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On Anabelian Properties of the Moduli Spaces of Curves of Genus Two

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Abstract. In this paper, we study the higher dimensional anabelian geometry. In particular, we show the "relative isomorphism version" of the Grothendieck conjecture for some finite étale coverings of the moduli stacks of curves of genus two in characteristic 0. The key point of the proof is to prove the Grothendieck conjecture for the configuration space of a hyperbolic curve over a hyperbolic polycurve under some conditions.

1. Introduction

Let k be a field and X and Y connected schemes/stacks over $\mathrm{Spec}(k)$. For k, X and Y, we consider the following two conditions

$$\text{(rel-Isom-GC)} \quad \mathrm{Isom}_k(X,Y) \xrightarrow{\sim} \mathrm{Isom}_{\mathrm{G}_k}(\pi_1(X),\pi_1(Y))/\mathrm{Inn}\left(\pi_1(X\otimes_k \bar{k})\right),$$

$$\text{(rel-Hom-GC)} \quad \mathrm{Hom}_k^{dom}(X,Y) \xrightarrow{\sim} \mathrm{Hom}_{\mathrm{G}_k}^{open}(\pi_1(X),\pi_1(Y))/\mathrm{Inn}\left(\pi_1(X\otimes_k \bar{k})\right).$$

Here G_K stands for the absolute Galois group of K for a field K and a separable closure $\bar{K}$ of K, $\pi_1(U)$ stands for the étale fundamental group of U for a connected scheme U and a geometric point of U, $\mathrm{Hom}_K^{dom}(U,V)$ stands for the set of all dominant K-morphisms from U to V, and $\mathrm{Hom}_{\mathrm{G}_K}^{open}(\pi_1(U),\pi_1(V))/\mathrm{Inn}\left(\pi_1(V\otimes_K \bar{K})\right)$ stands for the set of all open continuous homomorphisms from $\pi_1(U)$ to $\pi_1(V)$ over G_K, divided by the right action of $\pi_1(V\otimes_K \bar{K})$ on $\pi_1(V)$ by the conjugation, for connected schemes U, V, a field K and a separable closure $\bar{K}$ of K.

The definition of "anabelian" is not known yet. But, it is conjectured that (rel-Hom-GC) holds if X and Y are "anabelian" schemes over $\mathrm{Spec}(k)$.

Conjecture 1.1. *Let $\mathcal{M}_{g,r}$ be the moduli stack of (g,r)-curves in characteristic* 0. *It is conjectured that $\mathcal{M}_{g,r}$ is "anabelian" if $2g-2+r>0$. In fact, Grothendieck wrote in the letter to Faltings* [G] *as follows:*

Finally, my attention has been lately more and more strongly attracted by the moduli varieties (or better modular multiplicities) $\mathcal{M}_{g,\nu}$ of algebraic curves. I am rather convinced that these also may be approached as " anabelian ", namely that their relation with the fundamental group is just as tight as in the case of anabelian curves.

Definition 1.2 ((non-trivial) hyperbolic curve and hyperbolic polycurve)**.** [cf. [H], Definition 2.1]

(1) Let S be a scheme and X a scheme over S. We call $X\to S$ a *hyperbolic curve of type* (g,r) if and only if the following four conditions hold:

(i) there exist non-negative integers g and r such that $2g-2+r>0$,

(ii) there exists a proper smooth geometrically connected scheme Y over S of relative dimension 1 such that all the geometric fibers are curves of genus g,

(iii) there exists a closed subscheme D of Y which is finite and étale over S of degree r,

(iv) X is isomorphic to $Y \setminus D$ over S.

(2) Let $X \to S$ be a hyperbolic curve of type (g, r). In this paper, we call $X \to S$ a *non-trivial hyperbolic curve of type* (g, r) if and only if the following condition holds:

there exist some algebraically closed field Ω and distinct morphisms $\bar{s_1}$, $\bar{s_2}$ from $\mathrm{Spec}(\Omega)$ to S such that the smooth compactification of $X \times_S \bar{s_1}$ is not isomorphic to the smooth compactification of $X \times_S \bar{s_2}$.

