Preface

This volume of RIMS Kokyuroku Bessatsu is a collection of research articles which are contributed by invited speakers of the RIMS conference

"String theory, integrable systems and representation theory"

held at Research Institute for Mathematical Sciences (RIMS) of Kyoto University, from July 30 through August 2, 2013.

The aim of the conference was to overview recent developments in integrable systems and representation theory, mostly in related to mathematical aspects of string theory. There are 19 lectures and 65 participants during the conference.

It is our regret that Professor Kentaro Nagao's talk was cancelled because of his illness, and he passed away soon after the conference in October 2013. We thank his family for the efforts in looking for any of his written documents to be published here, though it was useless. We would like to dedicate this volume to his memory with us.

There are six articles here in this volume. They all have been refereed and are in their final forms. We would like to express our sincere gratitude to all the participants, invited speakers as well as the RIMS secretaries, who have made the conference successful. Last but not the least, we are sorry that this volume has not appeared earlier because of our editorial laziness.

January 2017

Editors
Koji Hasegawa (Tohoku)
Yasuhiko Yamada (Kobe)

京都大学数理解析研究所共同利用研究集会

RIMS symposium

超弦理論・表現論・可積分系の数理

String Theory, Integrable Systems and Representation theory

2013 July 30 - Aug 2

Room 420, Research Institute for Mathematical Sciences, Kyoto University

https://sites.google.com/site/rims2013sir/home

PROGRAM

60min each	9:45-	11:00-	13:30-	14:45-	16:00-
Jul 30 Tue	Nakayashiki	Iwaki	Naito	Saito	Ip
31 Wed	(Nagao) *	Okado	Yanagida	Nakajima	Feigin
Aug 1 Thu	Nakanishi	Iijima	Shiraishi	Hashimoto	Tsuchiya
2 Fri	Wood	Oshima	Shigyo	Kato	—

* This talk has been cancelled.

July 30(Tue)

09:45-10:45 Atsushi Nakayashiki (Tsudajuku)
Riemann surfaces and Schur functions

11:00-12:00 Kohei Iwaki (RIMS)
Wall-crossing like aspects in the exact WKB analysis

13:30-14:30 Satoshi Naito (Tokyo Institute of Technology)
A new path model for extremal weight modules over quantum affine algebras

14:45-15:45 Yousuke Saito (Tohoku)
Elliptic Ding-Iohara algebra and commutative families of the elliptic Macdonald operator

16:00-17:00 Ivan Ip (IPMU)
The braiding structure for positive representations of split real quantum groups

July 31(Wed)

09:45-10:45 NO TALK (Professor Nagao's talk has been cancelled)

11:00-12:00 Masato Okado (Osaka City U.)
PBW bases and quantized algebra of functions

13:30-14:30 Shintarou Yanagida (RIMS)
Classical and quantum vertex algebras

14:45-15:45 Hiraku Nakajima (RIMS)
Integral forms of W-algebras

16:00-17:00 Boris Feigin (Landau)
Constructions of the coordinate rings of zastava-like manifolds

Aug 1(Thu)

09:45-10:45 Tomoki Nakanishi (Nagoya)
Diagrammatic description of c-vectors and d-vectors of cluster algebras of finite type

11:00-12:00 Kazuto Iijima (Suzuka Colledge of Technology)
On a higher level extension of Leclerc-Thibon product theorem in q-deformed Fock spaces

13:30-14:30 Jun-ichi Shiraishi (Tokyo)
Elliptic hypergeometric series, Ruijsenaars operator and Heine's transformation formula

14:45-15:45 Yoshitake Hashimoto (Tokyo City University)
Conformal field theory for C2-cofinite VOAs and fusion tensor products

16:00-17:00 Akihiro Tsuchiya (IPMU)
On the extended W-algebra of type sl(2) at positive rational level, I

Aug 2(Fri)

09:45-10:45 Simon Wood (IPMU)
On the extended W-algebra of type sl(2) at positive rational level, II

11:00-12:00 Kazuyuki Oshima (Aichi Institute of Technology)
Elliptic quantum algebra $U_{q,p}(B_N^{(1)})$ and vertex operators

13:30-14:30 Yoko Shigyo (Tsudajuku)
On addition formulae of KP, mKP and BKP hierarchies

14:45-15:45 Shu Kato (Kyoto)
An algebraic study of extension algebras

Organizers:

