{Macro},$$

defined as the dual of embedding $\mathfrak{Z}(W^*(\mathfrak{A})) \simeq L^\infty(\widehat{\mathfrak{A}}) \hookrightarrow W^*(\mathfrak{A})$, can be interpreted as a *universal* $q{\to}c$ *channel*, transforming microscopic quantum states $\in E_{\mathfrak{A}}$ to macroscopic classical states $\in Prob(\widehat{\mathfrak{A}})$ identified with probabilities. This *basic* $q \to c$ *channel*,

$$E_{\mathfrak{A}} \ni \omega \longmapsto \mu_\omega = \omega'' \restriction_{\mathfrak{Z}(W^*(\mathfrak{A}))} \in E_{\mathfrak{Z}(W^*(\mathfrak{A}))} = M^1(Spec(\mathfrak{Z}(W^*(\mathfrak{A})))) = Prob(\widehat{\mathfrak{A}}),$$

describes the probability distribution μ_ω of sectors contained in the central decomposition of a state ω of $\mathfrak{A}$:

$$\widehat{\mathfrak{A}} \supset \Delta \longmapsto \omega''(\chi_\Delta) = \mu_\omega(\Delta) = Prob(\text{sector} \in \Delta \mid \omega),$$

where ω'' denotes the normal extension of $\omega \in E_{\mathfrak{A}}$ to $W^*(\mathfrak{A})$. While it tells us as to which sectors appear in ω, it cannot specify as to precisely which representative factor state appears within each sector component of ω.

ii) [**MASA**] To detect this *intrasectorial data*, we need to choose a *maximal abelian subalgebra* (MASA) $\mathfrak{N}$ of a factor $\mathfrak{M}$, defined by the condition

$\mathfrak{N}' \cap \mathfrak{M} = \mathfrak{N} \cong L^\infty(Spec(\mathfrak{N}))$. Using a tensor product $\mathfrak{M} \otimes \mathfrak{N}$ (acting on the Hilbert-space tensor product $c(\pi)\mathfrak{H}_u \otimes L^2(Spec(\mathfrak{N}))$) with a centre given by

$$\mathfrak{Z}(\mathfrak{M} \otimes \mathfrak{N}) = \mathfrak{Z}(\mathfrak{M}) \otimes \mathfrak{N} = \mathbf{1} \otimes L^\infty(Spec(\mathfrak{N})),$$

we find a *conditional sector structure* described by spectrum $Spec(\mathfrak{N})$ of a chosen MASA $\mathfrak{N}$.

iii) [**Measurement scheme** as **group duality**] Since the W*-algebra $\mathfrak{N}$ is generated by its unitary elements $\mathcal{U}(\mathfrak{N})$, the composite algebra $\mathfrak{M} \otimes \mathfrak{N}$ can be seen in the context of a certain group action which can be related with a coupling of $\mathfrak{M}$ with the *probe system* $\mathfrak{N}$ as seen in my simplified version [14] of Ozawa's measurement scheme [17]. To be more explicit, a reformulation in terms of a *multiplicative unitary* [1] can exhibit the universal essence of the problem. In the context of a *Hopf-von Neumann algebra* $M(\subset B(\mathfrak{H}))$ [7] with a coproduct $\Gamma : M \to M \otimes M$, a multiplicative unitary $V \in \mathcal{U}((M \otimes M_*)^-) \subset \mathcal{U}(\mathfrak{H} \otimes \mathfrak{H})$ implementing Γ, $\Gamma(x) = V^*(\mathbf{1} \otimes x)V$, is characterized by the pentagonal relation, $V_{12}V_{13}V_{23} = V_{23}V_{12}$, on $\mathfrak{H} \otimes \mathfrak{H} \otimes \mathfrak{H}$, expressing the coassociativity of Γ, where subscripts i, j of V_{ij} indicate the places in $\mathfrak{H} \otimes \mathfrak{H} \otimes \mathfrak{H}$ on which the operator V acts. It plays fundamental roles as an intertwiner, $V(\lambda \otimes \iota) = (\lambda \otimes \lambda)V$, showing the quasi-equivalence between tensor powers of the regular representation $\lambda : M_* \ni \omega \longmapsto \lambda(\omega) := (i \otimes \omega)(V) \in \hat{M}$, a generalized Fourier transform, $\lambda(\omega_1 * \omega_2) = \lambda(\omega_1)\lambda(\omega_2)$, of the convolution algebra M_*, $\omega_1 * \omega_2 := \omega_1 \otimes \omega_2 \circ \Gamma$. On these bases the duality for Kac algebras as a generalization of *group duality* can be formulated. In the case of $M = L^\infty(G, dg)$ with a locally compact group G with the Haar measure dg, the multiplicative unitary V is explicitly specified on $L^2(G \times G)$ by

$$(V\xi)(s,t) := \xi(s, s^{-1}t) \quad \text{for } \xi \in L^2(G \times G), s, t \in G, \tag{2}$$

or symbolically in the Dirac-type notation,

$$V|s,t\rangle = |s, st\rangle. \tag{3}$$

Identifying M with the Hopf-von Neumann algebra $L^\infty(G) = \mathfrak{N}$ corresponding to $G := \widehat{\mathcal{U}(\mathfrak{N})}$ given by the character group of our abelian group $\mathcal{U}(\mathfrak{N})$ (assumed to be *locally compact*), we adapt this machinery to the present context of the MASA $\mathfrak{N}$, by considering a crossed product $\mathfrak{M} \rtimes_\alpha G := [\mathbb{C} \otimes \lambda(G)''] \vee \alpha(\mathfrak{M})$ [9] defined as the von Neumann algebra generated by $\mathbb{C} \otimes \lambda(G)'' = \mathbb{C} \otimes \widehat{\mathfrak{N}}$ and by the image $\alpha(\mathfrak{M})$ of $\mathfrak{M}$ under an isomorphism α of $\mathfrak{M}$ into $\mathfrak{M} \otimes L^\infty(G) \simeq L^\infty(G, \mathfrak{M}) \simeq \mathfrak{M} \otimes \mathfrak{N}$ given by $[\alpha(B)](\gamma) := Ad_\gamma(B) = \phi_\gamma B \phi_\gamma^*$, $\gamma \in G$, $B \in \mathfrak{M}$ where ϕ_γ is an action of $\gamma \in G$ on $L^\infty(\mathfrak{M})$. By definition, $\mathfrak{M} \rtimes_\iota G = \mathfrak{M} \otimes \widehat{\mathfrak{N}}$ is evident for the trivial G-action ι with $\iota(\mathfrak{M}) = \mathfrak{M}$. The crossed product $\mathfrak{M} \rtimes_\alpha G$ is generated by the representation $\phi(V) = \int_G dE(\gamma) \otimes \lambda_\gamma$ of V on $L^2(\mathfrak{M}) \otimes L^2(G)$ with the spectral measure $E(\Delta) = E(\chi_\Delta)$ of $\mathfrak{N}$ (for Borel sets Δ in $Spec(\mathfrak{N})$) defined by the embedding homomorphism $E : \mathfrak{N} \cong L^\infty(G) \hookrightarrow \mathfrak{M}$ of $\mathfrak{N}$ into $\mathfrak{M}$, as seen from $(\omega \otimes i)(\phi(V)) = \lambda(E^*\omega) \in \mathbb{C} \otimes \widehat{\mathfrak{N}}$ and $(i \otimes \Omega)(\phi(V)) = \int_G dE(\gamma)\Omega(\lambda_\gamma) \in \alpha(\mathfrak{M})$. The action of $\phi(V)$ corresponding to Eq.(3) can be expressed by