(3) Let S be a scheme and X a scheme over S. If there exist a positive integer n and a [not necessary unique] factorization $X = X_n \to X_{n-1} \to \cdots \to X_1 \to X_0 = S$ of the structure morphism $X \to S$ such that X_i/X_{i-1} is a hyperbolic curve for each $i \in \{1, \cdots, n\}$, then X is called a *hyperbolic polycurve* over S.

When X and Y are the moduli stack of hyperbolic curves of genus 0 or 1, several affirmative results to the conjecture 1.1 are already known [cf., e.g., [N], Corollary 1.5.7 and [IN], Theorem C for genus 0, [K], Theorem 2.2 for genus 1]. Kinoshita's theorem was proved by showing the Grothendieck conjecture for the pure configuration spaces of a hyperbolic curve over a hyperbolic curve under some conditions [cf. [K], Theorem 2.1].

We have a connected finite étale covering $\mathcal{M}_{2,0} \leftarrow \mathcal{M}_{0,6}$, because $\mathcal{M}_{2,0} \xleftarrow{\sim} \mathcal{M}_{0,[6]}/\mathbb{Z}^{\times}$. Using this fact, we will prove the main theorem of this article by showing the Grothendieck conjecture for the pure configuration spaces of a hyperbolic curve over a hyperbolic polycurve under some conditions.

2. Main result and Proof

Theorem 2.1. *Let k be a sub-p-adic field for some prime number p and $\bar{k}$ an algebraic closure of k. For $\xi \in \{\alpha, \beta\}$, let C^{ξ} be a connected finite étale covering over $\mathcal{M}_{0,6}$, $X_n^{\xi} = \mathcal{M}_{2,n} \times_{\mathcal{M}_{2,0}} C^{\xi}$ $(n = 0, 1, \cdots)$, and r^{ξ} a non-negative integer.*

For $i \in \{0, 1, 2, 3\}$, write C_i^{ξ} for the normalization of $\mathcal{M}_{0,i+3}$ in C^{ξ}. For $i \in \{1, 2, 3\}$, write (g_i^{ξ}, r_i^{ξ}) for the type of the hyperbolic curve C_i^{ξ}/C_{i-1}^{ξ} [cf. Remark 2.1 below].

Suppose that $g_i^{\xi} \geq 1$ or $r_i^{\xi} > r^{\eta} + 4$ for any $i \in \{1, 2, 3\}$ and $\{\xi, \eta\} = \{\alpha, \beta\}$. Then the natural map

$$\mathrm{Isom}_k \left(X^{\alpha}_{r^{\alpha}}, X^{\beta}_{r^{\beta}} \right) \to \mathrm{Isom}_{G_k} \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \right), \pi_1 \left(X^{\beta}_{r^{\beta}} \right) \right) / \mathrm{Inn} \left(\pi_1 \left(X^{\beta}_{r^{\beta}} \times_k \bar{k} \right) \right).$$

is bijective.

Remark 2.1. By [H], Proposition 2.3, C^{ξ} is a hyperbolic polycurve and $C^{\xi} = C_3^{\xi} \to C_2^{\xi} \to C_1^{\xi} \to C_0^{\xi}$ is a sequence of parameterizing morphisms in the sense of [H], Definition 2.1 (ii). In particular, $C_i^{\xi} \to C_{i-1}^{\xi}$ is a hyperbolic curve for each $i \in \{1, 2, 3\}$.

Proof of Theorem 2.1. When $r^{\alpha} = 0$ or $r^{\beta} = 0$, $X^{\alpha}_{r^{\alpha}}$ or $X^{\beta}_{r^{\beta}}$ is a hyperbolic polycurve of dimension 3. So, the assertion holds thanks to [H], Theorem B.

Thus, we suppose that $r^\alpha > 0$ and $r^\beta > 0$. The injectivity of this map is proved by [H], Proposition 3.2, (ii). Let $\varphi \in \mathrm{Isom}_{G_k}\left(\pi_1\left(X^\alpha_{r^\alpha}\right), \pi_1\left(X^\beta_{r^\beta}\right)\right)$. Then φ induces a surjetive morphism $\varphi_{r^\alpha} : \pi_1\left(X^\alpha_{r^\alpha}\right) \twoheadrightarrow \pi_1\left(X^\beta_0\right)$. The proof of the surjectivity of the map in the statement of Theorem 2.1 is divided into four parts.

