Scaling relations
for percolation in the 2D high temperature Ising Model

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This talk is based on joint work with Yu Zhang (University of Colorado).
We consider the percolation problem for Ising model on the two-dimensional square lattice $\mathbb{Z}^2$. For $T > T_c$ and $h \in \mathbb{R}$, there exists a unique Gibbs measure $\mu_{T,h}$. The (+)-cluster containing the origin is denoted by $C_0^+$. For each $T > 0$, the critical external field is defined by

$$h_c(T) := \inf\{h : \mu_{T,h}(\#C_0^+ = \infty) > 0\}.$$

It is known that $h_c(T) > 0$ whenever $T > T_c$. Hereafter we fix a $T > T_c$, and abbreviate $\mu_{T,h}$ to $\mu_h$ and $h_c(T)$ to $h_c$, respectively. The expectation under $\mu_h$ is denoted by $E_h$.

The following power laws are widely believed to hold:

- **Percolation probability**
  $$\theta(h) := \mu_h(\#C_0^+ = \infty) \approx (h - h_c)^\beta$$
as $h \to h_c$.

- **Mean cluster size**
  $$\chi(h) := E_h[\#C_0^+ : \#C_0^+ < \infty] \approx |h - h_c|^{-\gamma}$$
as $h \to h_c$.

- **Correlation length**
  $$\xi(h) := \left(\frac{1}{\chi(h)} \sum_{v \in \mathbb{Z}^2} |v|^2 \mu_h(\text{O to } v \text{ in } C_0^+ < \infty)\right)^{1/2} \approx |h - h_c|^{-\nu}$$
as $h \to h_c$.

- **One-arm probability**
  $$\pi_{h_c}(n) := \mu_{h_c}(\text{O to } \partial S(n)) \approx n^{-1/\delta_c}.$$

- **Connectivity function**
  $$\tau_{h_c}(n) := \mu_{h_c}(\text{O to } (n,0)) \approx n^{-\eta}.$$

We derive some scaling relations, provided the exponents exist; for 2D Bernoulli percolation, these relations are proved by Kesten.