$$\phi(V)(\xi_\gamma \otimes |\chi\rangle) = \xi_\gamma \otimes |\gamma\chi\rangle \quad \text{for } \gamma, \chi \in G, \tag{4}$$

satisfying the modified version of the pentagonal relation, $\phi(V)_{12}\phi(V)_{13}V_{23} = V_{23}\phi(V)_{12}$, or equivalently, $V_{23}\phi(V)_{12}V_{23}^* = \phi(V)_{12}\phi(V)_{13}$. Under the assumption that $\mathcal{U}(\mathfrak{N})$ is locally compact, the spectral measure E constitutes an *imprimitivity system*, $\phi_\gamma(E_A(\Delta))\phi_\gamma^* = E_A(\gamma\Delta)$, w.r.t. a representations ϕ of G on $L^2(\mathfrak{M})$, from which the following intertwining relation follows: $\phi(V)(\phi_\gamma \otimes I) = (\phi_\gamma \otimes \lambda_\gamma)\phi(V)$, for $\gamma \in G$. While the role of a multiplicative unitary is to put an arbitrary representation ρ in quasi-equivalence relation $\approx$ with the regular representation λ by tensoring with λ: $\rho \otimes \lambda \cong U_\rho(\iota \otimes \lambda)U_\rho^* \approx \lambda$, the above relation allows us to proceed further to

$$\phi \approx \phi(V)(\phi \otimes \iota)\phi(V)^* = \phi \otimes \lambda \cong U_\phi(\iota \otimes \lambda)U_\phi^* \approx \lambda.$$

The important operational meaning of Eq.(4) can clearly be seen in the case where G is a *discrete* group which is equivalent to the *compactness* of the group $\mathcal{U}(\mathfrak{N})$ in its norm topology (or, the almost periodicity of functions on it). In the present context of *group duality* with G as an abelian group generated by $Spec(\mathfrak{N})$, the unit element $\iota \in G$ naturally enters to describe the **neutral position** of measuring pointer in addition to $Spec(\mathfrak{N})$, in contrast to the usual approach to measurements. Then Eq.(4) is seen just to create the required correlation ("perfect correlation" due to Ozawa [18]) between the states ξ_γ of microscopic system $\mathfrak{M}$ to be observed and that $|\gamma\rangle$ of the measuring probe system $\mathfrak{N}$ coupled to the former: $\phi(V)(\xi_\gamma \otimes |\iota\rangle) = \xi_\gamma \otimes |\gamma\rangle$ for $\forall\gamma \in G$. Applying it to a generic state[1] $\xi = \sum_{\gamma \in G} c_\gamma \xi_\gamma$ of $\mathfrak{M}$, an initial *uncorrelated* state $\xi \otimes |\iota\rangle$ is transformed by $\phi(V)$ to a *correlated* one:

$$\phi(V)(\xi \otimes |\iota\rangle) = \sum_{\gamma \in G} c_\gamma \xi_\gamma \otimes |\gamma\rangle.$$

The created perfect correlation establishes a *one-to-one* correspondence between the state ξ_γ of the system $\mathfrak{M}$ and the measured data γ on the pointer, which would not hold without the maximality of $\mathfrak{N}$ as an abelian subalgebra of $\mathfrak{M}$. On these bases, we can define the notion of an **instrument** $\mathfrak{I}$ unifying all the ingredients relevant to a measurement as follows:

$$\begin{aligned}\mathfrak{I}(\Delta|\omega_\xi)(B) &:= (\omega_\xi \otimes |\,\iota\rangle\langle\,\iota|)(\phi(V)^*(B \otimes \chi_\Delta)\phi(V)) \\ &= (\langle\,\xi| \otimes \langle\,\iota|)\phi(V)^*(B \otimes \chi_\Delta)\phi(V)(|\xi\rangle \otimes |\,\iota\rangle).\end{aligned}$$

[1]Note that any normal state of $\mathfrak{M}$ in the *standard form* can be expressed as a vectorial state without loss of generality.

In the situation with a state $\omega_\xi = \langle\, \xi |\ (-)\xi\rangle$ of $\mathfrak{M}$ as an initial state of the system, the instrument describes simultaneously the probability $p(\Delta|\omega_\xi) = \mathfrak{I}(\Delta|\omega_\xi)(\mathbf{1})$ for measured values of observables in $\mathfrak{N}$ to be found in a Borel set Δ and the final state $\mathfrak{I}(\Delta|\omega_\xi)/p(\Delta|\omega_\xi)$ realized through the detection of measured values [17]. While this measurement scheme of Ozawa's is formulated originally in quantum-mechanical contexts with *finite* degrees of freedom where $\mathfrak{M}$ is restricted to type I, its applicability to general situations without such restrictions is now clear from the above formulation which applies equally to non-type I algebras describing such general quantum systems with *infinite degrees of freedom* as QFT. Since instruments do not exclude "generalized observables" described by "positive operator-valued measures (POM)", it may be interesting to examine the possibility to replace the spectral measure $dE(\gamma)$ with such a POM as corresponding to a non-homomorphic completely positive map for embedding a commutative subalgebra $\mathfrak{N}$ into $\mathfrak{M}$.

