Analysis of basin of attraction for bipedal walking models from the viewpoint of hybrid dynamics and saddle dynamics

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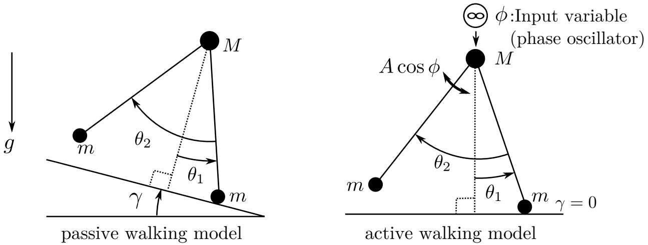
Outline

Introduction

- Studying the stability of bipedal walking models
  - Focus on the basin of attraction
- We consider compass-type bipedal walking models



Characteristics of bipedal walking

- Hybrid system
  - Combination of continuous-time dynamical systems (flows) and discrete-time dynamical systems (maps)
  - There are two states:
    - Single support phase
    - Double support phase
  - Multiple types equations of motion and condtiotions for state transition
- Inverted Pendulum Model
  - Bipedal walking is considered to be based on an inverted pendulum
  - Effective (low-energy) walking is realized by the mechanism

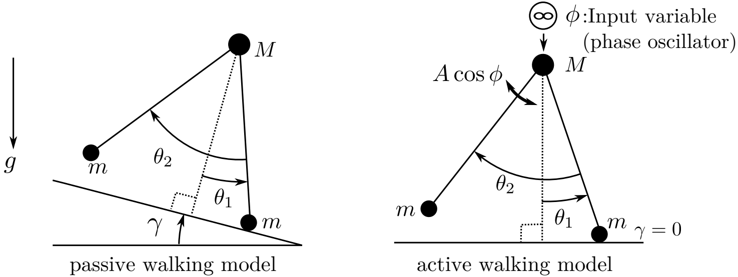
Prvious Results

There are many numerical studies by the researchers of robotics, biology, and biomechanics.

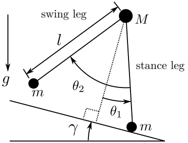
- Linear stablility
- Bifurcations
- The basin of attraction

There are many studies about how to improve the stablility, but the mathematical mechanism of formation of basin of attraction is not known.

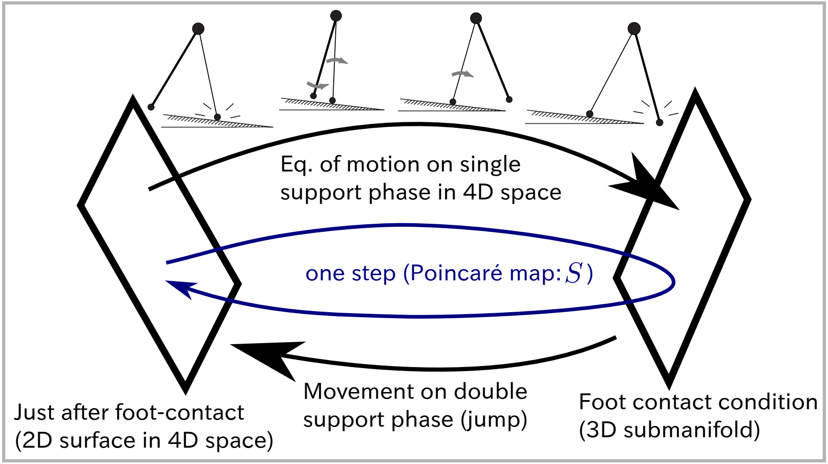
Models



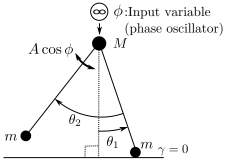
- The passive walking model
  - Without any input, walks on the shallow slope
- The active walking model
  - With periodic force (by a phase oscillator)
- These two models have different dimensional phase space since the passive walking model does not have phase oscillator



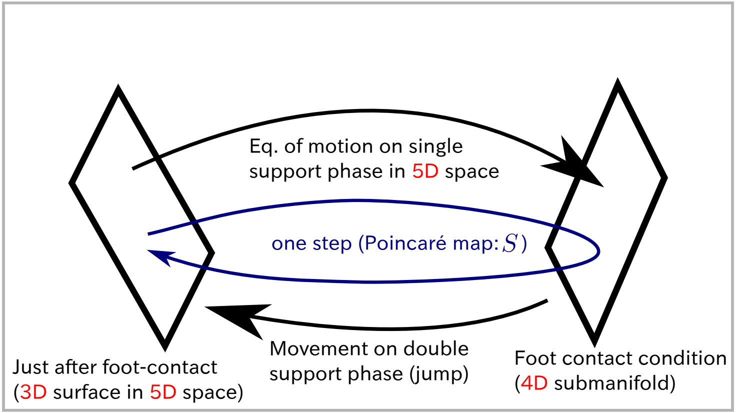
Typical hybrid system



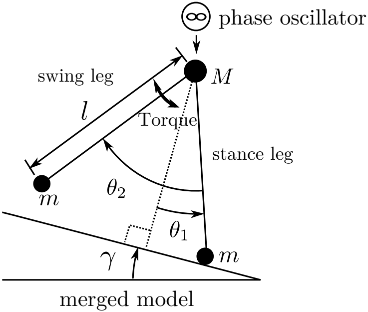
In case of the active walking model:



active walking model

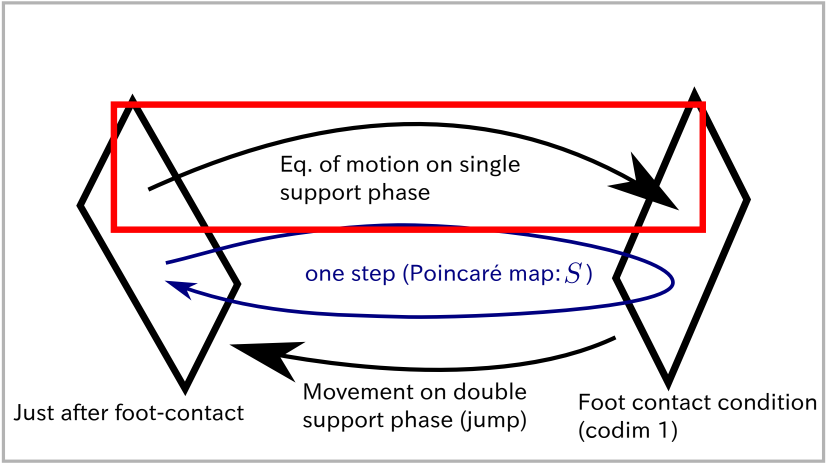


Model description



Use this “merged” model to describe the two models at once.

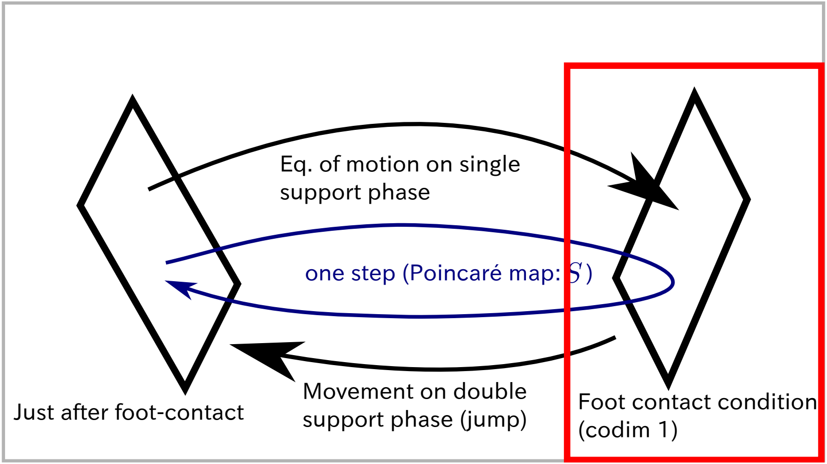
Equation of motion of single support phase



The single support phase is modeled by double pendulum. The following equation is given by Lagrange equations:

$$
\begin{pmatrix} ML^2 + 2mL^2(1-\cos(\theta_2)) & mL^2(-1+\cos(\theta_2)) \\ mL^2(-1+\cos(\theta_2)) & mL^2 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix}
+ \begin{pmatrix} mL^2(2\dot{\theta}_1 - \dot{\theta}_2 - \gamma)\dot{\theta}_2 \sin\theta_2 \\ -mL^2\dot{\theta}_1^2 \sin\theta_2 \end{pmatrix} + \begin{pmatrix} -gML\sin(\theta_1 - \gamma) \\ 0 \end{pmatrix}
+ \begin{pmatrix} gmL(\sin(\theta_1-\gamma) + \sin(\theta_2 - \theta_1 + \gamma)) \\ gmL\sin(\theta_2 - \theta_1 + \gamma) \end{pmatrix}
= \begin{pmatrix} 0 \\ A\cos\phi \end{pmatrix} \qquad \dot{\phi} = \omega
$$

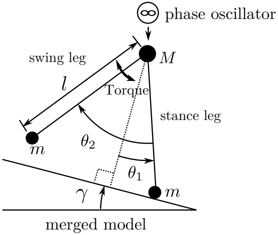
Foot-contact condition



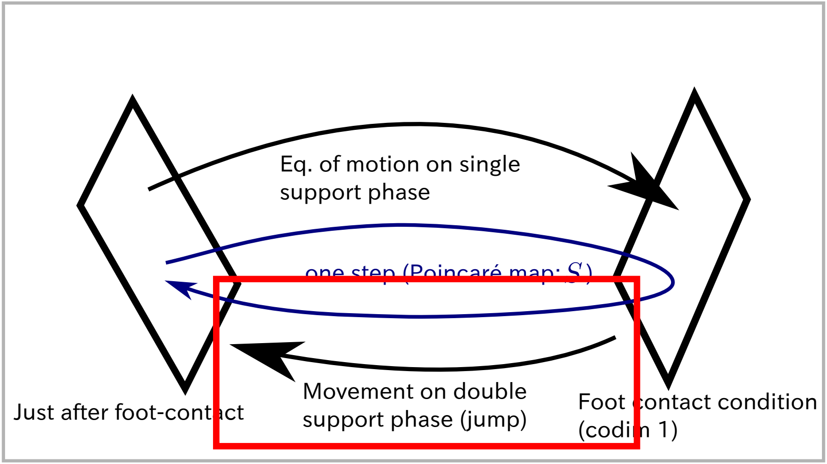
Foot-contact condition:

1. $2\theta_1 - \theta_2 = 0$
2. $\theta_1 < 0$
3. $2\dot{\theta}_1 - \dot{\theta}_2 < 0$

This is the transition condition from the single support phase to the double support phase.
Cond. 2 and 3 are required to avoid foot scuffing.