First, we claim that the following assertion holds:

Claim 2.2. $\varphi_{r^\alpha}\left(\pi_1\left(X^\alpha_{r^\alpha} \times_{X^\alpha_{r^\alpha-1}} \bar{x}\right)\right) = \{1\}$ *for any* $\bar{k}$*-valued point* $\bar{x} \to X^\alpha_{r^\alpha-1}$.

Proof of Claim 2.2. We shall begin with the following

Lemma 2.3. *Let K be a sub-p-adic field and $\bar{K}$ an algebraic closure of K. Let S be a connected separated normal scheme of finite type over* $\mathrm{Spec}(K)$*, $X \to S$ a non-trivial hyperbolic curve of type (g^X, r^X), and Y a hyperbolic curve over* $\mathrm{Spec}(K)$ *of type (g^Y, r^Y). Suppose that at least one of the following four conditions is satisfied:*
(i) $g^X \leq g^Y$,
(ii) $r^X = 0$ and $r^Y > 0$ and $2g^X < 2g^Y + r^Y - 1$,
(iii) $r^X > 0$ and $r^Y > 0$ and $2g^X + r^X < 2g^Y + r^Y$,
(iv) $r^X > 0$ and $r^Y = 0$ and $2g^X + r^X - 1 < 2g^Y$.
Then, for any surjective continuous homomorphism $\phi : \pi_1(X) \to \pi_1(Y)$ over G_K*,* $\phi\left(\pi_1(X \times_S \bar{s})\right) = \{1\}$ *for any $\bar{K}$-valued point $\bar{s} \to S$.*

Proof of Lemma 2.3. Under the assumption, by [H], Proposition 2.4, (i), we have the following exact sequence

$$0 \longrightarrow \pi_1(X \times_S \bar{s}) \longrightarrow \pi_1(X) \longrightarrow \pi_1(S) \longrightarrow 0 \quad \text{(exact)}$$

for any $\bar{K}$-valued point $\bar{s} \to S$ of S. Since K is of characteristic 0, $\pi_1(X \times_S \bar{s})$ is topologically finitely generated. Because ϕ is a surjective morphism over G_K, ϕ induces a morphism $\pi_1(X \times_S \bar{s}) \to \pi_1(Y \otimes_K \bar{K})$, which is denoted by the same notation. Thus, $\phi\left(\pi_1(X \times_S \bar{s})\right)$ is topologically finitely generated, closed normal subgroup of $\pi_1(Y \otimes_K \bar{K})$. Moreover, as Y is a hyperbolic curve, $\pi_1(Y \otimes_K \bar{K})$ is elastic [cf. [M2], Proposition 2.3, (i)]. Hence, $\phi\left(\pi_1(X \times_S \bar{s})\right)$ is trivial or open in $\pi_1(Y \otimes_K \bar{K})$.

Because each one of the conditions (ii)$\sim$(iv) implies that the rank of the abelianization of $\pi_1(X \times_S \bar{s})$ is less than the rank of the abelianization of $\pi_1(Y \otimes_K \bar{K})$, $\phi\left(\pi_1(X \times_S \bar{s})\right)$ cannot be open. So $\phi\left(\pi_1(X \times_S \bar{s})\right)$ is trivial for any $\bar{s}$ if one of the conditions (ii)$\sim$(iv) is satisfied. Next, we will show that $\phi\left(\pi_1(X \times_S \bar{s})\right) = \{1\}$ for any $\bar{s}$ if the condition (i) is satisfied. First, if $\phi\left(\pi_1(X \times_S \bar{s})\right)$ is open, then we have a dominant morphism $X \times_S \bar{s} \to Y \otimes_K \bar{K}$ by the Grothendieck conjecture [cf. [M1], Theorem A]. Hence, if $\phi\left(\pi_1(X \times_S \bar{s})\right)$ is open for some $\bar{s}$, then $g_X \geq g_Y$ by the Hurwitz formula. Namely, $\phi\left(\pi_1(X \times_S \bar{s})\right) = \{1\}$ for any $\bar{s}$ if $g^X < g^Y$. Second, suppose that $g^X = g^Y$ and $\phi\left(\pi_1(X \times_S \bar{s})\right)$ is open in $\pi_1(Y \otimes_K \bar{K})$ for some $\bar{s}$. Then the smooth compactification of $X \times_S \bar{s}$ and that of $Y \otimes_K \bar{K}$ are isomorphic over $\bar{K}$ by (the Grothendieck conjecture [cf. [M1], Theorem A] and) the Hurwitz formula. And, that holds for any $\bar{s}$, since the isomorphism class of $\pi_1(X \times_S \bar{s})$ is independent of the choice of $\bar{s}$. This is a contradiction, because $X \to S$ is non-trivial and S is of finite type over K. That is why $\phi\left(\pi_1(X \times_S \bar{s})\right) = \{1\}$ even if $g^X = g^Y$. This completes the proof of Lemma 2.3. □