In what follows, the above new formulation will be seen to provide a *prototype* of more general situations found in various contexts involving sectors, such as *Galois-Fourier duality* in the DHR sector theory and its extension to *broken symmetries* with *augmented algebras* (see below). It is important there to control such *couplings between Micro* ($\mathfrak{M}$) *and Macro* ($\mathfrak{N}$ as measuring apparatus) as $\phi(V) \in \mathfrak{M} \rtimes G$, whose Lie generators in infinitesimal version consist of $A_i \in \mathfrak{M}$ and their "conjugate" variables to transform $G \ni \chi \longmapsto \gamma_i\chi \in G$. This remarkable feature exhibited already in von Neumann's measurement model, is related with a Heisenberg group as a central extension of an abelian group with its dual and is found universally in such a form as Onsager's dissipation functions, (currents)$\times$(external forces), as a linearized version of general entropy production [11], etc. To be precise, what is described here is the state-changing processes caused by this type of interaction terms $\phi(V)$ between the observed system $\mathfrak{M}$ and the probing external system $\mathfrak{N}$, *with the intrinsic (="unperturbed") dynamics of the former being neglected.* While the validity of this approximation is widely taken for granted (especially in the context of measurement theory), the problem as to how to justify it seems to be a conceptually interesting and important issue which will be discussed elsewhere.

iv) [**Central measure** as a **c→q channel**] Here we note that, from the spectral measure in iii), a *central measure* μ is defined and achieves a central decomposition of $\mathfrak{M} \otimes \mathfrak{N} = L^\infty(Spec(\mathfrak{N}), \mathfrak{M}) = \int^{\oplus}_{Spec(\mathfrak{N})} \mathfrak{M}_a d\mu(a)$, where $\mu(\Delta) := \omega_0(E(\Delta))$ with ω_0 a state of $\mathfrak{N}$ supported by $Spec(\mathfrak{N})$ being faithful to ensure the equivalence $\mu(\Delta) = 0 \Longleftrightarrow E(\Delta) = 0$.

A central measure μ is characterized as a special case of orthogonal measures by the following relations according to a general theorem due to Tomita (see [3] Theorem 4.1.25): for a state $\omega \in E_{\mathfrak{A}}$ of a unital C*-algebra $\mathfrak{A}$ there is a 1-1 correspondence between the following three items, 1) (sub)central measures μ on $E_{\mathfrak{A}}$ s.t. $\omega = \int_{E_{\mathfrak{A}}} \omega' d\mu(\omega')$ and $\left[\int_{E_{\mathfrak{A}} \setminus S} \omega' d\mu(\omega')\right] \overset{\shortmid}{\circ} \left[\int_S \omega' d\mu(\omega')\right]$ for $\forall\Delta$: Borel set in $E_{\mathfrak{A}}$, 2) W*-subalgebras $\mathfrak{B}$ of the centre: $\mathfrak{B} \subset \mathfrak{Z}(W^*(\pi_\omega)) = \pi_\omega(\mathfrak{A})' \cap \pi_\omega(\mathfrak{A})''$, 3) projections P on $\mathfrak{H}_\omega$ s.t. $P\Omega_\omega = \Omega_\omega$, $P\pi_\omega(\mathfrak{A})P \subset \{P\pi_\omega(\mathfrak{A})P\}'$. If μ, $\mathfrak{B}$, P are in correspondence, they are related mutually as follows:

1. $\mathfrak{B} = \{P\}' \cap \mathfrak{Z}(W^*(\pi_\omega))$;
2. $P = [\mathfrak{B}\Omega_\omega]$;
3. $\mu(\hat{A}_1\hat{A}_2 \cdots \hat{A}_n) = \langle \Omega_\omega \mid \pi_\omega(A_1)P\, \pi_\omega(A_2)P \cdots P\pi_\omega(A_n)\Omega_\omega \rangle$, where $\hat{A} \in C(E_{\mathfrak{A}})$ for $A \in \mathfrak{A}$ denotes a map $\hat{A}(\varphi) = \varphi(A)$ for $\varphi \in E_{\mathfrak{A}}$;
4. $\mathfrak{B}$ is *-isomorphic to the image of $\kappa_\mu : L^\infty(E_{\mathfrak{A}}, \mu) \ni f \longmapsto \kappa_\mu(f) \in \pi_\omega(\mathfrak{A})'$ defined by $\langle \Omega_\omega \mid \kappa_\mu(f)\pi_\omega(A)\Omega_\omega \rangle = \int d\mu(\omega')f(\omega')\omega'(A)$, and, for $A, B \in \mathfrak{A}$, $\kappa_\mu(\hat{A})\pi_\omega(B)\Omega_\omega = \pi_\omega(B)P\pi_\omega(A)\Omega_\omega$.

When $\mathfrak{B} = \{P\}' \cap \mathfrak{Z}(W^*(\pi_\omega)) = \mathfrak{Z}(W^*(\pi_\omega))$, or equivalently, $\mathfrak{Z}(W^*(\pi_\omega)) \subset \{P\}'$, μ is called a **central measure**, for which we can derive the following result from the above fact:

Proposition 2 ([16]) *A map Λ_μ defined by*

$$\Lambda_\mu : \pi_\omega(\mathfrak{A})'' \ni \pi_\omega(A) \longmapsto \kappa_\mu(\hat{A}) \in \mathfrak{Z}(W^*(\pi_\omega))$$

is a ***conditional expectation*** *characterized by*

$$\Lambda_\mu(Z_1\pi_\omega(A)Z_2) = Z_1\Lambda_\mu(\pi_\omega(A))Z_2 \quad \text{for } Z_i \in \mathfrak{Z}(W^*(\pi_\omega)) \ (i = 1, 2).$$

To summarize, we have established the following logical connections:

1) As dual of embedding $\mathfrak{Z}(W^*(\mathfrak{A})) \hookrightarrow W^*(\mathfrak{A})$ of the centre, we obtain a basic $q \to c$ channel $E_{\mathfrak{A}} \twoheadrightarrow Prob(Spec(\mathfrak{Z}(W^*(\mathfrak{A}))) = Prob(\widehat{\mathfrak{A}})$ with a factor spectrum $\widehat{\mathfrak{A}} = F_{\mathfrak{A}}/\approx$ as the classifying space of sectors.