Tomoyuki Arakawa (RIMS)
Yasuhiko Yamada (Kobe University)
Koji Hasegawa (Tohoku University)

Abstracts of Talks

July 30(Tue)

Atsushi Nakayashiki (Tsudajuku)

Riemann surfaces and Schur functions

In this talk I will discuss the properties of Schur functions associated with partitions determined from a certain data on Riemann surfaces. We study the vanishing and non-vanishing properties of the derivatives of Schur functions. Next we show that those properties of Schur functions are lifted to the tau function of the KP-hierarchy. Finally we use the theta function expression of the tau function to derived the properties of the theta function. In this way a refined version of the Riemann's singularity theorem can be obtained. In 1999 Buchstaber-Enolski-Leykin have characterized the Schur function constructed from an (n, s) curve by the analogue of the Riemann's vanishing theorem. Their results and ours imply that Schur functions memorize well the properties of theta functions.

Kohei Iwaki (RIMS)

Wall-crossing like aspects in the exact WKB analysis

The exact WKB analysis, developed by Voros in 1980's, is a powerful tool for global analysis of ordinary differential equations. Using the method, the monodromy and Stokes data of second-oreder differential equations can be described explicitly in terms of "Voros coeficients". On the other hand, Kontsevich and Soibelman pointed out that, certain formulas concerning with the Voros coefficients closely resemble to their wall-crosiing formulas. More striking resemblances can be found in the work of Gaiotto, Moore and Neitzke for study of wall-crossing of BPS indices in 4d theory. In this talk we shall give an introduction to the exact WKB analysis, and explain the above points of similarity.

Satoshi Naito (Tokyo Institute of Technology)

A new path model for extremal weight modules over quantum affine algebras

In this talk, I explain a new path model for extremal weight modules over quantum affine algebras. This path model is given in terms of the generic Bruhat order, introduced by Lusztig in 1980, on the affine Weyl group, and is valid for extremal weight modules of arbitrary level-zero dominant integral extremal weight. Here I should mention that our previous path model, which is given in terms of Littelmann's level-zero weight poset, is valid only for extremal weight modules whose extremal weights are positive integer multiples of level-zero fundamental weights. Thus, the result of this talk generalizes our previous result very much.

This talk is based on joint work with D. Sagaki and M. Ishii.

Yousuke Saito (Tohoku)

Elliptic Ding-Iohara algebra and commutative families of the elliptic Macdonald operator

The Ding-Iohara algebra is a quantum group arising from the free field realization of the Macdonald operator. In 2009, Feigin, Hashizume, Hoshino, Shiraishi, and Yanagida

constructed two families of commuting operators which contain the Macdonald operator by using the Ding-Iohara algebra and the trigonometric Feigin-Odesskii algebra. On the other hand, in 2013 I obtained an elliptic analog of the Ding-Iohara algebra and the free field realization of the elliptic Macdonald operator. In this talk, I will show that from the elliptic Ding-Iohara algebra and the elliptic Feigin-Odesskii algebra, we can construct commutative families of the elliptic Macdonald operator.

Ivan Ip (IPMU)

The braiding structure for positive representations of split real quantum groups

Positive representations are introduced recently in a joint work with I. Frenkel to study split real quantum groups which seems to share many properties in parallel to the finite dimensional representation theory of compact quantum groups. In this talk, we construct the universal R operator for these representations that gives the braiding structure, generalizing the formula of compact quantum groups $U_q(g)$ by Kirillov-Reshetikhin and the formula in the case of $U_q(sl(2,R))$ by Faddeev and Teschner. The quantum Weyl element and Lusztig's isomorphism in the positive setting will be used in the construction. We also introduce a C^*-algebraic version of the split real quantum group in the language of multiplier Hopf algebras, so that the R operator can be realized as the canonical element of certain Drinfeld's Double construction.

July 31(Wed)

Kentaro Nagao (Nagoya)

cancelled

Masato Okado (Osaka City U.)

PBW bases and quantized algebra of functions

For a finite-dimensional simple Lie algebra g, let $U_q^+(g)$ be the positive part of the quantized universal enveloping algebra, and $A_q(g)$ be the quantized algebra of functions. We show that the transition matrix of the PBW bases of $U_q^+(g)$ coincides with the intertwiner between the irreducible $A_q(g)$-modules labeled by two different reduced expressions of the longest element of the Weyl group of g. This generalizes the earlier result by Sergeev on A_2 related to the tetrahedron equation and endows a new representation theoretical interpretation with the recent solution to the 3D reflection equation for C_2. Our proof is based on a realization of $U_q^+(g)$ in a quotient ring of $A_q(g)$.