We note that $\varphi_{r^\alpha} : \pi_1\left(X^\alpha_{r^\alpha}\right) \twoheadrightarrow \pi_1\left(X^\beta_0\right)$ induces $\tilde{\varphi}_i : \pi_1\left(X^\alpha_{r^\alpha}\right) \twoheadrightarrow \pi_1\left(C^\beta_i\right)$ for each $i \in \{1, 2, 3\}$ and $\tilde{\varphi}_3 = \varphi_{r^\alpha}$. By Lemma 2.3, $\tilde{\varphi}_1\left(\pi_1\left(X^\alpha_{r^\alpha} \times_{X^\alpha_{r^\alpha-1}} \bar{x}\right)\right) = \{1\}$ for any $\bar{k}$-valued point $\bar{x} \to X^{\alpha}_{r^{\alpha}-1}$ when $g^{\beta}_1 \neq 1$. (Because the condition (i) is satisfied when $g^{\beta}_1 > 1$ and the condition (iii) is satisfied when $g^{\beta}_1 = 0$ and $r^{\beta}_1 > r^{\alpha} + 4$.)

Next, we will show that $\tilde{\varphi}_1 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right) = \{1\}$ when $g^{\beta}_1 = 1$. Suppose that $\tilde{\varphi}_1 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right)$ is open in $\pi_1 \left(C^{\beta}_1 \times_k \bar{k}\right)$ for some $\bar{k}$-valued point $\bar{x} \to X^{\alpha}_{r^{\alpha}-1}$. Because $\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)$ are mutually isomorphic for any $\bar{k}$-valued point $\bar{x} \to X^{\alpha}_{r^{\alpha}-1}$, $\tilde{\varphi}_1 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right)$ is open in $\pi_1 \left(C^{\beta}_1 \otimes_k \bar{k}\right)$ for any $\bar{x} \to X^{\alpha}_{r^{\alpha}-1}$. So, for any $\bar{x} \to X^{\alpha}_{r^{\alpha}-1}$, we have a non-trivial morphism from $C^{\beta}_1 \otimes_k \bar{k}$ to the Jacobian of the smooth compactification of $X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}$ by the Grothendieck conjecture [cf. [M1], Theorem A]. This implies that the Jacobians of all (proper smooth) curves of genus 2 over $\bar{k}$ are not simple. On the contrary, there exists a (proper smooth) curves of genus 2 over $\bar{k}$ whose Jacobian is simple, because $\dim(\mathcal{A}_1 \times \mathcal{A}_1) = 2 < 3 = \dim(\iota(\mathcal{M}_2))$, where $\mathcal{A}_g$ is the moduli stack of principally polarized abelian varieties over $\bar{k}$ of dimension g and $\iota : \mathcal{M}_2 \to \mathcal{A}_2$ is the Torelli map. This is a contradiction. Hence, $\tilde{\varphi}_1 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right) = \{1\}$ for any $\bar{x} \to X^{\alpha}_{r^{\alpha}-1}$ even if $g^{\beta}_1 = 1$.