2) A central measure μ_ω with a barycentre $\omega = \int_{E_{\mathfrak{A}}} \omega' d\mu_\omega(\omega') \in E_{\mathfrak{A}}$ specifies a *conditional expectation* $\Lambda_{\mu_\omega} : W^*(\pi_\omega) \ni \pi_\omega(A) \longmapsto \kappa_{\mu_\omega}(\hat{A}) = [Spec(\mathfrak{Z}(W^*(\pi_\omega))) \ni \omega' \longmapsto \omega'(A)] \in \mathfrak{Z}(W^*(\pi_\omega))$, whose dual

$$\Lambda^*_{\mu_\omega} : Prob(Spec(\mathfrak{Z}(W^*(\pi_\omega))) \to E_{W^*(\pi_\omega)}$$

defines a *$c \to q$ channel* given by $Spec(\mathfrak{Z}(W^*(\pi_\omega)))[\subset \widehat{\mathfrak{A}}] \ni \gamma \longmapsto \omega_\gamma := \Lambda^*_{\mu_\omega}(\delta_\gamma) = \delta_\gamma \circ \Lambda_{\mu_\omega} \in supp(\mu_\omega) \subset F_{\mathfrak{A}}[\subset E_{\mathfrak{A}}]$ as a (local) section of the *bundle* $F_{\mathfrak{A}} \twoheadrightarrow [F_{\mathfrak{A}}/\approx] = \widehat{\mathfrak{A}}$.

3) Operationally, this corresponds just to a choice of a **selection criterion** to select out states of relevance and we have realized that the *more internal* structure to be detected, the *larger algebra* we need, which requires the *Galois extension scheme* just in parallel with *DHR sector theory* and with my propsal of general **augmented algebra**, as seen below.

2.2 Selection criteria to choose an appropriate family of sectors

Now we come to a "unified scheme for generalized sectors based on **selection criteria**" [13, 14], extracted from a new general formulation of local thermal states in relativistic QFT [2, 12]. What I have worked out so far in this direction can be summarized as follows:

$$
\begin{array}{ccc}
\left[\begin{array}{c}\text{A) Non-equilibrium}\\ \text{local states:}\\ \textit{continuous sectors}\end{array}\right] & & \left[\begin{array}{c}\text{B) DHR sector theory of}\\ \text{unbroken internal symmetry:}\\ \textit{discrete sectors}\end{array}\right]\\
\downarrow & \swarrow & \\
\left[\begin{array}{c}\text{C) Sector structure of}\\ \textit{broken symmetry:}\\ \textit{discrete}\ \&\ \textit{continuous}\end{array}\right] & \rightarrow & \left[\begin{array}{c}\text{D) Unified scheme for}\\ \text{Micro-Macro based}\\ \text{on } \textit{selection criteria}\end{array}\right]
\end{array}
$$

A) General formulation of *non-equilibrium local states* in QFT [2, 12, 13];

B) *Reformulation* [14] of DHR-DR sector theory [5, 6] of unbroken internal symmetry;

C) Extension of B) to *spontaneously or explicitly* **broken symmetry** [14, 15].

The results obtained in A), B) and C) naturally lead us to

D) **Unified scheme for describing Micro-Macro relations** based on **selection criteria** [12, 13, 14]:

$$
\begin{array}{ccc}
\text{i)}\left[\begin{array}{c}q:\text{generic states}\\ \text{of } \textit{object system}\end{array}\right] & \overset{}{\underset{\uparrow}{\Longrightarrow}}\text{ii)} & \left[\begin{array}{c}c:\textbf{reference model system}\text{ with}\\ \textbf{classifying space}\text{ of }\textbf{sectors}\end{array}\right]\\
 & \uparrow & \\
 & \text{iii) a map to } \textit{compare}\text{ i) with ii)} & \\
\Uparrow & & \Downarrow\\
\text{iv)}\left[\begin{array}{c}\text{state preparation \&}\\ \textbf{selection criterion}\text{:}\\ \text{ii)}\underset{c\text{-}q}{\Longrightarrow}\text{i)}\end{array}\right] & \overset{\textit{adjunction}}{\rightleftarrows} & \left[\begin{array}{c}\textit{classification}\ \&\\ \textit{interpretation}\text{ of}\\ \text{i) w.r.t. ii): i)}\underset{q\text{-}c}{\Longrightarrow}\text{ii)}\end{array}\right],
\end{array}
$$

which can be seen as a natural generalization of

Example 3 *The formulation of a manifold M based on local charts $\{(U_\lambda, \varphi_\lambda : U_\lambda \to \mathbb{R}^n)\}$ consisting of*
i)= local neighbourhoods U_λ of M constituting a covering $M = \cup U_\lambda$,
ii)= model space $\mathbb{R}^n$,
iii)= local homeomorphisms $\varphi_\lambda : U_\lambda \to \mathbb{R}^n$,
iv)= interpretation of the atlas in terms of geometrical invariants such as homology, cohomology, homotopy, K-groups, characteristic classes, etc., etc.

Example 4 *Non-equilibrium local states in A) [2, 12, 13] are characterized by localizing the following generalized equilibrium states with fluctuating thermal parameters:*
i) = the set E_x of states ω at a spacetime point x satisfying certain energy bound locally [$\omega((\mathbf{1}+H_{\mathcal{O}})^m) < \infty$ with "local Hamiltonian" $H_{\mathcal{O}}$],
ii) = the space B_K of thermodynamic parameters (β, μ) to distinguish among different thermodynamic pure phases and the space $M_+(B_K) =: Th$ of probability measures ρ on B_K to describe fluctuations of (β, μ),
iii) = comparison of an unknown state ω with members of standard states $\omega_\rho = \mathcal{C}^*(\rho) = \int_{B_K} d\rho(\beta,\mu)\omega_{\beta,\mu}$ with parameters ρ belonging to reference system, in terms of the criterion $\omega \underset{\mathcal{T}_x}{\equiv} \mathcal{C}^*(\rho)$ through "quantum fields at x" $\in \mathcal{T}_x$ (justified by energy bound in i)).
iv) = adjunction

$$E_x/\mathcal{T}_x(\omega, \mathcal{C}^*(\rho)) \overset{q \rightleftarrows c}{\simeq} Th/\mathcal{C}(\mathcal{T}_x)((\mathcal{C}^*)^{-1}(\omega), \rho)$$

with $q \to c$ channel $(\mathcal{C}^*)^{-1}$ as a "left adjoint" to the $c \to q$ channel $\mathcal{C}^*$ (from the classical reference system to generic quantum states): as a localized form of the zeroth law of thermodynamics, this adjunction achieves simultaneously the two goals of identifying generalized equilibrium local states and of giving the thermal interpretation $(\mathcal{C}^*)^{-1}(\omega) \underset{\mathcal{C}^*(\mathcal{T}_x)}{\equiv} \rho$ of a selected generic state ω in the vocabulary of a standard known object $\rho \in Th$.