Motion of double support phase



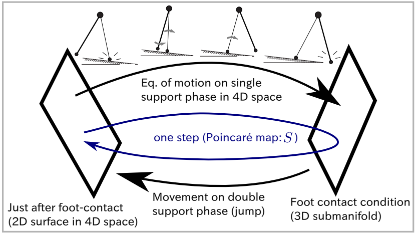
The motion of double support phase:

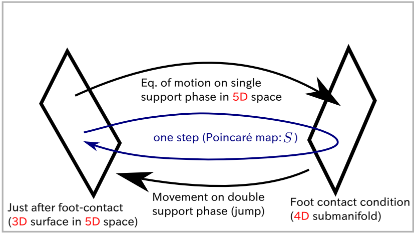
- Assume that the collision is fully inelastic
- We assume that the double support phase completes in a blink.
- The motion is described by the map.

$$(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \phi) \mapsto (-\theta_1,\ -\theta_2, h_1, h_2, \phi - \pi)$$

where

$$h_1 = 2M\dot{\theta}_1 \cos\theta_2 / (2M + m(1 - 2\cos\theta_2))$$
$$h_2 = 2M\dot{\theta}_1 (2\cos\theta_2 - \cos 2\theta_2 - 1)/(2M + m(1 - 2\cos\theta_2))$$

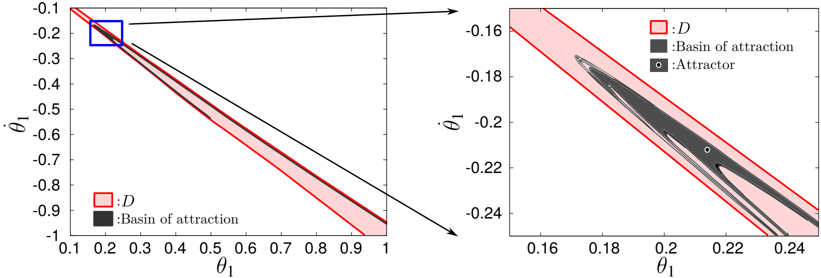




- The equations of motion on single support phase
- The foot contact condition
- The motion of double support phase

Basin of attraction(passive model)

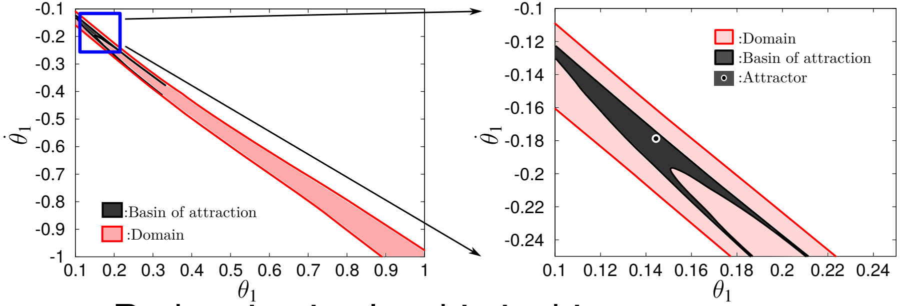
The Poincaré map S has an attracting fixed point when $\beta = 0$, $\gamma = 0.011$.



- Red region (The region of initial points that can take at least one step; i.e. the domain D of the Poincaré map S) is very thin
- The basin of attraction is V-shaped, and has slits and stripe patterns

Basin of attraction(active model)

The Poincaré section is 3D, so we take a slice at $\phi = 0.0535$.

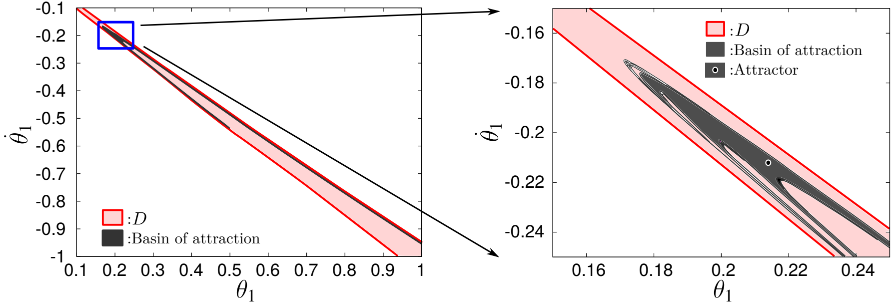


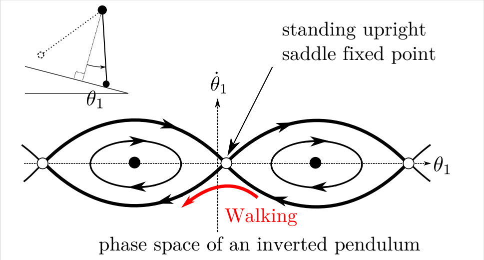
- Red region is also thin in this case.
- The basin of attraction has a “horn-like” shape.

