IUT III

11/Dec/2015 at Oxford

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In short, in IUT II, we performed "Galois evaluation"

\[ \text{theta} \xrightarrow{\text{fct}} \text{theta values} \]

"env" labels \[ \xrightarrow{\text{MF-objects}} \] "gau" labels

(filtered \( \eta \)-modules)
Two Problems

1. Unlike "theta fits", "theta values" DO NOT admit a multiradial alg'm in a NAIVE way.
2. We need ADDITIVE nth for (log-) height at
\( n \log \)

On 1.
theta fits $\rightarrow$ theta values

(admit multival. alg'ns)

(c.f. [IUT II, Prop 3.4 (i)])

requires cyc. rig via

LCFT

(c.f. [IUT II, Prop 3.4 (ii), Cor 3.7 (ii), Rem 4.10.2 (ii)])
Recall cyc. mg. via LCFT

\[ \Theta^\Delta = (\text{unit portion}) \times (\text{value gap}) \]

We DO NOT share it in both sides of \( \Theta \)-link!

theta values

DO NOT admit a multiradial alg. in a NAIVE way.
cf.
cycl. nig. via mono-theta env.
cycl. nig. via $\mathbb{Z} \times \mathbb{D}_2 = 414$

use only "m-portion"

form
form

core

form
form

$\uparrow$ do not interfere

admit a multidad, algo
To overcome these problems,

use log link!

(& allowing mild vendor’s)

non-interference

etc.

(later)
Note $\mathbb{F}_2$-symm. isom's are compatible with log-links, can pull-back via log-links.
We consider the infinite chain of log-links:

\[ \text{this is invariant by one shift!} \]
Important Fact

$k/\mathcal{D}_p \rightarrow \text{fin.} \quad \log \text{shell}

\[ \log_0^* \leq \frac{1}{2p} \log \hat{0}_k^* = \mathcal{I}_k \]

\[ \log \leq \mathcal{C} \text{ the domain & codomain} \]

\[ \log \hat{0}_k^* \]

\[ \text{upper semi-compatibility} \]

(Note also: log-shells are rigid)
Besides theta values, we need another thing:

we need NF (:= number field)

To convert □-line bolles into □-line bolles and vice versa.
\[ \text{line bundles} \]
- def'd in terms of torsors

\[ \text{line bundles} \]
- def'd in terms of fractional ideals

natural (at. equiv.) in a scheme theory
- line bundles
  - def'd only in terms of \( \mathbb{Z} \)-rings
  - admits precise log-Kummer corrs. But, difficult to compute log-volumes

- line bundles
  - def'd by both of \( \mathbb{Z} \)-rings
  - only admits upper semi-continuous log-Kummer corrs.
  - But, suited to explicit estimates.
We also include NFs as data

\[(\alpha, \text{NF}) \in \mathbb{T} \log (\Theta^+)]

theta values / NFs

story goes in a parallel way in some sense

(of course, essential difference)

17. [IUT III, Rem 2.3.2, 2.3.3]
To obtain the final multivar. algo:

Frob-like data assoc. to $F$-prime-strips

\[ \downarrow \quad \text{Kummer theory} \]

\[ \text{étale-like data assoc. to } \mathcal{O}_p \text{-prime-strips with } \mathbb{Z}_p \]

\[ \downarrow \quad \text{forget with } \mathbb{Z}_p \text{-data} \]

\[ \text{mon- } \mathbb{Z}_p \text{-data assoc. to } \mathcal{O}_p^{+} \text{-prime-strips} \]
Frobenius picture

Kummer theory

(cycl. sig.'s)

permutable

rad. ideal

rad. ideal
3 portions of $\mathbb{G}$-link

\[ + \mathbb{G}_n \rightarrow \mathbb{G}_n \]

\[ + \mathbb{G}_n \rightarrow \mathbb{G}_n \]

\[ + \mathbb{G}_n \rightarrow \mathbb{G}_n \]

\[ \circ \text{unit} \]

\[ \circ \text{value } \mathbb{G} \]

\[ \text{local} \]

\[ \text{global realified } (\mathbb{R}_\mathbb{Z})^n \sim (\cdots, \cdots) \sim [\mathbb{R}_\mathbb{Z}]_n \log q \]

\[ \text{d-charge} \]

\[ \text{strictly charge } \frac{3}{2 \pm \text{fact}} \]
Kummer theory

unit portion

\[ G_{\nu} \otimes C_{\nu} \to \mathbb{G}_m^\nu \]

\[ \mathbb{Q}^\times /\mu \] \( \mathbb{Q}_p \)-module

+ integral at \( \nu \), i.e., \( \text{Im}(\mathbb{Q}_p^{\times \nu}) \subseteq (\mathbb{Q}_p^{\times \nu})^H \)

\[ \mathbb{Q}^{\times \nu} \]

wall

\( \mathbb{Q}^{\times \nu} \) is monogenic (computable) log-mod.
\( (\gamma_{n} \cdot \alpha_{\gamma} \cdot \beta_{\gamma}) \xrightarrow{\text{Hummer}} (\gamma_{n} \cdot \alpha_{\gamma} \cdot \beta_{\gamma}(\gamma_{n})) \)

