

## 研究集会 Intelligence of Low-dimensional Topology

京都大学数理解析研究所 RIMS 共同研究（公開型）として、また、大阪市立大学数学研究所から後援をうけて、トポロジープロジェクトの一環として、標記の研究集会を開催いたします。また、この研究集会は科学研究費補助金 基盤研究 B 「グラフィクスとカントル理論の観点からの 4 次元トポロジーの研究」( 課題番号 26287013、研究代表者 鎌田聖一氏 ( 大阪市立大学 )) と科学研究費補助金 基盤研究 A 「結び目と 3 次元多様体の量子トポロジー」( 課題番号 16H02145、研究代表者 大槻知忠 ( 京都大学 )) と科学研究費補助金 挑戦的萌芽研究 「ゲージ理論に関連する結び目と 3 次元多様体の不変量と量子トポロジー」( 課題番号 16K13754、研究代表者 大槻知忠 ( 京都大学 )) の援助を受けています。

日程： 2018 年 5 月 30 日 (水) ~ 6 月 1 日 (金)

場所： 京都大学 数理解析研究所 420 大講演室

アクセス： <http://www.kurims.kyoto-u.ac.jp/ja/access-01.html>

### 5 月 30 日 (水)

13:20 ~ 14:10 市原 一裕 (日本大学文理学部)

Minimal coloring numbers of  $\mathbb{Z}$ -colorable links

14:30 ~ 15:20 木村 直記 (早稲田大学基幹理工学研究科)

A generalization of the Dijkgraaf-Witten invariant

15:40 ~ 16:30 Neil Hoffman (Oklahoma State University)

The 3D index as a sum over surfaces

### 5 月 31 日 (木)

10:00 ~ 10:50 安井 弘一 (大阪大学大学院情報科学研究科)

Nonexistence of twists and surgeries generating exotic 4-manifolds

11:10 ~ 12:00 石橋 典 (東京大学大学院数理科学研究科 / 日本学術振興会特別研究員 DC)

Cluster Dehn twists in cluster modular groups

13:20 ~ 14:10 佐伯 修 (九州大学)

Simplified broken Lefschetz fibrations and trisections of 4-manifolds

14:30 ~ 15:20 Oliver Dasbach (Department of Mathematics, Louisiana State University)

Turaev surfaces and invariants of links

15:40 ~ Problem Session

6月1日(金)

10:00 ~ 10:50 粕谷 直彦 (京都産業大学)

Knots and links of complex tangents

11:10 ~ 12:00 和田 康載 (早稲田大学 / 日本学術振興会特別研究員 PD)

An obstruction to trivializing links by  $n$ -moves

13:20 ~ 14:10 片山 拓弥 (広島大学)

Virtual embeddability between surface mapping class groups

14:30 ~ 15:20 高尾 和人 (京都大学)

Singularities of product maps for low-dimensional topology

組織委員：河内明夫、河野俊丈、金信泰造、鎌田聖一、大槻知忠

世話人：大槻知忠(京大 数理研)、伊藤哲也(京大 理学研究科)

# Intelligence of Low-dimensional Topology

May 30 – June 1, 2018

Room 420, RIMS, Kyoto University

Access: <http://www.kurims.kyoto-u.ac.jp/en/access-01.html>

## Program

### May 30 (Wed)

13:20–14:10 Kazuhiro Ichihara (College of Humanities and Sciences, Nihon University)  
Minimal coloring numbers of  $\mathbb{Z}$ -colorable links

14:30–15:20 Naoki Kimura (Graduate School of Fundamental Science and Engineering, Waseda University)

A generalization of the Dijkgraaf-Witten invariant

15:40–16:30 Neil Hoffman (Oklahoma State University)

The 3D index as a sum over surfaces

### May 31 (Thu)

10:00–10:50 Kouichi Yasui (Graduate School of Information Science and Technology, Osaka University)

Nonexistence of twists and surgeries generating exotic 4-manifolds

11:10–12:00 Tsukasa Ishibashi (Graduate School of Mathematical Sciences, University of Tokyo / JSPS research fellow DC)

Cluster Dehn twists in cluster modular groups

13:20–14:10 Osamu Saeki (Kyushu University)

Simplified broken Lefschetz fibrations and trisections of 4-manifolds

14:30–15:20 Oliver Dasbach (Department of Mathematics, Louisiana State University)

Turaev surfaces and invariants of links

15:40– Problem Session

**June 1 (Fri)**

10:00–10:50 Naohiko Kasuya (Kyoto Sangyo University)

Knots and links of complex tangents

11:10–12:00 Kodai Wada (Waseda University / JSPS Research Fellow PD)

