Intelligence of Low-dimensional Topology

May 24–26, 2017
Room 420,RIMS, Kyoto University
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Program

May 24 (Wed)
13:20–14:10 Kouki Sato (School of Science, Tokyo Institute of Technology)
A full-twist inequality for the ν^+ invariant

14:30–15:20 Delphine Moussard (RIMS, Kyoto university, JSPS)
Splitting formulas for the rational lift of the Kontsevich integral

15:40–16:30 Scott Carter (University of South Alabama)
Foams, Polytopes, Abstract Tensors, and Homology

May 25 (Thu)
10:00–10:50 Masaki Taniguchi (Graduate School of Mathematical Sciences, University of Tokyo)
Instanton moduli spaces on 4-manifolds with periodic ends and an obstruction of existence of embeddings

11:10–12:00 Naoko Kamada (Nagoya City University)
The double covering method for twisted knots

13:20–14:10 Qingtao Chen (ETH, Zurich)
Recent progress of various Volume Conjectures for links as well as 3-manifolds

14:30–15:20 Anh Tran (Tohoku University/JSPS & The University of Texas at Dallas)
Some conjectures about the colored Jones polynomial

15:40– Problem Session

May 26 (Fri)

10:00–10:50 Shin Hayashi (AIST-TohokuU Mathematics for Advanced Materials-OIL)
On some topological invariants related to localized wave functions

11:10–12:00 Kengo Kishimoto (Osaka Institute of Technology)
Simple-ribbon fusions and Alexander polynomials

13:20–14:10 Jieon Kim (Osaka city University, JSPS)
Presentations of (immersed) surface-knots by marked graph diagrams

14:30–15:20 Hiroshi Goda (Tokyo University of Agriculture and Technology) Lifts of holonomy representations and the volume of a link complement

RIMS

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Abstracts

Scott Carter (University of South Alabama)

Foams, Polytopes, Abstract Tensors, and Homology

In this talk, I want to discuss a number of structural consequences that follow by considering the local crossings of n-foams. These will be interpreted from several combinatorial and algebraic points of view. The ideas originate by considering four relations between group multiplication $(\cdot)$ and conjugation $(\triangleleft)$. Specifically, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$; $(a \cdot b) \triangleleft c = (a \triangleleft c) \cdot (b \triangleleft c)$; $(a \triangleleft b) \triangleleft c = a \triangleleft (b \cdot c)$; and $(a \triangleleft b) \triangleleft c = (a \triangleleft b) \triangleleft (b \triangleleft c)$;

An *n-foam* is a topological space that is locally modeled on the n-skeleton that is obtained by removing the neighborhoods of the vertices of an $(n+1)$-simplex and deforming the resulting space to its core. For example, a 1-foam is a trivalent graph, and the model is the trinian represent by the alphabetic character Y. The tetrahedron can be decomposed as the union of four cubes — one at each vertex. Each face of the tetrahedron intersects a cube in a square; three intersect at a face in a Y, and the remaining six faces in the interior for the model of a 2-foam.

The crossings of foams are expressed in the interior of the products of simplices. Using the competing technologies of movie moves (and their higher dimensional analogues), singularities, and dualizations, we obtain a variety of polytopal descriptions. These lead to formulations of a variety of analogues of the Yang-Baxter equations. In addition, there are homological interpretations that combine aspects of group and rack homology.

The polytopes that are involved include products of simplices, permutohedra, and the Stasheff polytope.

Qingtao Chen (ETH, Zurich)

Recent progress of various Volume Conjectures for links as well as 3-manifolds

The original Volume Conjecture of Kashaev-Murakami-Murakami predicts a precise relation between the asymptotics of the colored Jones polynomials of a knot in S^3 and the hyperbolic volume of its complement. I will discuss two different directions that lead to generalizations of this conjecture.