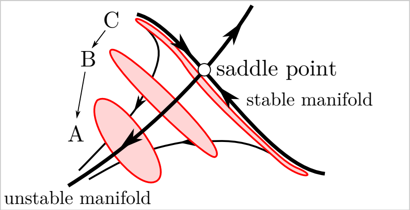
Parameter: $\beta = m/M = 0.15, \omega = 2.09, A = -12.0$

The shape of the basin of attraction(Passive)

We discuss why the shape is like this.

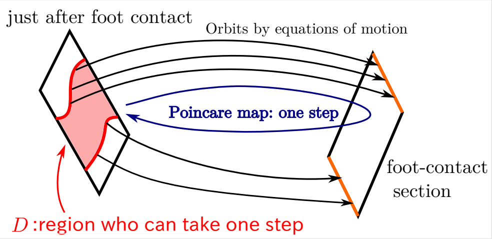




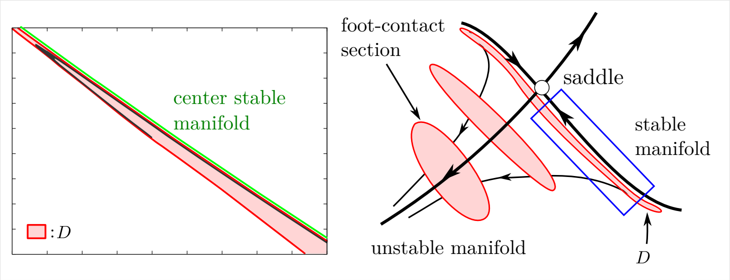


- An inverted pendulum is a part of the model.
  - Ignoring the motion of the swing leg
- An inverted have a saddle fixed point (The fixed point means upright standing)
- From λ-lemma, the region A moves to B and C by the time-backward of the equations of motions.

Thin domain



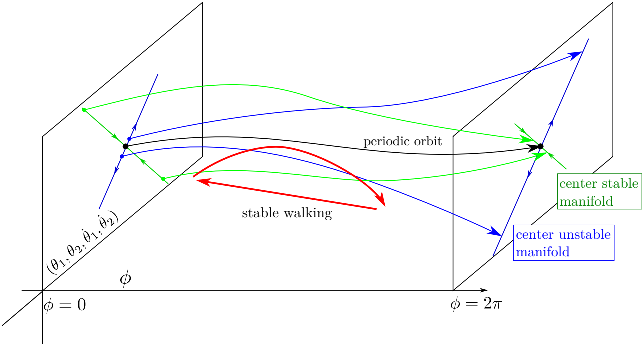
D is determined by the time backward of equation of motion



- The center stable manifold in the phase space is cartesian product of the stable manifold of the simple inverted pendulum and $\mathbb{R}^2$
- Codimension 1 center stable manifold plays an important role to form the thin domain, and the basin of attraction
    - The upright equilibrium point is saddle-center, The point has 1D unstable subspace, 1D stable subspace and 2D center subspace.
    - The manifold is a separatrix of “falldown forward” and “falldonw backward”

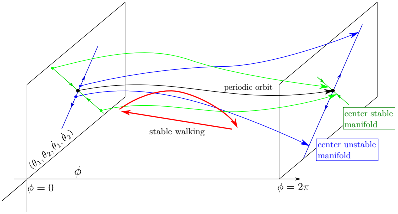
In case of active model

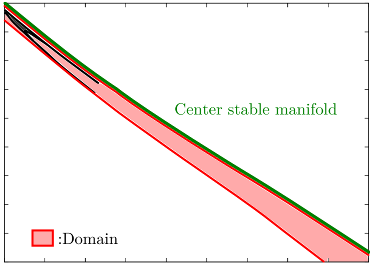
In case of the active model, we can find a similar structure.



- The eq. of motion has a saddle-center periodic orbit instead of an equilibrium point(Movie).

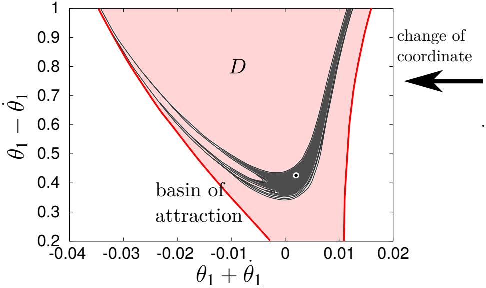
The saddle-center periodic orbit plays an similar role as the equilibrium point in the passive walking model.