An obstruction to trivializing links by  $n$ -moves

13:20–14:10 Takuya Katayama (Hiroshima University)

Virtual embeddability between surface mapping class groups

14:30–15:20 Kazuto Takao (Kyoto University)

Singularities of product maps for low-dimensional topology

Scientific Committee: Akio Kawauchi, Toshitake Kohno, Taizo Kanenobu,  
Seiichi Kamada, Tomotada Ohtsuki

Organizers: Tomotada Ohtsuki (RIMS, Kyoto University),  
Tetsuya Ito (Dept of Math, Kyoto University)

# Intelligence of Low-dimensional Topology

May 30 – June 1, 2018

RIMS, Kyoto University

## Abstract

**Oliver Dasbach (Department of Mathematics, Louisiana State University)**  
**Turaev surfaces and invariants of links**

Turaev constructed for a given link diagram a closed, orientable surface on which the link embeds alternatingly. The Turaev genus of a link is the minimal genus among the Turaev surfaces of that link. It defines a natural filtration on knots where Turaev genus zero knots are precisely the alternating knots. We discuss its relation to other link invariants.

**Neil Hoffman (Oklahoma State University)**  
**The 3D index as a sum over surfaces**

The 3D index is an invariant of cusped, atoroidal 3-manifold introduced by Dimofte, Gaiotto and Gukov, which takes the form of a formal Laurent series. After providing the necessary background and introduction, I will discuss some of the difficulties of computing the 3D index from an arbitrary triangulation of the manifold, as well as a connection between this invariant and normal surfaces. My goal is to keep this talk expository, but it will be based in part on previous joint work with Stavros Garoufalidis, Craig Hodgson, and Hyam Rubinstein.

**Kazuhiro Ichihara (College of Humanities and Sciences, Nihon University)**  
**Minimal coloring numbers of  $\mathbb{Z}$ -colorable links**

One of the most well-known invariants of knots and links would be the Fox  $n$ -coloring for an integer  $n$ . On the other hand, there exist certain links known to admit a non-trivial  $n$ -coloring for any integer  $n$ . For such a link, one can define a  $\mathbb{Z}$ -coloring, which is a natural generalization of the Fox  $n$ -coloring. I will report on recent results on the  $\mathbb{Z}$ -coloring on links, in particular, the minimal number of colors for non-trivial  $\mathbb{Z}$ -colorings on a link. This talk is based on joint works with Eri Matsudo (Nihon University).

**Tsukasa Ishibashi (Graduate School of Mathematical Sciences, University of Tokyo / JSPS research fellow DC)**

### **Cluster Dehn twists in cluster modular groups**

A cluster modular group, which is introduced by Fock-Goncharov, is an automorphism group of a cluster algebra. The cluster modular group acts on a pair  $(A, X)$  of contractible manifolds called a cluster ensemble. These objects are associated with a combinatorial data called a seed. For the seed associated with an ideal triangulation of a punctured surface, it is known that  $(A-/X)$ -space coincides with the (decorated/ enhanced) Teichmüller space respectively, and the cluster modular group coincides with the tagged mapping class group. Taking a suitable seed, we can also describe various objects: double Bruhat cells of algebraic groups, higher Teichmüller spaces, etc.

In this talk, we introduce the concept of “cluster Dehn twists” for a general cluster modular group, which is a generalization of Dehn twists and half-twists in the mapping class group of a surface. We show that orbits of the action of a cluster Dehn twist on the  $A$ -space have the similar asymptotic behavior as those of (half) Dehn twists. Moreover, for several seeds of finite mutation type, we show that the corresponding cluster modular group is generated by cluster Dehn twists. It is a generalization of the classical fact that the mapping class group of a surface is generated by Dehn twists and half-twists.

**Naohiko Kasuya (Kyoto Sangyo University)**

### **Knots and links of complex tangents**

We show that a link in a closed orientable 3-manifold can be realized as the complex tangents of a smooth embedding of the 3-manifold into the complex 3-space if and only if it represents the trivial integral homology class in the 3-manifold. In the proof, we use Saeki’s theorem on singularities of stable maps. This is a joint work with Masamichi Takase.