What we have discussed so far can be summarized as follows:

1. Classification of quantum states/representations by *quasi-equivalence* (= unitary equivalence *up to multiplicity*): achieved by means of **sectors** labelled by **macroscopic order parameters** as points in the spectrum of **centre**, where a sector is defined by a **quasi-equivalence class of factor states** $\omega \in F_{\mathfrak{A}}$ with *trivial centres* $\mathfrak{Z}(W^*(\pi_\omega)) := W^*(\pi_\omega) \cap W^*(\pi_\omega)' = \mathbb{C}\mathbf{1}_{\mathfrak{H}_\omega}$. In short, a sector = all density-matrix states within a factor representation = a *folium* of a *factor* state.
$\Downarrow$

2. A **mixed phase** = *non-factor* state = *non-trivial centre* $\mathfrak{Z}(W^*(\mathfrak{A})) \neq \mathbb{C}\mathbf{1}_{\mathfrak{H}}$: allows "simultaneous diagonalization" as a *central decomposition* arising from non-trivial sector structure.
$\Longrightarrow \mathfrak{Z}(W^*(\mathfrak{A}))$: the set of all **macroscopic order parameters** to distinguish among different sectors;
$Spec(\mathfrak{Z}(W^*(\mathfrak{A})))$: a **classifying space** to parametrize sectors completely in the sense that *quasi-equivalent sectors correspond to one and the same point* and that *disjoint sectors to the different points.*
$\Downarrow$

3. **Micro-macro relation**:
   Intersector level controlled by $\mathfrak{Z}(W^*(\mathfrak{A}))$: *macroscopic situations* prevail, which are macroscopically observable and controllable;

   Inside a sector: *microscopic situations* prevail (e.g., for a pure state in a sector, as found in the vacuum situations, it represents a "*coherent subspace*" with *superposition principle* being valid).

4. **Selection criterion** = *physically* and *operationally meaningful* characterization as to *how and which sectors* should be picked up for discussing a *specific physical domain.* E.g., *DHR criterion* for states ω with localizable charges (based upon "Behind-the-Moon" argument) $\pi_\omega \upharpoonright_{\mathfrak{A}(\mathcal{O}')} \cong \pi_0 \upharpoonright_{\mathfrak{A}(\mathcal{O}')}$ in reference to the vacuum representation π_0.

   A suitably set up criterion determines the associated *sector structure* so that natural *physical interpretations* of a theory are provided in a physical domain specified by it.

3 Sectors and symmetry: Galois-Fourier duality

To control the relations among algebras with group actons, their extensions and corresponding representations, we need the **Galois-Fourier duality** as an important variation of our main theme Micro-Macro Duality. The essence of DHR-DR theory [5, 6] of sectors associated with an unbroken internal symmetry can be seen in this duality which enables one to reconstruct a field algebra $\mathfrak{F}$ as a dynmaical system $\mathfrak{F} \curvearrowleft G$ with the action of an internal symmetry group G from its fixed-point subalgebra $\mathfrak{A} = \mathfrak{F}^G$ consisting of G-invariant observables in combination with data of a family $\mathcal{T}$ of states $\in E_{\mathfrak{A}}$ specified by the above DHR **selection criterion**:

$$\begin{array}{ccc} \text{Invisible micro} & & \text{Visible macro} \\ \left[\begin{array}{l} G \cong Rep\mathcal{T} \\ \curvearrowright \\ \mathfrak{F} \cong \mathfrak{A} \rtimes \hat{G} \end{array}\right] & \begin{array}{c} \text{Fourier duality} \\ \rightleftarrows \\ \rightleftarrows \\ \text{Galois duality} \end{array} & \left[\begin{array}{l} \mathcal{T} \cong RepG \\ \curvearrowright \\ \mathfrak{A} = \mathfrak{F}^G \cong \mathfrak{F} \rtimes G \end{array}\right] \end{array}.$$

In my recent reformulation, its applicability range restricted to unbroken symmetries has been extended to not only *spontaneously* but also *explicitly broken symmetries.*

In B) DHR-DR sector theory, we see

1. Sector structure:

$$\mathfrak{H} = \bigoplus_{\gamma \in \hat{G}} (\mathfrak{H}_\gamma \otimes V_\gamma);$$
$$\pi(\mathfrak{A})'' = \bigoplus_{\gamma \in \hat{G}} (\pi_\gamma(\mathfrak{A})'' \otimes \mathbf{1}_{V_\gamma}) = U(G)',$$
$$U(G)'' = \bigoplus_{\gamma \in \hat{G}} (\mathbf{1}_{\mathfrak{H}_\gamma} \otimes \gamma(G)'') = \pi(\mathfrak{A})'.$$

2. $\mathfrak{Z}(\pi(\mathfrak{A})'') = \underset{\gamma\in\hat{G}}{\oplus}\mathbb{C}(\mathbf{1}_{\mathfrak{H}_\gamma}\otimes\mathbf{1}_{V_\gamma}) = l^\infty(\hat{G})$; $\hat{G} = Spec(\mathfrak{Z}(\pi(\mathfrak{A})'')) \Longrightarrow$ *vocabulary for interpretation* of sectors in terms of G*-charges*.

3. $(\pi_\gamma, \mathfrak{H}_\gamma)$: sector of $\mathfrak{A} \overset{1-1}{\longleftrightarrow} (\gamma, V_\gamma) \in \hat{G}$: equiv. class of irred. unitary representations of a *compact Lie* group G of *unbroken* internal symmetry of field algebra $\mathfrak{F} := \mathfrak{A} \underset{\mathcal{O}_d^G}{\otimes} \mathcal{O}_d$ with a Cuntz algebra generated by isometries.

4. $(\pi, U, \mathfrak{H})$: covariant irred. vacuum representation of C*-dynamical system $\mathfrak{F} \underset{\tau}{\curvearrowleft} G$, s.t. $\pi(\tau_g(F)) = U(g)\pi(F)U(g)^*$.