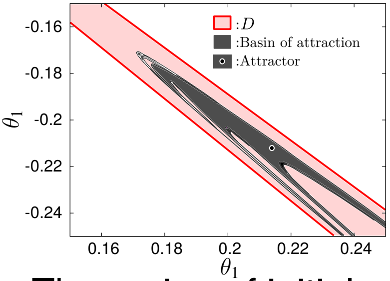


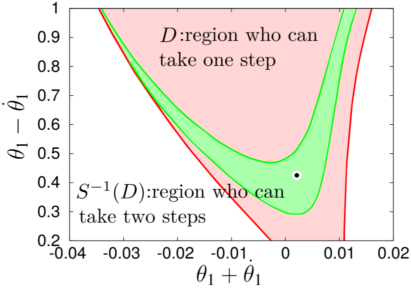


- The red region (domain) is thin along the center stable manifold

V-shaped basin of attraction

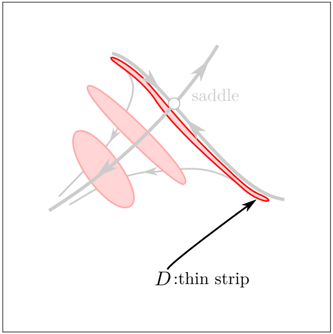


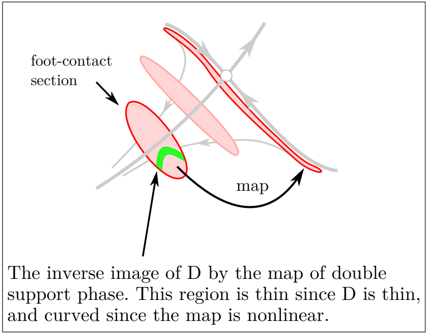


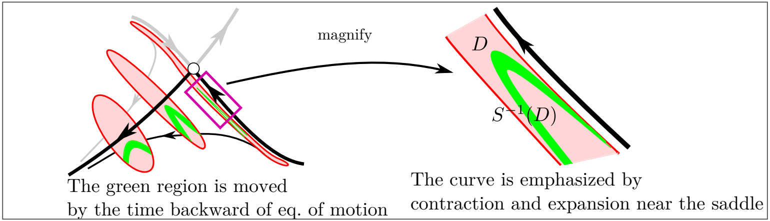


- The region of initial points who can take at least two steps (the green region) is V-shaped
- This causes V-shaped basin of attraction

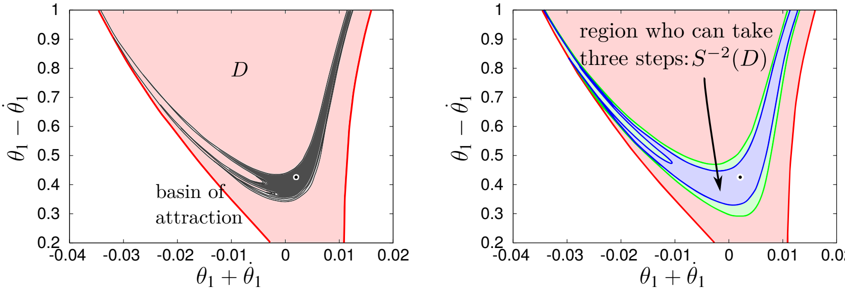
V-shaped region and the saddle





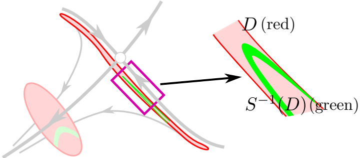


Slits in the basin of attraction

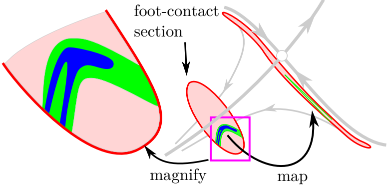


- "Three steps region" has a slit
- The slit is constructed by a similar mechanism of the construction of V-shaped region
- The stripe pattern is constructed by the repeated application of the mechanism
- As a result, the basin of attraction has slits and stripe patterns

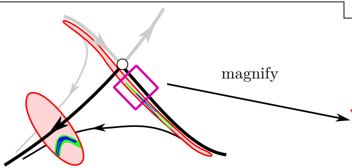
Construction of the slit



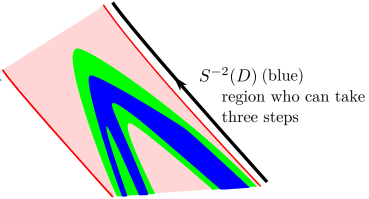
$S^{-1}(D)$ is V-shaped in the thin region D



The blue region is the inverse image of $S^{-1}(D)$ by the map of double support phase

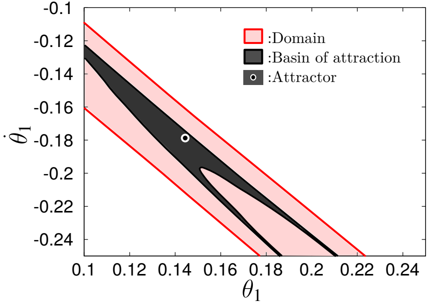


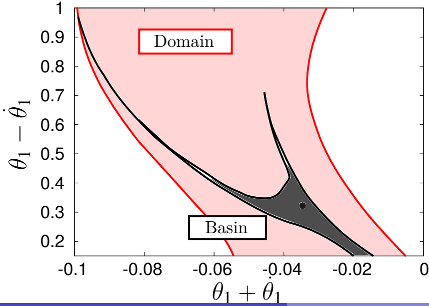
The blue region is moved by the time backward of eq. of motion and $S^{-2}(D)$ is formed in $S^{-1}(D)$