The first direction concerns different quantum invariants of knots, arising from the colored $SU(n)$ (with the colored Jones polynomial corresponding to the case $n = 2$). I will first display subtle relations between congruence relations, cyclotomic expansions and the original Volume Conjecture for colored Jones polynomials of knots. I will then generalize this point of view to the colored $SU(n)$ invariant of knots. Certain congruence relations for colored SU(n) invariants, discovered in joint work with K. Liu, P. Peng and S. Zhu, lead us to formulate cyclotomic expansions and a Volume Conjecture for these colored $SU(n)$ invariants. If time permits, I will briefly discuss similar ideas for the superpolynomials that arise in HOMFLY-PT homology.

Another direction for generalization involves the Witten-Reshetikhin-Turaev and (modified) Turaev-Viro quantum invariants of 3-manifolds. In a joint work with T. Yang, we formulated a new Volume Conjecture for the asymptotics of these 3-manifolds invariants evaluated at certain roots of unit, and numerically checked it for many examples. Interestingly, this conjecture uses roots of unity that are different from the one usually considered in literature. This may indicate that the understanding of this new phenomenon requires new physical and geometric interpretations that go beyond the usual quantum Chern-Simons theory.

Thanks to the new methods provided by T. Ohtsuki, many examples of hyperbolic knots of original Volume Conjecture have been solved. Such new methods are also employed by T. Ohtsuki himself and J. Murakami & me to study the above new Volume Conjectures proposed by T. Yang and me.

Shin Hayashi (AIST-TohokuU Mathematics for Advanced Materials-OIL)
On some topological invariants related to localized wave functions

In condensed matter physics, a correspondence between two topological invariants defined for a gapped Hamiltonian (bulk-edge correspondence) is well-known. One is defined for such a Hamiltonian on a lattice (bulk invariant), and the other is defined for its restriction onto some subsemigroup (edge invariant). The edge invariant is defined by counting the wave functions localized near the edge. Such a system with edge can be seen as a system with boundary (codimension is 1). In this talk, I will introduce a variant of such correspondence related to the boundary of the boundary (codimension is 2) . We consider a periodic Hamiltonian on a three dimensional lattice (bulk) and its restrictions onto two subsemigroups (edges) and their intersection (corner). If our Hamiltonian is "gapped" in some sense, we can define a topological invariant for the bulk and edges. Another topological invariant related to the wave functions localized near the corner is also defined. I will show that there is a relation between these two topological invariants by using the six-term exact sequence of K-theory for C*-algebras associated to the quarter-plane Toeplitz extension obtained by E. Park.

Hiroshi Goda (Tokyo University of Agriculture and Technology)
Lifts of holonomy representations and the volume of a link complement

Let M be an oriented, complete, hyperbolic 3-manifold of finite volume. The hyperbolic structure of M yields the holonomy representation: $\mathrm{Hol}_M : \pi_1(M, p) \to \mathrm{Isom}^+\mathbb{H}^3$. $\mathrm{Isom}^+\mathbb{H}^3$ is naturally identified with $\mathrm{PSL}(2, \mathbb{C}) \cong \mathrm{SL}(2, \mathbb{C})/\{\pm 1\}$. It is known that Hol_M can be lifted to $\mathrm{SL}(2, \mathbb{C})$; moreover, such lifts are in canonical one to one correspondence with spin structures on M. In this talk, we discuss the lifts of the holonomy representations, and then we give a volume formula of a hyperbolic link complement using the twisted Alexander invariants.

Naoko Kamada (Nagoya City University)
The double covering method for twisted knots

Twisted knot theory is an extension of virtual knot theory. A virtual knot corresponds to a stable equivalence class of knot diagrams on closed oriented surfaces, and a twisted knot corresponds to a stable equivalence class of knot diagrams on closed surfaces which are not necessary orientable or oriented. We consider double coverings of twisted knot diagrams and their applications. The first non-trivial example of twisted knot invariants was the twisted knot group defined by twisted knot diagrams. It turns out that the twisted knot group is equal to the virtual knot group of virtual knot diagrams which are double coverings of the twisted knot diagrams. Another application is that we can convert virtual knot diagrams to normal virtual knot diagrams. In this talk, we introduce the double covering method for twisted knot diagrams and its applications.

