

この講究録別冊は、2015年7月20日から7月24日まで東京大学玉原国際セミナーハウスにて行われた京都大学数理解析研究所合宿型セミナー「数論的アルゴリズムの自然拡大と S -adic システム」の記録である。セミナー参加者は27名（海外機関から7名）であった。本合宿型セミナーは数論と力学系が非常に密接に関連する置換規則（Substitution）関連分野の世界のトップクラスの研究者を集め、合宿セミナー形式で最新研究の進展を学習することを目標とした。

数論と力学系の関係は古く、連分数などのディオファントス近似のアルゴリズムに関する研究は境界領域にあり現在も活発な研究が行われている。一次元連分数のガウスによる不変測度は周知であるが、高次元の同時近似アルゴリズムでは知られていることは少ない。例えば基本単数系を決定する高次元連分数は知られていないし、不変測度を具体的に記述できる例もわずかである。自然拡大は不変測度を具体的に求めるための強力な手段であり、それ自身が数論的、エルゴード理論的研究を行う上での基礎となる。連分数アルゴリズムは無理回転を適宜小区間に誘導していく過程を具体化したものであり基本単数はその自己誘導構造の拡大係数となる。対応する記号システムは複数の自己誘導構造で不変な体系である S -adic システム（さらに一般には Bratteli-Vershik のダイアグラム）に抽象化、一般化され様々な方向に発展している。無理回転と連分数のように誘導力学系を通じて親子関係になっている力学系はエントロピー零の加法的作用と、それを加速したエントロピー正の乗法的作用を兼備しており、きわめて重要である。このような対象として平行移動作用と拡大的行列の作用を併せ持つ置換規則力学系および、その時間作用を \mathbf{R}^d に一般化したタイル張り力学系が近年大きな注目を集めている。

本セミナーでは以上の状況を踏まえ関連する研究者に主にサーベイ講演をお願いした。T.Schmidt 氏と P.Arnoux 氏は様々な連分数の自然拡大および不変測度の構成に関し近年著しい進展を得ており、今回招聘できたことは幸運であった。また V.Berthé 氏に日進月歩の S -adic システムの概説をお願いした。関連して線形再帰性と S -adic システムの關係の解説を湯浅久利氏に依頼し、Bratteli-Vershik ダイアグラムの基本的解説では杉崎文亮氏に無理をお願いした。無理回転のコードを用いた加法機械の構成に関する吉田雅通氏の講演もいただいた。タイル張り力学系の基本的な解説を永井康史氏をお願いした。D.H.Kim 氏はグラフの複雑度と無理回転のコード化の類似研究を紹介した。この分野でよく用いられる道具であるフラクタル解析の解説を D.J.Feng 氏にいただいた。より数論に近い研究では、B.Vallée 氏に数論的アルゴリズムの統計的挙動を推移作用素による研究のサーベイをいただいた。安富真一氏、田村純一氏から連分数と関連する置換規則の複素指数への一般化、大音智弘氏からは p 進超越数論の話題提供があった。

開催週は関東では特別の猛暑であったが、参加者は玉原高原の自然の冷気のなか多くの最新の進展を学習できた。周囲の自然環境も素晴らしく食事もおいしく参加者に大変好評であった。本セミナーを具体化するにあたり下記のオーガナイザーの皆さまに、事前調整、資材準備、交通手段、集金等、運営全般に関して言葉で尽くせないほどお世話になりました

た。この場を借りて深くお礼を申し上げます。またセミナーハウス管理者と関係の方々には快適に過ごせるよう細かくご配慮いただき有難うございました。最後にこの機会を与えてくださった京都大学数理解析研究所に感謝いたします。

2016 年 6 月 14 日

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This volume collects papers based on lectures delivered at RIMS Camp Style Seminar “Natural extension of arithmetic algorithms and S-adic system” held at Tambara International Seminar House on 20-24 July 2015. The seminar was funded by RIMS, Research Institute of Mathematical Science, Kyoto University, and organized as a summer school aiming at studying recent developments from world top researchers in the area related to substitution, a bridge between number theory and dynamical system. It was quite successful with 27 participants including 7 oversea researchers.

The interplay between number theory and dynamical system has a long history. The research on Diophantine approximation algorithm like continued fraction belongs to both subjects and is quite productive still today. Though Gauss measure is a well-known invariant measure of continued fraction, we know little on higher dimensional versions of continued fractions. For instance, we do not know any continued fraction which surely gives the system of fundamental units, and explicit absolutely continuous invariant measures are known in rare cases. Natural extension is the strong tool to make explicit the invariant measure, and also gives a fundamental understanding of the algorithm from number theoretic and dynamical aspects. Continued fraction algorithm is a concrete description of recursive induction of irrational rotation to smaller intervals and the fundamental unit appears as a dilation constant of self-inducing structure appear in the irrational rotation. The associated symbolic system could be described by the system closed by actions of several substitutions, that is, S-adic system (and Bratteli-Vershik diagram in general), and developed in diverse directions. The irrational rotation and continued fraction are intimately connected by dynamical induction and forms a very important pair of dynamical systems. It is a kind of mother (irrational rotation) and child (continued fraction) relation, the former with zero entropy and the later with positive entropy. Substitutive dynamical systems have the same nature, having translation action on one hand, self-affine expansion on the other hand. Together with its \mathbf{R}^d action version, self-affine tiling dynamical systems, they attract a lot of attention of researchers.

