

山下剛

$$B\mu_l \longrightarrow \mathcal{O}(-l)_{\mathbb{P}^\infty} \longrightarrow \text{Th}_{\mathbb{P}^\infty}(\mathcal{O}(-l))$$

$$\begin{aligned} \sim \rightarrow \quad & \mathbb{Z}/l\text{-coc!} \\ 0 \rightarrow \tilde{H}^{*,*}(k)[\sigma] & \xrightarrow{\quad} H^{*,*}(B\mu_l) \rightarrow (\tilde{H}(k)[\sigma])^{*-1, *-1} \rightarrow 0 \\ & \downarrow \quad \quad \quad \downarrow \\ & 0 \longmapsto v \quad \quad \quad \sigma \in [\mathcal{O}(-1)] \in H^{2,1}(\mathbb{P}^\infty) \end{aligned}$$

$v := c(\text{taut. line bundle on } B\mu_l) \in H^{2d}$

Lemma
$$H^{1,1}(B\mu_l, \mathbb{Z}/l) \xrightarrow{\mathcal{J}} H^{2,1}(B\mu_l, \mathbb{Z})$$

$$\begin{matrix} \downarrow & & \downarrow \\ \exists! u & \longmapsto & v \end{matrix} \quad u|_x = 0$$

Prop $v^i, uv^i (i \geq 0)$ forms a basis of $\tilde{H}^{*,*}(F, \wedge(B\mathbb{S}^1)_+, \mathbb{Z}/l)$ over $\tilde{H}^{*,*}(F, \mathbb{Z}/l)$ □

$u^2 = ? \quad l > 2 \Rightarrow \text{graded commutativity} \quad u^2 = 0$

$l=2$
$$H^{2,2}(B\mu_l, \mathbb{Z}/2) = H^{0,1}(k, \mathbb{Z}/2) \downarrow v \oplus H^{1,1}(k, \mathbb{Z}/2) \downarrow u \oplus H^{2,2}(k, \mathbb{Z}/2)$$

$$u^2 = \begin{matrix} \uparrow (2,1) \\ x v \end{matrix} + \begin{matrix} \uparrow (1,1) \\ y u \end{matrix} + 0 \quad \leftarrow u|_x = c$$

Fact $x = \tau \in H^{0,1} \simeq \mu_2 \quad y = \rho \in H^{1,1} \simeq k^* / (k^*)^2$

Prop
$$\tilde{H}^{*,*}(F, \wedge(B\mu_l)_+, \mathbb{Z}/l) \simeq \begin{cases} \tilde{H}^{*,*}(F, \mathbb{Z}/l)[u, v] / (v^2) & l > 2 \\ \tilde{H}^{*,*}(F, \mathbb{Z}/l)[u, v] / (u^2 - \tau v - \rho u) & \end{cases}$$

□

→ $B\mathbb{G}_l$ ξ_l : taut. vect. bundle on $B\mathbb{G}_l$
 $(\xi_l \xrightarrow{\text{incl}} \mathbb{G}_l)$
 $d := c(\xi_l / \mathcal{O}) \in H^{2(l-1), l-1}(B\mathbb{G}_l, \mathbb{Z}/l)$

Thm $\exists!$ $c \in H^{2l-3, l-1}(B\mathbb{G}_l)$ $\delta(c) = d$, $c|_* = 0$ \square

graded commutative

$l > 2 \Rightarrow c^2 = 0$

$l = 2 \Rightarrow BU_2 = B\mathbb{G}_2$

$$\left(\begin{array}{l} k \geq 3 \\ BU_k \xrightarrow{P_\xi^*} B\mathbb{G}_k \\ \downarrow \xi \\ \cdot \end{array} \right) \quad \begin{array}{l} P_\xi^*(d) = \frac{l-1}{i-1} (i \cup v) = -v^{l-1} \\ P_\xi^*(c) = -u v^{l-2} \end{array}$$

$l > 2$

$\tilde{H}(F.)[[c, d]]/(c^2) \longrightarrow \tilde{H}^{*,*}(F. \wedge (B\mathbb{G}_l)_+, \mathbb{Z}/l)$

$\xleftarrow{P_\xi^*} \tilde{H}^{*,*}(F. \wedge (BU_l)_+, \mathbb{Z}/l)^{\text{Aut}(u, \xi)}$