**Takuya Katayama (Hiroshima University)**

### **Virtual embeddability between surface mapping class groups**

Applying the Birman–Hilden theory to hyper-elliptic involutions of closed hyperbolic surfaces, Perron–Vannier in 1999 proved that  $B_{2g}$  is embedded in the mapping class group  $\text{Mod}(\Sigma_g)$ , where  $B_{2g}$  is the braid group on  $2g$  strands and  $\Sigma_g$  is a closed orientable surface of genus  $g$ . Therefore,  $B_n$  is (virtually) embedded in  $\text{Mod}(\Sigma_g)$  if  $n \leq 2g$ . In addition, for  $g \geq 2$ , the Birman–Hilden theory also implies that  $\text{Mod}(\Sigma_{0,p})$  is virtually embedded in  $\text{Mod}(\Sigma_g)$  if  $p \leq 2g + 2$ , where  $\Sigma_{0,p}$  is a sphere with  $p$  punctures. In this talk, by investigating “symmetric” right-angled Artin groups embedded in mapping class groups, we prove the following:

- (1)  $B_n$  is virtually embedded in  $\text{Mod}(\Sigma_g)$  only if  $n \leq 2g$ ,
- (2)  $\text{Mod}(\Sigma_{0,p})$  is virtually embedded in  $\text{Mod}(\Sigma_g)$  only if  $p \leq 2g + 2$ .

This talk contains joint work with Erika Kuno.

**Naoki Kimura (Graduate School of Fundamental Science and Engineering, Waseda University)**

**A generalization of the Dijkgraaf-Witten invariant**

The Dijkgraaf-Witten invariant is a topological invariant for compact oriented 3-manifolds in terms of a finite group and its 3-cocycle. The invariant is a state sum invariant constructed by using a triangulation, likewise the Turaev-Viro invariant. In this talk, we consider a generalization of the Dijkgraaf-Witten invariant for cusped 3-manifolds by using an ideal triangulation. We present some calculation examples of the generalized Dijkgraaf-Witten invariants for orientable cusped hyperbolic 3-manifolds.

**Osamu Saeki (Kyushu University)**

**Simplified broken Lefschetz fibrations and trisections of 4-manifolds**

We present explicit algorithms for simplifying the topology of indefinite fibrations on 4-manifolds, which include broken Lefschetz fibrations and indefinite generic maps, from the viewpoint of singularity theory. These algorithms allow us to give purely topological and constructive proofs of the existence of simplified broken Lefschetz fibrations on general 4-manifolds, and a theorem of Auroux-Donaldson-Katzarkov on the existence of simplified broken Lefschetz pencils on near-symplectic 4-manifolds. We moreover establish a correspondence between broken Lefschetz fibrations and trisections of 4-manifolds, and show the existence of simplified trisections on all 4-manifolds. Based on this correspondence, we provide several new constructions of trisections.

**Kazuto Takao (Kyoto University)**

**Singularities of product maps for low-dimensional topology**

Fairly many sorts of fundamental objects in low-dimensional topology correspond to some sorts of smooth maps. For example, handle decompositions are represented by Morse functions, and fibrations are determined by their projections. For given two such objects in one manifold, the product map of the corresponding smooth maps can be assumed to be a generic smooth map. In fact, the singularities of the product map hold much information of the relationship between the original two objects. In this talk, I would like to survey some known results and pending problems in this context.

**Kodai Wada (Waseda University / JSPS Research Fellow PD)**

**An obstruction to trivializing links by  $n$ -moves**

Let  $n$  be a positive integer. Dabkowski and Przytycki introduced the  $n$ th Burnside group of links which is preserved by  $n$ -moves, and proved that for any odd prime  $p$  there exist links which are not equivalent to trivial links up to  $p$ -moves by using their  $p$ th Burnside groups. This gives counterexamples for the Montesinos-Nakanishi 3-move conjecture. In general, it is hard to distinguish  $p$ th Burnside groups of a given link and a trivial link. In this talk, we give a necessary condition for which  $p$ th Burnside groups are isomorphic to those of trivial links. The necessary condition gives us an efficient way to distinguish  $p$ th Burnside groups of a given link and a trivial link. As an application, we show that there exist links, each of which is not equivalent to a trivial link up to  $p$ -moves for any odd prime  $p$ . This is a joint work with Haruko A. Miyazawa and Akira Yasuhara.

**Kouichi Yasui (Graduate School of Information Science and Technology, Osaka University)**

**Nonexistence of twists and surgeries generating exotic 4-manifolds**

In dimension four, there are still no manifolds whose all (exotic) smooth structures are found. On the other hand, it is well known that, under a certain condition, infinitely many exotic smooth structures on a 4-manifold are produced by twisting a neighborhood of an embedded torus. Thus, it is natural to ask whether a (compact) 4-manifold admits a submanifold such that the set of all smooth structures on the 4-manifold is generated by twisting the submanifold. In this talk, we give a partial negative answer to this problem. In particular, we show that there exists no ‘universal’ compact 4-manifold  $W$  such that, for any simply connected closed 4-manifold  $X$ , the set of all smooth structures on  $X$  is generated by twisting a fixed embedded copy of  $W$ . Moreover, we give similar results for more general surgeries.