Jieon Kim (Osaka city University)
Presentations of (immersed) surface-knots by marked graph diagrams

An immersed surface-knot is a generically immersed closed and connected surface in the 4-space $\mathbb{R}^4$. When it is embedded, it is called a surface-knot. K. Yoshikawa studied surface-knots by using marked graph diagrams. Similar to the surface-knot case, immersed surface-knots can be represented by marked graph diagrams. In this talk, we introduce marked graph diagrams of (immersed) surface-knots. An immersed 2-knot is a generically immersed 2-sphere in $\mathbb{R}^4$. An immersed 2-knot with essential singularity is an immersed 2-knot which is not equivalent to the connected sum of any 2-knot and any unknotted immersed 2-sphere. We show that there are infinitely many immersed 2-knots with essential singularity. This is a joint work with S. Kamada, A. Kawauchi, and S. Y. Lee.

Kengo Kishimoto (Osaka Institute of Technology)
Simple-ribbon fusions and Alexander polynomials

We introduce a special kind of fusion of a link and a trivial link, called a simple-ribbon fusion. We call a knot obtained from the trivial knot by a finite sequence of simple ribbon fusions a simple-ribbon knot. For example, all ribbon knots with no more than 9 crossings, Kinoshita-Terasaka knot, and Kanenobu knots are simple-ribbon knots. We determine the Alexander polynomials of simple-ribbon knots. This is a joint work with Tsuneo Ishikawa, Tetsuo Shibuya and Tatsuya Tsukamoto.

Delphine Moussard (RIMS, Kyoto university)
Splitting formulas for the rational lift of the Kontsevich integral

Kricker defined an invariant of knots in homology 3-spheres which is a rational lift of the Kontsevich integral, and proved with Garoufalidis that this invariant satisfies splitting formulas with respect to a surgery move called null-move. Following the Cheptea-Habiro-Massuyeau's construction of a functorial LMO invariant for Lagrangian cobordisms, we define a functorial extension of the Kricker invariant. As an application, we obtain splitting formulas for this invariant with respect to null Lagrangian-preserving surgeries, a generalization of the null-move.

Kouki Sato (School of Science, Tokyo Institute of Technology)
A full-twist inequality for the ν^+ invariant

Hom and Wu introduced a knot concordance invariant called nu+, which dominates many concordance invariants derived from Heegaard Floer homology. In this talk, we give a full-twist inequality for ν^+. By using the inequality, we extend Wu's cabling formula for ν^+ (which is proved only for particular positive cables) to all cables in the form of an inequality. In addition, we also discuss ν^+-equivalence, which is an equivalence relation on the knot concordance group. We introduce a partial order on ν^+-equivalence classes, and study its relationship to full-twists.

Masaki Taniguchi (Graduate School of Mathematical Sciences, University of Tokyo)
Instanton moduli spaces on 4-manifolds with periodic ends and an obstruction of existence of embeddings

For a certain class of pairs of 3- and 4-manidfolds, we construct an obstruction in the filtered instanton Floer homology of the existence of an embedding with some homological conditions between them. In order to achieve that goal, we study the compactness of the ASD-moduli spaces over 4-manifolds with periodic ends. This work is a generalization of the Taubes's in 1987.

Anh Tran (Tohoku University/JSPS & The University of Texas at Dallas)

Some conjectures about the colored Jones polynomial

We will discuss some old and new conjectures about the colored Jones polynomial. These include the volume conjecture, AJ conjecture, slope conjecture, and strong slope conjecture. The volume conjecture of Kashaev-Murakami-Murakami relates the colored Jones polynomial of a knot and the hyperbolic volume of the knot complement in S^3. The AJ conjecture of Garoufalidis relates the A-polynomial and the colored Jones polynomial of a knot. The A-polynomial was introduced by Cooper et al. in 1994 and has been fundamental in geometric topology. A similar conjecture to the AJ conjecture was also proposed by Gukov from the viewpoint of the Chern-Simons theory. The slope conjecture of Garoufalidis and the strong slope conjecture of Kalfagianni-Tran assert that certain boundary slopes and Euler characteristics of essential surfaces in a knot complement can be read off from the degree of the colored Jones polynomial.