$\tilde{H}^{*,*}(F.)[[x, y]]/(x^2)$
 $x = u v^{l-2}, y = v^{l-1}$

Thm $\tilde{H}^{*,*}(F. \wedge (B\mathbb{G}_l)_+, \mathbb{Z}/l)$

$$\begin{cases} \tilde{H}^{*,*}(F., \mathbb{Z}/l)[[c, d]]/(c^2) & l > 2 \\ \tilde{H}^{*,*}(F., \mathbb{Z}/l)[[c, d]]/(c^2 - \tau d - pc) & l = 2 \end{cases}$$

$\deg c = (2l-3, l-1)$
 $\deg d = (2l-2, l-1)$

\square

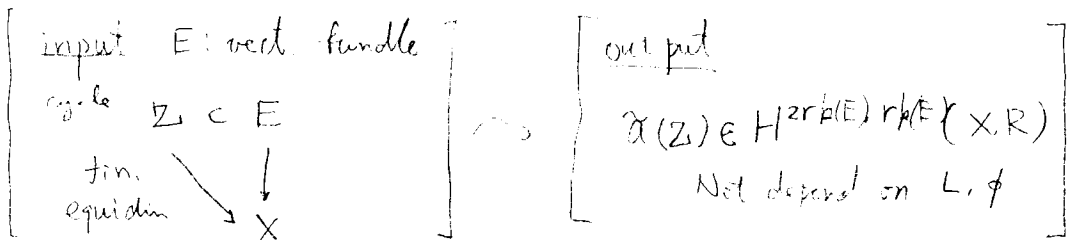
total power operation

$$K_{N,R} = k(\mathbb{C}^n) \otimes R, 2n = R \otimes \mathbb{Z}_{tr}((\mathbb{P}^1, \omega)^{\wedge n})$$

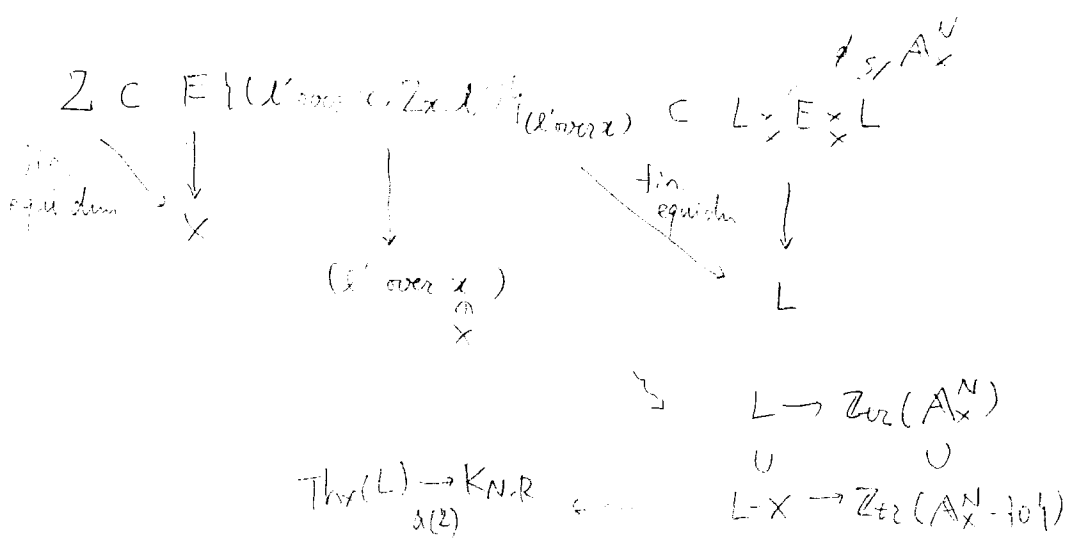
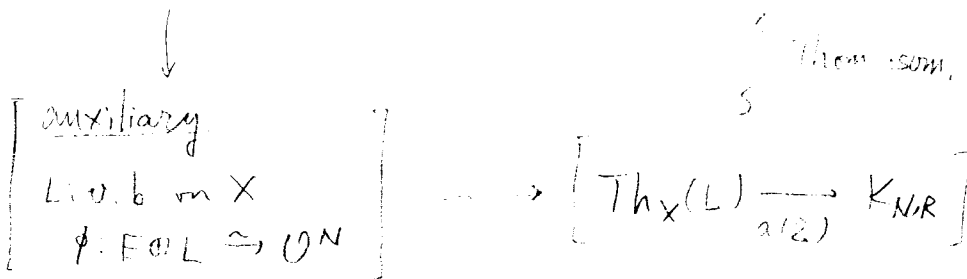
$$= R \otimes \mathbb{Z}_{tr}(A^n / (A^2 - 1))$$