Unlike the case of \( \Theta_{\frac{D_{n}}{2}} \),

\( \Omega_{\frac{D_{n}}{2}} \)-indef. occurs

\( \text{log mol. is invariant under this } \Omega_{\frac{D_{n}}{2}} \)-indef. \)

\( \text{OK} \)

\( \gamma_{n} \), \( \gamma_{n} \), \( M(\gamma_{n}) \xrightarrow{\text{via LEFT}} M(\Theta_{\frac{D_{n}}{2}}) \)

\( \text{does not hold.} \)
We want to protect

\{ value sp portion
  \} global real'd portion

from this \(2^6\)-inlet!

\[
\text{shaving } f^6 \to f^1 \text{ w/ int. etc.}
\]  
\(\text{\arrowvert (Ind 2)}\)

\(\text{\arrowvert \ \text{horizontal inlet}}\)
value gp portion

mono-theta cycl. rig

only \( M \) is involved

\( \hat{2} \times \Omega_{30} = 415 \) cycl. rig.

MF

( unlike ECFT cycl. rig.)

\( \Phi \)

( do not obstruct each other

( multiward. on the function level )

\( n \)
Note also

mono-theta cyc. vis.


\[ \text{Fe} \xrightarrow{\text{sym.}} (\text{conj.}, \text{synchro.}) \]

is compact w/ log-links

\[ \text{can pull-back coric (diagonal) obj} \]

\[ \text{via log-links} \]

\[ \text{GAP monoid [logarithmic Gaussian Processes]} \]
\[ \log \mu = 0 \]

- No new action appears by the iterations of \( \log \mu \)'

No interference
Note also

\[ M^{\log_\mu} (\log_\mu (A)) = M^{\log_\mu} (A) \]

\[ M \stackrel{\sim}{\to} \log_\mu (A) \]

(compatibility of log-volumes w/ log-links)

As not need to care about how many times log's are applied.
In the Archimedean case, we now a system (cf. [EUC], Def. I.10). Let $h$ be killed in $O(h)$. We construct $h$ from $O(h)$, not of $O(h)$, acting as $O(h)$. Let us put "weight $W" in the $h$-volume (cf. [EUC], Prop. 1, 2, 11).
as well, consider the actions of \( (F_{mod})_n \)
after \( (Kammer) \circ (log)^n \) \( (n \geq 0) \)

By \( F_{mod} \cap \Omega_n = \emptyset \)

No new action appears in the iteration of \( log^n \)

No interference
<table>
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<tr>
<th>( \theta )</th>
<th>( \text{cyl. &amp; nig.} )</th>
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<td>mono-theta</td>
<td>( \text{cyl. &amp; nig.} )</td>
<td>( \text{no interference by const. multi. nig.} )</td>
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<tr>
<td>( \text{MF} )</td>
<td>( \text{cyl. &amp; nig.} )</td>
<td>( \text{no interference by } E_{\text{modulation}} )</td>
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</table>
local & transcendental\( \mathbb{Q} \subset \mathbb{Q}(\theta) \quad q = p^2 \quad \text{comp. w/ prof. torp.} \)

\[ \text{theta fit} \]

\[ \text{zero of order} = 1 \]

\[ \text{at each cycle} \]

\[ \text{"only one valuation"} \]

\[ \text{cycl. rig.} \]

\[ \text{Note} \]

\[ \text{theta fts/ theta values} \]

\[ \text{does not have } \mathbb{Q}_x \text{-sym.} \]

\[ \text{But, the cycl. rig. } \]

\[ \text{7 does} \]

\[ \text{not} \]

\[ \text{for alg. rad., fts} \]

\[ \text{Never} \]

\[ \text{in comp. w/ prof. Ap.} \]

\[ \mathbb{Q} \cap \mathbb{Q} = \mathbb{Q} \]

\[ \text{"many valuations" - global} \]

\[ \mathbb{Q}_x \cap \mathbb{Q}_x = \mathbb{Q}_x \]
Note also that eval or use labels

theta eval & Kummer

correct w/ labels

NF

the output Find does not depend on labels

global real & monoids are non-analytic nature (units are killed)
do not depend on 
hat. etc.
Stable Transport full poly
$fG \rightarrow fG$

(Ind 1)

In permutations

we can transport the data
over the G-mall
Another thing

\[ \exists \text{gan } C \left[ \left( \text{const. monoids} \right) \implies \left( \text{labels come from arithmetic, hol. etc.} \right) \right] \]
cannot transport the labels for $\omega$-link

$F^* \subseteq 1$

$L^* \left( : \frac{p-1}{2} \right) \rightarrow ?$
\[
\frac{l^+}{\tau} = \frac{\omega}{1 + 1} = \frac{\omega}{2}
\]
use processes

