Comments on
“The internal operads of combinatory algebras”

Masahito Hasegawa

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After submitting the final version of our paper “The internal operads of combinatory algebras” [1] for the post-conference proceedings of MFPS2022, we noticed several points in that paper should be corrected. Below we shall list some of most important ones:

1. While the internal operad construction was shown to give a left adjoint to the forgetful functor from closed operads to extensional combinatory algebras (Theorem 4.7, 5.4, 5.8 and 5.12), it turned out that it is not just an adjunction but actually an *equivalence*. In the paper, only the cartesian case (the comment after Theorem 5.12) was stated as an equivalence (as shown by Hyland [2]), but the same applies to all the other cases. Thus for each extensional combinatory algebra there exists (up to equivalence) a unique closed operad, which can be given as the internal operad.

2. The counterexample for the uniqueness of closed operads for an extensional combinatory algebras described in Section 4.2 was wrong (otherwise we cannot have the equivalence mentioned above): the operad $\mathcal{P}_{\mathcal{A}}$ of planar polynomials given there is *not* closed.

3. Consequently, the overall story of the paper (as given in the introduction) should be greatly simplified: while the present paper says that the internal operad is the canonical one among *many* possible choices of polynomials on an extensional combinatory algebra, now we can say that the internal operad gives a description of the *unique* meaningful notion of polynomials.

In the forthcoming expanded version, we will correct all these issues.

References

[1] Hasegawa, M., *The internal operads of combinatory algebras*, in Proc. MFPS2022. DOI: `https://doi.org/10.48550/arXiv.2211.11118`

[2] Hyland, J.M.E., *Classical lambda calculus in modern dress*, Mathematical Structures in Computer Science **27** (2017), 762–781, DOI: `https://doi.org/10.1017/S0960129515000377`