京都大学大学院理学研究科数学・数理解析専攻数理解析系
数理解析特別講義１　計算機科学（集中講義）

Categorical Algebra and Coalgebra

An Instance of Category Theory in Computer Science

講　師:蓮尾　一郎 氏　（東京大学大学院情報理工学系研究科 講師）
日　時:2012年7月19日(木)・20日(金)・31日(火)・8月1日(水)　14時～
場　所:総合研究2号館4階 478号室

Abstract

Category theory is an abstract mathematical language that is used in many different branches of mathematics. It has also found successful applications in computer science - in fact, in many different ways. The classic example is in the semantics of functional programming, where types are objects and programs are arrows. In this course we focus on another eminent use of categories in computer science, namely *categorical algebra and coalgebra*.

The bottom-line here is: a coalgebra is a categorical abstraction of *dynamics*, i.e. a state-based system like an automaton; and an algebra (especially an initial one) is an abstraction of *syntax*, i.e. the set of well-formed programs. Plotkin's *structural operational semantics* − connecting syntax and dynamics − also allows an elegant categorical modeling via a distributive law.

After exhibiting these basics of the (co)algebraic modeling in computer science, we proceed to a more advanced categorical structure of *presheaf categories*. We introduce the necessary categorical machineries - (co)end, Kan extension, Yoneda lemma, etc. - as well as demonstrate their applications in name-passing calculi like the pi-calculus.

No preliminary knowledge in category theory is assumed. The course materials will be announced at the course website.

Course Structure

- Categorical Algebra and Coalgebra
  - Introduction to category theory I: the category of sets and functions
  - System as coalgebra; final coalgebras
  - Syntax via algebra; initial algebras
- Categorical Structural Operational Semantics
  - Distributive law; monads, comonads
- (Co)Algebra in a Presheaf Category
  - Introduction to category theory II: working with presheaves
  - Name-passing calculi

Keywords

Theoretical computer science; category theory; algebra; coalgebra; automaton; semantics of programming language; presheaf

Course Website

http://www-mmm.is.s.u-tokyo.ac.jp/~ichiro/　(The lecturer's website) ➔ "Teaching"