\{ 0, 1, 2, \ldots \} \times \{ 0, 1, 2, \ldots \}

\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow

\{ ?, ?, ?, \ldots \} \times \{ ?, ?, ?, \ldots \} \times \{ ?, ?, \ldots \}

\begin{itemize}
  \item then, in total \((2^\pm)^n\)-possibilities
  \item gives more strict inequality than the former case
\end{itemize}
By this multivariate reg's the compatibility w/ each

\[
\mathbf{\beta} = \mathbf{X}'\mathbf{X}^{-1}\mathbf{X}'\mathbf{Y}
\]

\[
\mathbf{m}_0 \leq \mathbf{m}_1 \leq \mathbf{m}_2
\]

We cannot distinguish them.
(Ind 1) permutative rindet. \[ \begin{array}{c} \Leftrightarrow \\ \text{in the \textit{etale transport}} \end{array} \]

(Ind 1\1/2) horizontal rindet. \[ \Rightarrow \]

\[ t \in H \in \text{the Kummer detach.} \]

(Ind 1\2/3) vertical rindet. \[ \Rightarrow \log \]

\[ \log(Q^x) C \Rightarrow \begin{array}{c} 7 \log(10^y) C \Rightarrow \\ 2 \log(10^y) \end{array} \]

can be considered as a kind of "decent data from \[ \mathbb{Z} \] to \[ \text{IF} \]"
possible image $\mathcal{A}$, $\mathcal{A}^*$ contains $a$.

Container $X$.

Log-shell mono-analytic.
Recall \( q^2 \; q^2 \rightarrow q \)

\[ 0 \preceq - (ht) + (\text{midet}) \]

\[ \preceq (ht) \preceq 13 \text{ log-diff } \]

\[ \preceq \text{ log-diff } + \text{ log-cmd} \]

Calculation in Hodge-Arakelov

\[ \sum i^2 \left[ i, i \right] \approx \frac{e^2}{24} \left[ \cdot, \cdot \right] \]

 Miracle equality \( \sum i \left[ i, i \right] \approx \frac{e^2}{20} \left[ \cdot, \cdot \right] \)
cf. Hodge-Arakelov

IF a global multi-subspace existed

\[ q \circ \supset q^0 \circ \]

\[ \Rightarrow \quad \text{deg} = 0 \quad \text{deg} < 0 \]

\[ \Rightarrow \quad - (\text{large}) \geq 0 \]
<table>
<thead>
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<th>IUT III, Th3, 11)</th>
<th>In summary,</th>
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<td>(i) (objects)</td>
<td>(ii) (log-Kummer)</td>
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<td>ν = unit</td>
<td>invariant after</td>
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<td>only μ is involved</td>
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<td>ell. cup. spin</td>
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<tr>
<td></td>
<td>+ hidden endom.</td>
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<td>quadratic str.</td>
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<td>Heisenberg sp</td>
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| F* × F^x: mod    | no interference |
|                  | by F_{mod} ∩ T^0 \nu = μ |
|                  | protected from |
|                  | Z^x × Q > 0 = 1/3 |

- [coord. proc. after admitting (Ind1)]
- (coord. proc. after admitting (Ind1))
Some questions
How about the following variants of $\Phi$-link?

\begin{align*}
  \text{i) } \left\{ q^2 \right\}_0 & \rightarrow q^N \\
  \text{ii) } \left\{ \left(q^2 \right)^N \right\}_1 & \rightarrow q^N \quad (N \geq 1)
\end{align*}
\[ \frac{1}{2} q^{2} g \frac{1}{2} q^{-1} = q^{N} \]

A works

\[ \deg = 0 \quad \left( \frac{l \approx \delta + \ell}{\ell + \delta \ll \ell} \right) \]

\[ N \cdot 0 \leq -(\text{ht}) + (\text{indet}) \]

(As for \( N \ll \ell \))

(When \( N \gg \ell \) the inequality is weak)
ii) \( \left\{ 9 \left( 0^2 \right)^2 \right\}_5 - 9 \)

It DOES NOT work!
Because

\[ \mathbb{D}^n \sim (\mathbb{D}^n)^{\text{mon-\thetaeta}} \text{ cycl. Mig.} \]

\[ \text{mono-\thetaeta cycl. Mig. comes} \]

from the quadracity

\[ \left[ , \right] \]

\[ \text{cf. [E+76, Rem 2.19.2]} \]

\[ C^n (N > 1) \sim \text{Kummer compact.} \]
2) mono-theta
constant/multiple rig.

as well

extn thm. 1

\[ 0 \to \mathcal{O}^\times(-1) \to \text{Aut}_e((-1)) \to \text{Aut}_{D0}((-1)^{bs}) \to 1 \]

cf. [EtTh, Rem 5.12.5]
3. Vicious cycles

\[ 2^N \text{ zero of order } N > 1 \]

Various

Kummer theory

Fat-bid M \subseteq \text{ stable-like } M \text{ at cusps}

\[ \text{cf. [IUT III, Rem. 2.3.3 [vii]]} \]

C loops - one loop gives one N-power
IF IT WORKED

\[ 0 \leq -N(\lambda t) + \left( \text{in det} \right) \]

\[ \left( \lambda t \right) \leq \frac{1}{N} \left( 1 + \varepsilon \right) \left( \text{log-diff} + \text{log-cond} \right) \]

contradiction

To a lower bound
given by analytic number theory

(Masser, Stewart-Tijdeman)