This seminar consists mainly of invited survey lectures to learn basics as well as recent developments on the above described subjects. T.Schmidt and P.Arnoux recently obtained a way to construct natural extensions and absolutely continuous invariant measures of many types of continued fractions. We are lucky that they accepted our invitation. V.Berthé gave an invited lecture on the recent development on S-adic systems intimately related to Diophantine algorithms. For further development, we asked H.Yuasa to talk on linearly recurrent system and S-adic system and F.Sugisaki on the unique ergodicity of Bratteli-Vershik diagram. M.Yoshida gave a talk on the adding

machine based on irrational rotation. For tiling dynamical system, Y.Nagai gave an introductory survey. D.H.Kim talked on the complexity of graphs and an analogy of Sturmian words. We asked D.J.Feng to talk on methods in fractal geometry which often appear in the related works. Coming to number's theoretical aspect, we asked B.Vallée to deliver a survey on the study of stochastic behavior of arithmetic algorithms using transfer operators. J.Tamura and S.Yasutomi talked on the generalization of substitution with complex indices and T.Ooto discussed Mahler's classification on transcendence of p -adic numbers.

The seminar week was exceptionally warm around Tokyo but we could enjoy nice up-to-date lectures in cool mountain area, good foods and great natural environment. To make this seminar in practice, I owe almost everything to the seminar organizers listed below. It is really impossible without them to organize this seminar and I would like to express my deepest gratitude to these four. Thanks are also due to related people on Tambara Seminar House as well as Research Institute of Mathematical Science to realize this seminar in this cozy nice way.

14 June 2016

Shigeki AKIYAMA

Institute of Mathematics, University of Tsukuba.

Seminar Organizers

Shin'ichi Yasutomi (Toho University)

Hisatoshi Yuasa (Osaka Kyoiku University)

Masamichi Yoshida (Osaka City University)

Hiromi Ei (Hirosaki University)

RIMS Camp Style Seminar
***Natural extension of arithmetic algorithms
and S -adic system***

July 20-24, 2015

Organizers:

Shigeki Akiyama (*Univ. Tsukuba*)

Shin'ichi Yasutomi (*Toho Univ.*)

Hisatoshi Yuasa (*Osaka Kyoiku Univ.*)

Masamichi Yoshida (*Osaka City Univ.*)

Hiromi Ei (*Hirosaki Univ.*)

Monday 20

15:00-17:30 **Thomas Schmidt** (Oregon State Univ.)
Continued Fraction: Old and New

Tuesday 21

9:00-11:30 **Pierre Arnoux** (Univ. Aix-Marseille, Inst. Luminy)
Invariant densities for continued fractions, models of natural extensions
and fixed point theorems

11:30-12:00 **Thomas Schmidt** (Oregon State Univ.)
Q&A: Farey Map, Tent Map, Minkowski ? function and Kneading
sequence

15:00-17:30 **Yasushi Nagai** (Keio Univ.)
Distribution of patches in tilings, tiling spaces and tiling dynamical
systems

Wednesday 22

9:00-11:10 **Valérie Berthé** (LIAFA, Paris 7 Univ.)
S-adic systems related to continued fractions

11:20-12:00 **Dong-Han Kim** (Dongguk Univ.)
Subball complexity and Sturmian colorings of regular trees

- 15:00-17:00 **Hisatoshi Yuasa** (Osaka Kyoiku Univ.)
Linearly recurrent sequences and S-adic sequences
- 17:15-18:00 **Fumiaki Sugisaki** (Kumamoto Univ.)
Necessary and Sufficient condition that Bratteli-Vershik adic system is uniquely ergodic

Thursday 23

- 9:00-12:15 **Brigitte Vallée** (GREYC, CNRS and University of Caen))
Lattice Reduction Algorithms: Euclid, Gauss, LLL; Description and Probabilistic Analysis
- 15:00-15:50 **De-Jun Feng** (Chinese Univ. Hongkong)
Hausdorff dimension of self-similar sets and measures
- 16:00-16:50 **Tomohiro Ooto** (Univ. Tsukuba)
Mahler's classification and a certain class of p -adic numbers
- 17:00-17:50 **Jun'ichi Tamura**
The Kolakoski word and transformations on the set of words over $\mathbb{Z}_{\geq 0}$

Friday 24

- 9:00-9:30 **Shin'ichi Yasutomi** (Toho Univ.)
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