5. $\mathfrak{A}$, G, $\mathfrak{F}$: triplet of Galois extension $\mathfrak{F}$ of $\mathfrak{A} = \mathfrak{F}^G$ by Galois group $G = Gal(\mathfrak{F}/\mathfrak{A})$, determining *one* term from *two*.
How to solve two unknowns G & $\mathfrak{F}$ from $\mathfrak{A}$?: DHR **selection criterion** $\Longrightarrow \mathcal{T}$ ($\subset End(\mathfrak{A})$): DR tensor category $\cong RepG \underset{\text{duality}}{\overset{\text{Tannaka-Krein}}{\Longrightarrow}} G \Longrightarrow$ $\mathfrak{F} \cong \mathfrak{A} \rtimes \hat{G}$.

Similar schemes hold also for C) with spontaneously and/or explicitly broken symmetries. For instance, in the case of SSB, we have [14]

$$\begin{bmatrix} \text{broken: } G \supset H\text{: unbroken} \\ \curvearrowright \qquad\quad \curvearrowright \\ \widehat{\mathfrak{F}} \supset \mathfrak{F} \cong \mathfrak{A}^d \rtimes \hat{H} \\ \| \\ \mathfrak{F} \rtimes (\widehat{H\backslash G}) = \mathfrak{A}^d \rtimes \hat{G} \end{bmatrix} \begin{matrix} \rightleftarrows \\ \circlearrowright \\ \rightleftarrows \end{matrix} \begin{bmatrix} \mathcal{T} \cong RepH \sim \hat{H} & \hookrightarrow \amalg_{gH\in G/H} g\hat{H}g^{-1} \\ \curvearrowright & \downarrow \\ \mathfrak{A}^d = \mathfrak{F}^H & G/H\text{: degenerate vacua} \\ \cup & \because \\ \mathfrak{A} = \mathfrak{F}^G & \text{sector bundle} \end{bmatrix},$$

with $\mathfrak{Z}_{\bar{\pi}}(\mathfrak{A}^d) = L^\infty(H\backslash G; d\dot{g})\otimes\mathfrak{Z}_\pi(\mathfrak{A}^d) = L^\infty(H\backslash G; d\dot{g})\otimes l^\infty(\hat{H})$ and the base space G/H of the sector bundle, $Spec(\mathfrak{Z}_{\bar{\pi}}(\mathfrak{A}^d)) = \amalg_{gH\in G/H} g\hat{H}g^{-1} \twoheadrightarrow G/H$, corresponds mathematically to the "roots" in Galois theory of equations and physically to the degenerate vacua characteristic to SSB.

3.1 Hierarchy of symmetry breaking patterns and augmeneted algebras

Extension of B) to **broken symmetries** [14, 15]: In my attempts to extend DHR-DR sector theory with unbroken symmetries to the broken cases, the adjunction,

$$Broken \overset{\textbf{augmented algebra}}{\rightleftarrows} Unbroken,$$

has been important, as seen in my criterion of symmetry breaking:

Definition 5 ([14]) *A symmetry described by a (strongly continous) automorphic G-action τ: $G \underset{\tau}{\curvearrowright} \mathfrak{F}$(: field algebra), is* ***unbroken*** *in a given representation $(\pi, \mathfrak{H})$ of $\mathfrak{F}$ if the spectrum Spec$(\mathfrak{Z}_\pi(\mathfrak{F}))$ of centre $\mathfrak{Z}_\pi(\mathfrak{F}) := \pi(\mathfrak{F})'' \cap \pi(\mathfrak{F})'$ is pointwise invariant (μ-a.e. w.r.t. the central measure μ which decomposes π into factor representations) under the G-action induced on Spec$(\mathfrak{Z}_\pi(\mathfrak{F}))$. If the symmetry is not unbroken in $(\pi, \mathfrak{H})$, it is said to be* ***broken*** *there.*

Remark 6 *Since macroscopic order parameters Spec$(\mathfrak{Z}_\pi(\mathfrak{F}))$ emerge in low-energy infrared regions, a symmetry breaking means the "****infrared(=Macro) instability****" along the direction of G-action.*

Remark 7 *Since a representation π with broken symmetry can still contain unbroken and broken subrepresentations, further decomposition of Spec$(\mathfrak{Z}_\pi(\mathfrak{F}))$ is possible into G-invariant domains. A minimal G-invariant domain is characterized by G-ergodicity which means* ***central ergodicity****. $\Longrightarrow$ π is decomposed into a direct sum (or, direct integral) of* ***unbroken factor representations*** *and* ***broken non-factor representations****, each component of which is centrally G-ergodic. $\Longrightarrow$ phase diagram on Spec$(\mathfrak{Z}_\pi(\mathfrak{F}))$.*

Thus the essence of broken symmetry is found in the **conflict between factoriality** *and* **unitary implementability**. In the usual approaches, the former is respected at the expense of the latter. Taking the opposite choice to respect implementability, we encounter a *non-trivial centre* which provides convenient tools for analyzing *sector structure* and flexible treatment of macroscopic **order parameters** to distinguish different sectors. Namely, the adjunction holds between [Broken $\overset{\textbf{non-trivial centre}}{\rightleftarrows}$ Unbroken[2]], controlled by a canonical homotopy η from [$\mathfrak{F} \curvearrowleft G$ with non-implementable broken symmetry G in a pure phase] to [$\widehat{\widehat{\mathfrak{F}}} \curvearrowleft G$ with unitarily implemented symmetry $G \to U(G)$ in a mixed phase with a *non-trivial centre*], where $\widehat{\widehat{\mathfrak{F}}}$ is an **augmented algebra** [14] defined by $\widehat{\widehat{\mathfrak{F}}} := \mathfrak{F} \rtimes \widehat{(H\backslash G)}$, as a crossed product of $\mathfrak{F}$ by the coaction of $H\backslash G$ (: degenerate vacua) arising from the symmetry breaking from G to its unbroken subgroup H.

Note here that the above criterion does not touch upon the relation between the symmetry group G and the dynamics of the physical system described by the algebra $\mathfrak{F}$ in relation with spacetime; if the latter is preserved by the former, the breakdown of symmetry G is called *spontaneous* (SSB for short). Otherwise, it is *explicit*, associated with some *parameter changes* involving changes of physical constants appearing in the specification of a physical system. For instance, we can formulate such an **explicitly broken symmetry** as *broken scale invariance* associated with

[2]To be precise, "unbroken" should be understood as "unitarily implemented".

temperature β as order parameter [15], where augmented algebra of observables $\hat{\mathfrak{A}} = \mathfrak{A} \rtimes \widehat{(SO(3)\backslash(\mathbb{R}_+ \rtimes L_+^{\uparrow})}$ is the *scaling algebra* due to Buchholz and Verch [4] to accommodate the notion of renormalization group (in combination with components arising from SSB of Lorentz boost symmetry due to thermal equilibrium [10] to accommodate relative velocity $u^\mu := \beta^\mu/\beta \in SO(3)\backslash L_+^{\uparrow}$). *What is scaled here is actually Boltzmann constant k_B!!* In this way, we are led to the **hierarchy of symmetry breaking patterns** ranging from unbroken symmetries, spontaneous and explicit breakdown of symmetries, the latter of which would be related with more general treatments of transformations, such as semigroups or groupoids.