Thus, $\tilde{\varphi}_1 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right) = \{1\}$ for any $\bar{x} \to X^{\alpha}_{r^{\alpha}-1}$ when $g^{\beta}_1 \geq 1$ or $r^{\beta}_1 > r^{\alpha} + 4$.

Hence, $\tilde{\varphi}_2 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right) \subset \mathrm{Ker}\left(\pi_1(C^{\beta}_2) \to \pi_1(C^{\beta}_1)\right)$. So, by the same argument as above replacing $\tilde{\varphi}_1$ by $\tilde{\varphi}_2$ and $\pi_1 \left(C^{\beta}_1 \times_k \bar{k}\right)$ by $\mathrm{Ker}\left(\pi_1(C^{\beta}_2) \to \pi_1(C^{\beta}_1)\right)$, we can prove $\tilde{\varphi}_2 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right) = \{1\}$ when $g^{\beta}_2 \geq 1$ or $r^{\beta}_2 > r^{\alpha} + 4$.

Hence, $\tilde{\varphi}_3 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right) \subset \mathrm{Ker}\left(\pi_1(C^{\beta}_3) \to \pi_1(C^{\beta}_2)\right)$. So, by the same argument as above replacing $\tilde{\varphi}_1$ by $\tilde{\varphi}_3$ and $\pi_1 \left(C^{\beta}_1 \times_k \bar{k}\right)$ by $\mathrm{Ker}\left(\pi_1(C^{\beta}_3) \to \pi_1(C^{\beta}_1)\right)$, we can prove $\tilde{\varphi}_3 \left(\pi_1 \left(X^{\alpha}_{r^{\alpha}} \times_{X^{\alpha}_{r^{\alpha}-1}} \bar{x}\right)\right) = \{1\}$ when $g^{\beta}_3 \geq 1$ or $r^{\beta}_3 > r^{\alpha} + 4$. This completes the proof of Claim 2.2. □

Second, we claim that the following assertion holds:

Claim 2.4. *φ induces* $\varphi_0 : \pi_1\left(X^{\alpha}_0\right) \xrightarrow{\sim} \pi_1\left(X^{\beta}_0\right)$.

Proof of Claim 2.4. By Claim 2.2, $\varphi_{r^{\alpha}}$ determines $\varphi_{r^{\alpha}-1} : \pi_1\left(X^{\alpha}_{r^{\alpha}-1}\right) \to \pi_1\left(X^{\beta}_0\right)$. If $r^{\alpha} - 1 = 0$, then we have $\varphi_0 : \pi_1\left(X^{\alpha}_0\right) \twoheadrightarrow \pi_1\left(X^{\beta}_0\right)$. If $r^{\alpha} - 1 > 0$, then we can show that $\varphi_{r^{\alpha}-1}$ determines $\varphi_{r^{\alpha}-2} : \pi_1\left(X^{\alpha}_{r^{\alpha}-2}\right) \to \pi_1\left(X^{\beta}_0\right)$ by applying the same argument to $\varphi_{r^{\alpha}-1} : \pi_1\left(X^{\alpha}_{r^{\alpha}-1}\right) \to \pi_1\left(X^{\beta}_0\right)$. By repeating this procedure, we can prove that φ determines $\varphi_0 : \pi_1\left(X^{\alpha}_0\right) \twoheadrightarrow \pi_1\left(X^{\beta}_0\right)$. By applying the same argument $\varphi^{-1} : \pi_1\left(X^{\beta}_{r^{\beta}}\right) \twoheadrightarrow \pi_1\left(X^{\alpha}_{r^{\alpha}}\right)$, we can prove that φ^{-1} determines $\varphi^{-1}_0 : \pi_1\left(X^{\beta}_0\right) \twoheadrightarrow \pi_1\left(X^{\alpha}_0\right)$. Thus, we can prove that φ determines $\varphi_0 : \pi_1\left(X^{\alpha}_0\right) \xrightarrow{\sim} \pi_1\left(X^{\beta}_0\right)$ over G_k. This completes the proof of Claim 2.4. □

Third, we claim that the following assertion holds:

Claim 2.5. *φ_0 induces a k-isomorphism $f_0 : X_0^\alpha \xrightarrow{\sim} X_0^\beta$.*