construction D

$X \in \text{Sm}/k$, R : commutative ring



Thom isom.
S



input. Gltin. grp.
 $r: G \rightarrow \mathbb{S}^n$ hom
 $U \in \text{Sm}/k$
 \downarrow
 \cong freely
 \downarrow
 $(\mathbb{S}^n \text{ vector bundle on } U/G)$

output
 $P: K_{iR} \wedge (U/G)_+$
 $\rightarrow K_{iR}$
 Not depend on ℓ

$\left\{ \begin{array}{l} G = \mathbb{S}^1, n=1, r = \text{id} \\ U = A^{n\ell} \otimes R = 4\ell \quad U/G \sim B\mathbb{S}^1 \end{array} \right.$

auxiliary.
 Lined l. on U/G
 $\phi: \mathbb{S}^n \oplus L \xrightarrow{\cong} \mathcal{O}_N$

$\tilde{P}: K_{iR} \wedge \text{Th}_{U/G}(L^i)$
 $\rightarrow K_{iR}$

auxiliary
 $Z \subset X = A^i$
 \downarrow
 X
 $\rightarrow X_{+n} \text{Th}(L^i)$
 $\xrightarrow{a(Z)} K_{iR}$

$Z \subset X = A^i$
 \downarrow
 X
 fin equidim

$p^*(Z^{x^n}) \subset (X \times A^i)^n \times U$
 \downarrow
 $Z' \subset (X \times A^i)^n \times U / G$

$X \times \sum_{\mathbb{S}^1}^i$
 \downarrow
 $(X^n \times U) / G$
 fin equidim

$Z'' \subset X \times (A^i \times U) / G$
 \downarrow
 $X \times U / G$
 fin equidim

Δ^*
 $(\Delta: X \rightarrow X^n)$

$a(Z''): X_{+n} \text{Th}_{U/G}(L^i)$
 $\rightarrow K_{iR}$
 construction?

$$G = \mathbb{Z}/l \quad n=l \quad r=id \quad U = A^{\lambda^m} - \Delta, \quad R = \mathbb{Z}/l$$

$(m \rightarrow \infty)$

$$P: \tilde{H}^{2d,d}(F., \mathbb{Z}/l) \rightarrow \tilde{H}^{2dl,dl}(F. \wedge \mathbb{B}\mathbb{S}^1)_+, \mathbb{Z}/l)$$

total power operation

$$\rightsquigarrow P^i, B^i$$

property of $P \rightsquigarrow$ property of P^i, B^i

$$P(a \wedge b) \rightsquigarrow \text{Cartan formula}$$

$$= \Delta^*(P(a) \wedge P(b))$$

$$\text{symm. then} \rightsquigarrow \text{Adem relation}$$

$$\beta P_i = 0 \rightsquigarrow \beta B^i = 0, \beta P^i = B^i$$

$$m < n \Rightarrow \hat{H}^{*m}(K_{n,p}, B) = 0 \rightsquigarrow p^i = 0 \quad B^i = 0 \quad i < 0$$

$$\mathcal{A}^{*,*} := \langle \beta, P^i (i \geq 0), \alpha \in H^{*,*}(k) \rangle_{\text{left } H^{*,*}(k)\text{-alg}}$$

\subset (biscable coh. operation)

$$\cong \text{Hom}_{\text{sym}}(H_{\mathbb{Z}/l}, \Sigma_s^* \Sigma_t^* H_{\mathbb{Z}/l})$$

$$\mathcal{A}^{*,*} = \text{Hom}_{\text{left } H^{*,*}(\text{al-mod})}(\mathcal{A}^{*,*}, H^{*,*}(k))$$

$$H^{*,*} \text{ cup prod} \rightsquigarrow \mathcal{A}^{*,*} : \text{asprod.} \rightsquigarrow \mathcal{A}^{*,*} \cdot \text{prod.}$$