An eminent feature emerging through the hierarchy of symmetry breaking patterns is the phenomena of *externalization* of internal degrees of freedom in the form of order parameters and breaking parameters, along which external degrees of freedom coupled to the system are incorporated through Galois extension into the **augmented algebra**: it describes a composite system consisting of the microscopic object system and its macroscopic "environments", which canonically emerge at the macroscopic levels consisting of **macroscopic order parameters** classifying different *sectors* and of **symmetry breaking terms** such as mass m and k_B, etc. This formulation allows us to describe the **coupling between the system and external fields** in a universal way (e.g., measurement couplings).

4 From [thermality $\rightleftarrows$ geometry] towards [history of Nature]

Although the *modular structure* of a W*-algebra in standard form has not been explicitly mentioned so far, it plays fundamental roles almost everywhere in the above discussion, responsible for the **homotopical extension** mechanism: this is crucial, for instance, in the formulation of group duality and of scaling as well as conformal aspects. From the viewpoint that the notion of *quasi-equivalence* fundamental to our whole discussion is just a form of homotopy, we show here the Galois-theoretical aspects of modular structure $\mathfrak{M} \rightleftarrows \mathfrak{M}'$ arising from canonical homotopy $\eta_\pi : \pi \to \pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}}$ to move to **standard form**.

Theorem 8 ([16]) *i) In the universal representation $(\pi_u, \mathfrak{H}_u = \bigoplus_{\omega \in E_{\mathfrak{A}}} \mathfrak{H}_\omega)$ of a C*-algebra $\mathfrak{A}$, we define the maximal representation $\pi^{\overset{\shortmid}{\circ}}$ disjoint from a representation $\pi = (\pi, \mathfrak{H}_\pi) \in Rep_{\mathfrak{A}}$ by*

$$\pi^{\overset{\shortmid}{\circ}} := \sup\{\rho \in Rep_{\mathfrak{A}}; \rho \leq \pi_u, \rho \overset{\shortmid}{\circ} \pi\}.$$

Then we have the following relations in terms of the projection $P(\pi) \in$

$W^*(\mathfrak{A})'$ on the representation space $\mathfrak{H}_\pi$ of π and its central support $c(\pi)$:

$$\pi_1 \le \pi_2 \Longrightarrow \pi_1^{\overset{\shortmid}{\circ}} \ge \pi_2^{\overset{\shortmid}{\circ}},\ \pi^{\overset{\shortmid}{\circ}} = \pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}} \quad \textit{and} \quad \pi \le \pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}},$$

$$P(\pi^{\overset{\shortmid}{\circ}}) = c(\pi)^{\perp} := \mathbf{1} - c(\pi),$$

$$P(\pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}}) = c(\pi)^{\perp\perp} = c(\pi) = \bigvee_{u\in\mathcal{U}(\pi(\mathfrak{A})')} uP_\pi u^* \in \mathcal{P}(\mathfrak{Z}(W^*(\mathfrak{A}))).$$

ii) Quasi-equivalence $\pi_1 \approx \pi_2 (\Longleftrightarrow \pi_1(\mathfrak{A})'' \simeq \pi_2(\mathfrak{A})'' \Longleftrightarrow c(\pi_1) = c(\pi_2) \Longleftrightarrow W^*(\pi_1)_* = W^*(\pi_2)_*)$ is equivalent to

$$\pi_1^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}} = \pi_2^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}}.$$

*iii) The representation $(\pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}}, c(\pi)\mathfrak{H}_u)$ of W^*-algebra $W^*(\pi) \simeq \pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}}(\mathfrak{A})''$ in the Hilbert space $c(\pi)\mathfrak{H}_u = P(\pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}})\mathfrak{H}_u$ gives the **standard form** of $W^*(\pi)$ associated with a normal faithful semifinite weight φ and the corresponding Tomita-Takesaki **modular structure** $(J_\varphi, \Delta_\varphi)$. It is characterized by the universality:*

$$Std(\pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}}, \sigma) \simeq Rep_{\mathfrak{A}}(\pi, \sigma),$$

where Std denotes the caterogy of representations of $\mathfrak{A}$ in standard form; according to this relation, any intertwiner $T : \pi \to \sigma$ to a representation $(\sigma, \mathfrak{H}_\sigma)$ in standard form of $W^*(\sigma)$ is uniquely factored $T = T^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}} \circ \eta_\pi$ through the canonical homotopy $\eta_\pi : \pi \to \pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}}$ with a uniquely determined intertwiner $T^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}} : \pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}} \to \sigma$.
*iv) The quasi-equivalence relation $\pi_1 \approx \pi_2$ defines a classifying groupoid $\Gamma_\approx$ consisting of **invertible intertwiners** in the category $Rep_{\mathfrak{A}}$ of representations of $\mathfrak{A}$, which reduces on each $\pi \in Rep_{\mathfrak{A}}$ to $\Gamma_\approx(\pi,\pi) \simeq Isom(W^*(\pi)_*)$, the group of isometric isomorphisms of predual $W^*(\pi)_*$ as a Banach space. The modular structure in iii) of W^*-algebra $W^*(\pi) =: \mathfrak{M}$ in the standard form in $(\pi^{\overset{\shortmid}{\circ}\overset{\shortmid}{\circ}}, c(\pi)\mathfrak{H}_u)$ can be understood as the minimal **implemention** by the unitary group $\mathcal{U}(\mathfrak{M}')$ of a normal subgroup $G_{\mathfrak{M}} := Isom(\mathfrak{M}_*)_{\mathfrak{M}} \lhd Isom(\mathfrak{M}_*)$ fixing $\mathfrak{M}$ pointwise: namely, for $\gamma \in G_{\mathfrak{M}}$, there exists $U'_\gamma \in \mathcal{U}(\mathfrak{M}')$ s.t.*