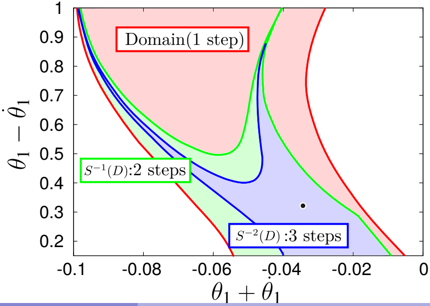


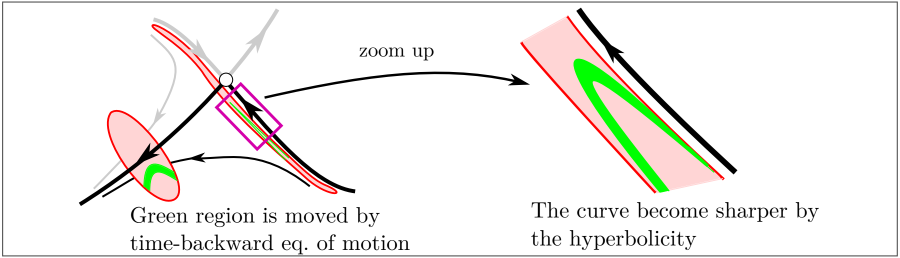
The blue region with a slit is formed in the green V-shaped region

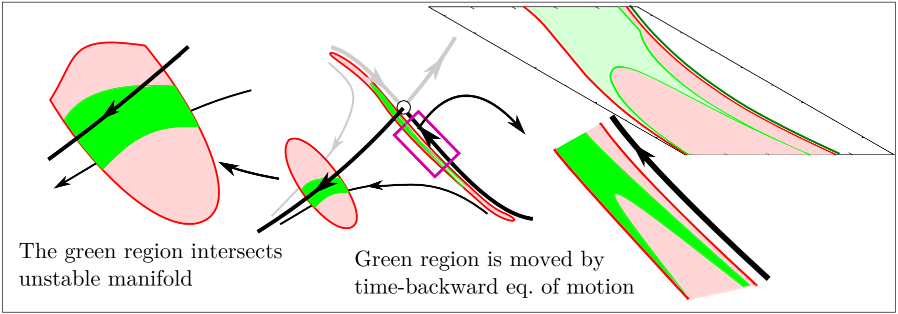
"Horns" in active walking model








zoom up
Green region is moved by
time-backward eq. of motion
The curve become sharper by
the hyperbolicity


The green region intersects
unstable manifold
Green region is moved by
time-backward eq. of motion

Summary

- The saddle property plays an important role to form the basin of attraction.
  - Codim 1 center stable and center unstable manifolds are important.
- The instability of the saddle makes the thin region.
  - This instability is important for bipedal walking
  - We consider the phenomenon is quite common among many bipedal walking models.
  - The fact suggest that bipedal walking is essentially not so stable from the mechanical viewpoint

- V-shaped, slits, strip patterns, “horns”
  - The position relation of the domain, the center-stable and center-unstable manifolds, and foot-contact section constructs these structures.
  - Hybridness is important
  - Other model has other structure
  - The mechanism itself is probably common among many models
- For a continuous-time dynamical system, a codim 1 invariant manifold is a kind of obstacles, and any orbit cannot pass throught the manifold, but for a hybrid system, an orbit can jump over such a manifold.

Future works

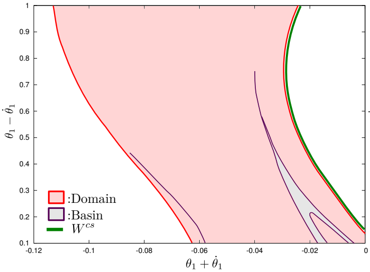
- Comparison with experimental data (in progress)
    - We expect that human walking uses this mechanism
- Examine more complicated models
    - We believe that the mechanism desribed here is common for complicated models, and we need to show it
    - Spring-mass models, 3D models
    - We can find other dynamical mechanisms?
- Find how to imporve the stablility of bipedal walking models using the mechanism

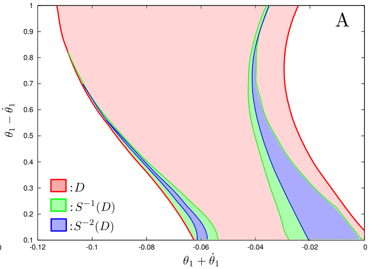
Future works (Dreams)

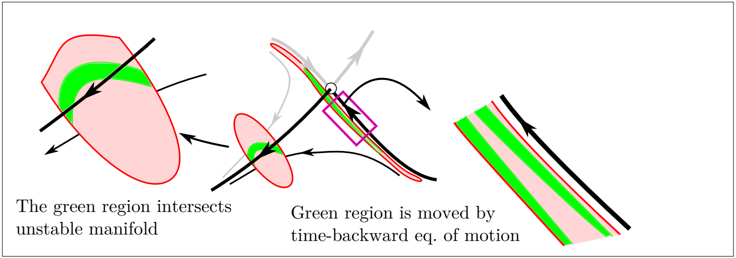
- Designing robots
  - It is difficult to directly apply the idea to the real robots since such robots have higher dimensional phase space and complex geometric structures.
  - Can we design a robot with a simpler geometric structure and apply the idea?
- Beyond bipedal walking
  - In our theory, the fact is important that bipedal walking is a hybrid system based on an inverted pendulum
  - Other hybrid system with a saddle?