Proof of Claim 2.5. Since both X_0^α and X_0^β are hyperbolic polycurves of dimension 3, we obtain the conclusion by [H], Theorem B. This completes the proof of Claim 2.5. $\square$

Finally, we claim that the following assertion holds:

Claim 2.6. *f_0 induces a k-isomorphism $f : X_{r_\alpha}^\alpha \xrightarrow{\sim} X_{r_\beta}^\beta$.*

Proof of Claim 2.6. For each $\xi \in \{\alpha, \beta\}$, let $\eta_0^\xi \to X_0^\xi$ be the generic point. Then, by Claim 2.5, we have an isomorphism $\eta_0^\alpha \xrightarrow{\sim} \eta_0^\beta$. Thus, we have an isomorphism $\pi_1(X_{r_\alpha}^\alpha \times_{X_0^\alpha} \eta_0^\alpha) \xrightarrow{\sim} \pi_1(X_{r_\beta}^\beta \times_{X_0^\beta} \eta_0^\beta)$ that is compatible with $\pi_1(\eta_0^\alpha) \xrightarrow{\sim} \pi_1(\eta_0^\beta)$, by [H], proposition 2.4 (ii), because we have an isomorphism $\varphi_0 : \pi_1(X_0^\alpha) \xrightarrow{\sim} \pi_1(X_0^\beta)$, $\pi_1(\eta_0^\alpha) \xrightarrow{\sim} \pi_1(\eta_0^\beta)$ and $\varphi : \pi_1(X_{r^\alpha}^\alpha) \xrightarrow{\sim} \pi_1(X_{r^\beta}^\beta)$.

By [MT], Corollary 6.3, this isomorphism preserves all fiber subgroups. So this isomorphism preserves the length of each fiber. Hence, $r_\alpha = r_\beta$. Moreover, as k is a sub-p-adic field, k is ℓ-cyclotomically full for any prime ℓ. Thus, by [HM], Corollary 2.6, we have an isomorphism $X_{r_\alpha}^\alpha \times_{X_0^\alpha} \eta_0^\alpha \xrightarrow{\sim} X_{r_\beta}^\beta \times_{X_0^\beta} \eta_0^\beta$ that is compatible with $\eta_0^\alpha \simeq \eta_0^\beta$. This together with [H], Lemma 2.10 brings us to the conclusion of Claim 2.6. $\square$

This completes the proof of Theorem 2.1. $\square$

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References

[G] A. Grothendieck, *Letter to G. Faltings (June 1983), Geometric Galois Actions, 1. London Math. Soc. Lect. Note Ser.* **242** (1997), pp. 285-293.

[H] Y. Hoshi, *The Grothendieck conjecture for hyperbolic polycurves of lower dimension, J. Math. Sci. Univ. Tokyo* **21** (2014), **no. 2**, pp.153-219.

[HM] Y. Hoshi and S. Mochizuki, *Topics surrounding the combinatorial anabelian geometry of hyperbolic curves II: Tripods and combinatorial cuspidalization, RIMS preprint* **1762** (2012, November).

[IN] Y. Ihara and H. Nakamura, *Some illustrative examples for anabelian geometry in high dimensions, Geometric Galois Actions, 1. London Math. Soc. Lect. Note Ser.* **242** (1997), pp.127-138.

[K] R. Kinoshita, *The Grothendieck Conjecture for the Moduli Spaces of Hyperbolic Curves of Genus One, preprint* (2015).

[M1] S. Mochizuki, *The local pro-p Grothendieck conjecture for hyperbolic curves, Invent. Math.* **138** (1999), pp.319-423.

[M2] S. Mochizuki, *Topics in absolute anabelian geometry I: Generalities, J.Math. Sci. Univ. Tokyo* **19** (2012), **no. 2**, pp.139-242.

[MT] S. Mochizuki and A. Tamagawa, *The algebraic and anabelian geometry of configuration spaces, Hokkaido Math. J.* **37** (2008), **no. 1**, pp.75-131.

[N] H. Nakamura, *Galois rigidity of pure sphere braid groups and profinite calculus, J. Math. Sci. Univ. Tokyo* **1** (1994), pp.71-136.

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