$$\langle \gamma\omega, x\rangle = \langle \omega, \gamma^*(x)\rangle = \langle \omega, U'^*_\gamma x U'_\gamma\rangle \quad \textit{for } \omega \in \mathfrak{M}_* \ ,$$

and $U'^*_\gamma x U'_\gamma = x \Longleftrightarrow x \in \mathfrak{M}$. For $\mathfrak{M}$ of type III, we can verify Galois-type relations involving crossed product by a coaction of the group $G_{\mathfrak{M}} \simeq \mathcal{U}(\mathfrak{M}')$ as follows:

$$\mathfrak{Z}(\mathfrak{M})' = \mathfrak{M} \vee \mathfrak{M}' = \mathfrak{M} \rtimes \widehat{G_{\mathfrak{M}}}\text{: Galois extension of } \mathfrak{M},$$

$$\mathfrak{M} = (\mathfrak{M} \vee \mathfrak{M}')^{G_{\mathfrak{M}}}\text{: fixed-point subalgebra under } G_{\mathfrak{M}},$$

$$G_{\mathfrak{M}} = Gal(\mathfrak{Z}(\mathfrak{M})'/\mathfrak{M})\text{: Galois group of } \mathfrak{M} \hookrightarrow \mathfrak{Z}(\mathfrak{M})',$$

according to which factoriality $\mathfrak{Z}(\mathfrak{M}) = \mathbb{C}\mathbf{1}$ *of* $\mathfrak{M}$ *can be seen as the* ***ergodicity*** *of* $\mathfrak{M}$ *under* $Aut(\mathfrak{M})$ *or* $G_{\mathfrak{M}}$*:*

$$\mathbb{C}\mathbf{1} = \mathfrak{M} \cap \mathfrak{M}' = \mathfrak{M}' \cap \mathcal{U}(\mathfrak{M}')' = (\mathfrak{M}')^{G_{\mathfrak{M}}} \supset (\mathfrak{M}')^{Aut(\mathfrak{M})}.$$

In view of the dominant roles of thermal or modular-theoretical notions mentioned above, this theorem suggests possible paths *from thermality to geometry* to explain different geometries at macroscopic classical levels emerging from the invisible microscopic quantum world; it would explain the origin of *universality* of Macro put in Micro-Macro Duality in our theoretical descriptions of physical worlds. A typical example of this sort can be seen in the formulation of group duality which exhibits its essence as a **homotopical duality** involving interpolation spaces [8]. Moreover, we can develop a framework to go into a step from the above modular homotopy to the generalized version of classifying spaces or classifying toposes [16]. Along this line of thoughts, we can envisage such a perspective that theoretical descriptions of physical nature can be mapped into a "categorical bundle of physical theories" over a base category consisting of *selection criteria* to characterize each theory as a fibre, which are mutually connected by *metamorphisms* of intertheory deformation arrows parametrized by fundamental physical constants like $\hbar$, c, k_B; κ, e, etc., controlled by the "method of variations of natural constants" (work in progress). One of the most important virtues of the above augmented algebra is found in the possibility that such physical constants can be treated on the same footing as various physical variables responsible for changing the symmetry properties of the systems; in such contexts, they represent controlling parameters of deformations among different *selection criteria* to determine *theories corresponding to stabilized hierarchical domains*. Then the most crucial step will be to formulate each selection criterion as an integrability condition in terms of generalized categorical connections, through which the framework can accommodate such an adjunction as

$$\begin{bmatrix} \text{irreversible} \\ \text{historical process} \end{bmatrix} \overset{\text{homotopical dilation}}{\rightleftarrows} \begin{bmatrix} \text{stabilized hierarchical domains} \\ \text{with reversible dynamics} \end{bmatrix}$$

to be found among such adjunctions as to put a generic category with non-invertible arrows (describing an irreversible open system in a historical process) in a relation adjoint to a groupoid with invertible arrows (corresponding to a reversible closed system with repeatable dynamics in a specific hierarchical domain). This kind of theoretical framework would provide an appropriate stage on which the natural *history* of cosmic *evolution* be developed.

References

[1] Baaj, S. and G. Skandalis, Ann. Scient. Ècole Norm. Sup. **26** (1993), 425-488.

[2] Buchholz, D., Ojima, I. and Roos, H., Ann. Phys. (N.Y.) **297** (2002), 219 - 242.

[3] Bratteli, O. and Robinson, D.W., *Operator Algebras and Statistical Mechanics*, vol. 1, Springer-Verlag (1979).

[4] Buchholz, D. and Verch, R., Rev. Math. Phys. **7** (1995), 1195-1240.

[5] Doplicher, S., Haag, R. and Roberts, J.E., Comm. Math. Phys. **13** (1969), 1-23; **15** (1969), 173-200; **23** (1971), 199-230; **35** (1974), 49-85.

[6] Doplicher, S. and Roberts, J.E., Comm. Math. Phys. **131** (1990), 51-107; Ann. Math. **130** (1989), 75-119; Inventiones Math. **98** (1989), 157-218.

[7] Enock, M. and Schwartz, J.-M., *Kac Algebras and Duality of Locally Compact Groups*, Springer, 1992.

[8] Maumary, S. and Ojima, I., in preparation.

[9] Nakagami, Y. and Takesaki, M., Lec. Notes in Math. **731**, Springer, 1979.

[10] Ojima, I., Lett. Math. Phys. **11** (1986), 73-80.

[11] Ojima, I., J. Stat. Phys. **56**(1989), 203-226; Lec. Notes in Phys. **378**, pp.164-178, Springer, 1991.

[12] Ojima, I., pp. 48-67 in Proc. of Japan-Italy Joint Workshop on Fundamental Problems in Quantum Physics, Sep. 2001, eds. Accardi, L. and Tasaki, S., World Scientific (2003).

[13] Ojima, I., pp.365-384 in "*A Garden of Quanta*", World Scientific (2003); e-print: cond-mat/0302283.

[14] Ojima, I., Open Systems and Information Dynamics, **10** (2003), 235-279; math-ph/0303009.

[15] Ojima, I., Publ. RIMS **40**, 731-756 (2004).

[16] Ojima, I., in preparation.

[17] Ozawa, M., J. Math. Phys. **25**, 79-87 (1984); Publ. RIMS, Kyoto Univ. **21**, 279-295 (1985); Ann. Phys. (N.Y.) **259**, 121-137 (1997).

[18] Ozawa, M., quant-phys/0310072, to appear in Phys. Lett. A.