Appendix

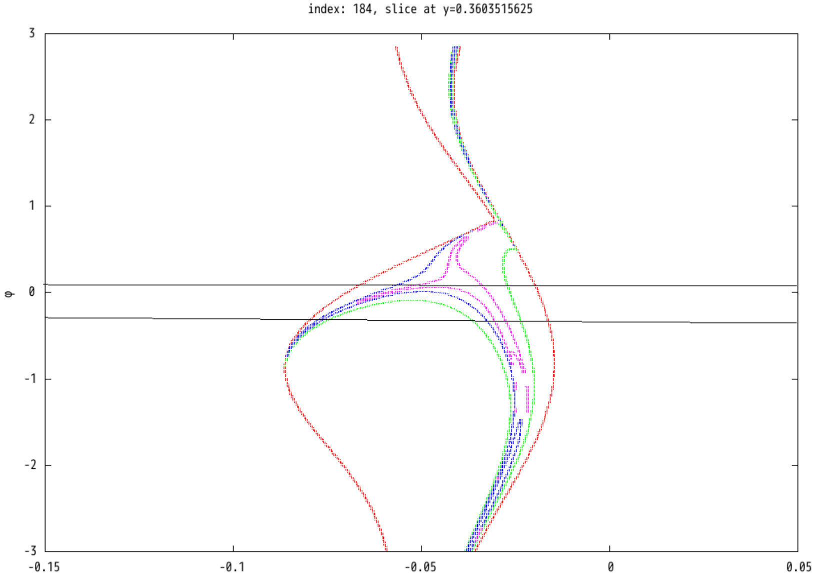
Another slice? ($\phi = 5.94(\sim -0.34) \pmod{2\pi}$)







Another slice($\theta_1 - \dot{\theta}_1 = 0.45$). The coordinate uses $\theta_1 + \dot{\theta}_1$ and ϕ



Comparison with experimental data

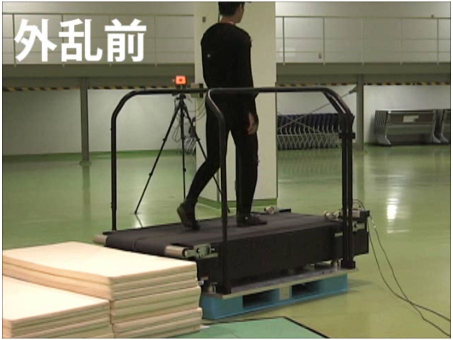
For comparison with experimental data, We assume the following:

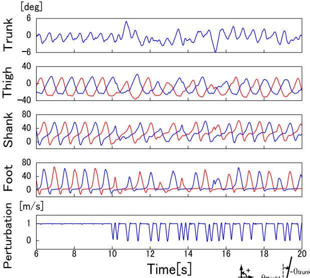
- A bipedal walker use the mechanism described in this talk
- A motion passing throught the *thin* region is better for stable walking
  - So the walker try to adjust the walking motion
- We regard an walker as an inverted pendulum and stable and unstable subspace is computable

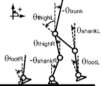
We measured a human walking on a treadmill with disturbances and ploted $(\theta_1, \dot{\theta}_1)$ (the angle and angular velocity of the stance leg just after lift off)

- Without disturbances, the walking motion will quickly converge to the attractor

実験状況





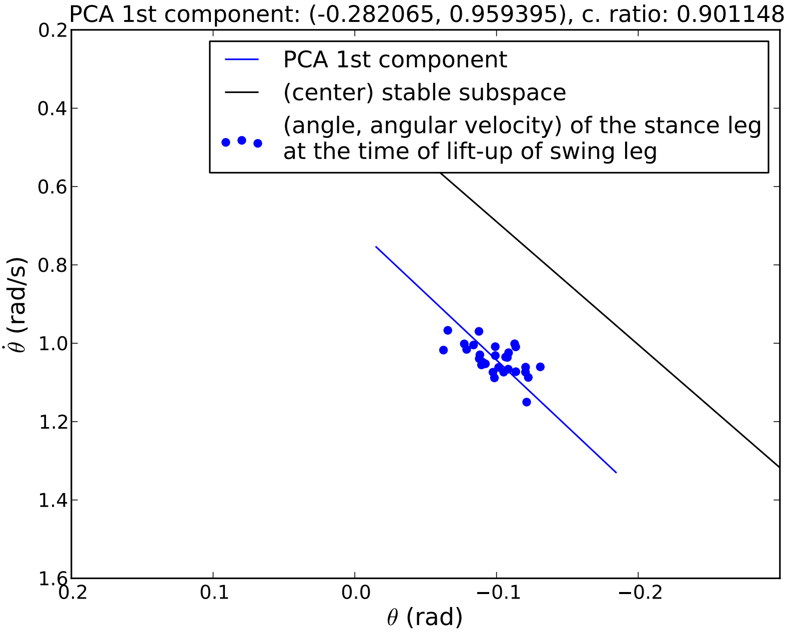


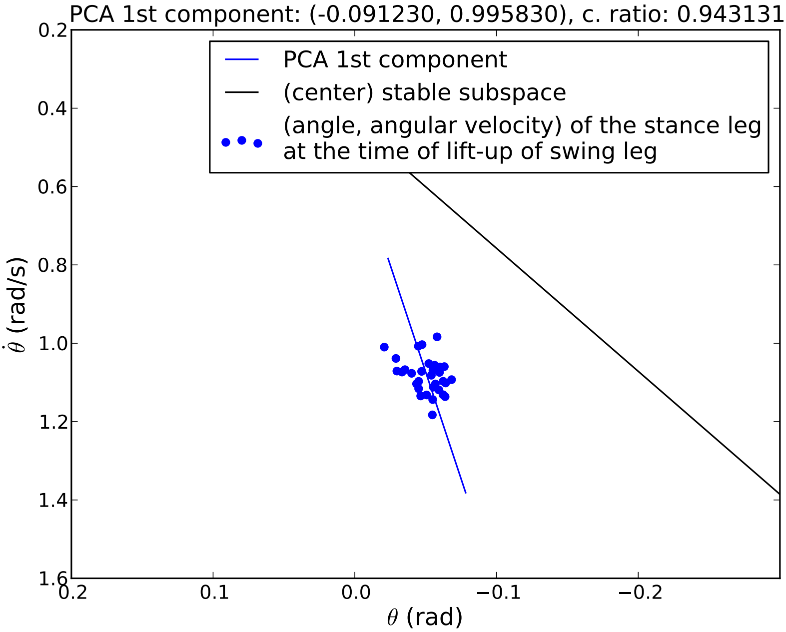
- 被験者 5名
- 定常速度 :1.0[m/s]
- 外乱：
  - 速度増加外乱 :+1.2[m/s]
  - 速度減少外乱 :-1.0[m/s]

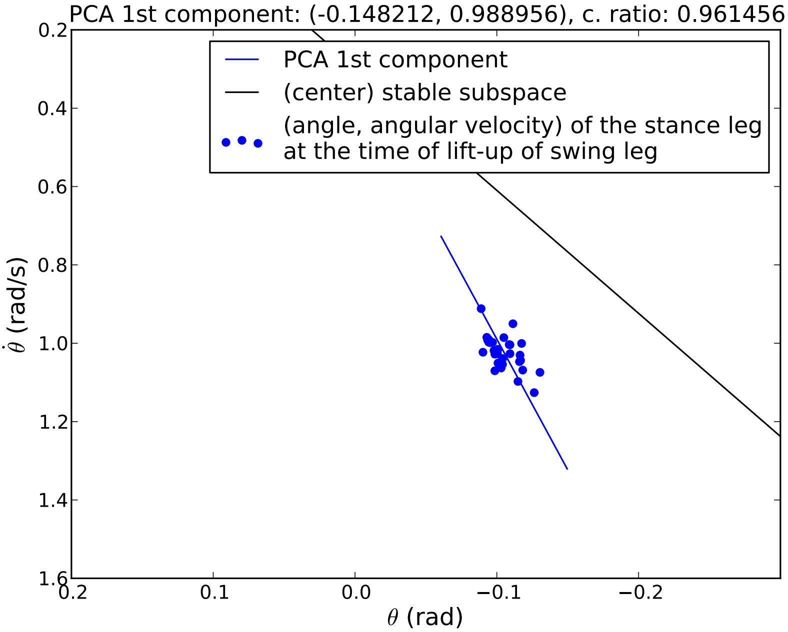
  を0.1秒で変化させ、0.1秒で戻す
- 一回(試行)の計測時間：180秒 (10秒後に外乱開始)
- 一人の被験者 15試行
- 運動計測周波数 :500[Hz], 外乱計測周波数 :1000[Hz]

By Prof. Funato (in UEC).

- A disturbance per five seconds (by the treadmill)
- Some preprocesses
- Plotting $(\theta_1, \dot{\theta}_1)$ for each 180 seconds trial, and find 1-dim structure with PCA


PCA 1st component: (-0.282065, 0.959395), c. ratio: 0.901148
PCA 1st component
(center) stable subspace
(angle, angular velocity) of the stance leg at the time of lift-up of swing leg
$\dot{\theta}$ (rad/s)
θ (rad)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0.2
0.1
0.0
−0.1
−0.2


PCA 1st component: (-0.091230, 0.995830), c. ratio: 0.943131
PCA 1st component
(center) stable subspace
(angle, angular velocity) of the stance leg at the time of lift-up of swing leg
$\dot{\theta}$ (rad/s)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0.2
0.1
0.0
−0.1
−0.2
θ (rad)


PCA 1st component: (-0.148212, 0.988956), c. ratio: 0.961456
PCA 1st component
(center) stable subspace
(angle, angular velocity) of the stance leg at the time of lift-up of swing leg
$\dot{\theta}$ (rad/s)
θ (rad)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0.2
0.1
0.0
−0.1
−0.2

Summary

We have found a *thin* structure in experimental data of human walking.