This is a joint work with Atsuo Kuniba and Yasuhiko Yamada.

Shintarou Yanagida (RIMS)

Classical and quantum vertex algebras

In the last 30 years, vertex algebra and its cousins have been introduced by several people in various frameworks, and the whole picture of those theories look quite complicated. The (A, H, S)-vertex algebra and its quantum version, introduced in 1998 by Borcherds for the purpose of simplification of the complexity, seem to give a unified way of understanding various notions of (quantum or deformed) vertex algebras. In this talk I will give several

remarks on Borcherds' framework. One of them is an observation of the relationship between the framework of (A, H, S)-vertex algebras and the deformation theory of algebraic varieties.

Hiraku Nakajima (RIMS)

Integral forms of W-algebras

During the study of the AGT conjecture, I am lead to introduce an integral form of the W-algebra, defined over $\mathbb{Q}[\varepsilon_1, \varepsilon_2]$, where $k+h^\vee = -\varepsilon_2/\varepsilon_1$. The definition does not involve the AGT, and is based on the BRST cohomological definition of the W-algebra. I will explain the construction, and the specialization at $\varepsilon_1 = 0$.

16:00-17:00 Boris Feigin (Landau)

Troidal algebras for $\mathfrak{gl}_n$, moduli spaces, decomposition of $\widehat{\mathfrak{gl}}_n$- modules (title changed)

Aug 1(Thu)

Tomoki Nakanishi (Nagoya)

Diagrammatic description of c-vectors and d-vectors of cluster algebras of finite type

We provide an explicit Dynkin diagrammatic description of the c-vectors and the d-vectors (the denominator vectors) of any cluster algebra of finite type with principal coefficients and any initial exchange matrix. We use the surface realization of cluster algebras for types A_n and D_n, then we apply the folding method to D_{n+1} and A_{2n-1} to obtain types B_n and C_n. Exceptional types are done by direct inspection with the help of a computer algebra software. This is a joint work with Salvatore Stella.

Kazuto Iijima (Suzuka Colledge of Technology)

On a higher level extension of Leclerc-Thibon product theorem in q-deformed Fock spaces

The q-deformed Fock spaces of higher levels were introduced by Jimbo-Misra-Miwa-Okado. Uglov defined a canonical bases in q-deformed Fock spaces of higher levels. Leclerc-Thibon showed a product theorem in q-deformed Fock spaces of level one. The product theorem is regarded as a formal q-analogue of the tensor product theorem of level one. In this talk, we show a higher level analogue of Leclerc-Thibon product theorem under a suitable multi charge condition.

Jun-ichi Shiraishi (Tokyo)

Elliptic hypergeometric series, Ruijsenaars operator and Heine's transformation formula

Yoshitake Hashimoto (Tokyo City University)

Conformal field theory for C_2-cofinite VOAs and fusion tensor products

Akihiro Tsuchiya (IPMU)

On the extended W-algebra of type sl(2) at positive rational level

Aug 2(Fri)

Simon Wood (IPMU)

Coset constructions in log CFT

One of the best understood families of logarithmic conformal field theories is that consisting of the $(1,p)$ models $(p = 2, 3, ...)$ of central charge $c_{1,p} = 1 - 6(p-1)^2/p$. This family includes the theories corresponding to the singlet algebras $M(p)$ and the triplet algebras $W(p)$. The $W^(2)_n$ algebra of level k was introduced by Feigin and Semikhatov as a (conjectured) quantum hamiltonian reduction of affine $sl(n)_k$, generalising the Bershadsky-Polyakov algebra $W^(2)_3$. It is possible to construct certain vertex algebras B_p as subalgebras of the kernel of certain screening charges acting on a rank 2 lattice vertex algebra of indefinite signature. For $p \leq 5$, the algebra B_p is a homomorphic image of $W^(2)_{p-1}$ at level $-(p-1)^2/p$ and there are strong reasons to believe that that is true for $p > 5$ as well. The triplet algebra $W(p)$ can then be realised as a coset inside the full kernel of the screening operator, while the singlet algebra $M(p)$ is similarly realised inside B_p.

Kazuyuki Oshima (Aichi Institute of Technology)

Elliptic quantum algebra $U_{q,p}(B_N^{(1)})$ and vertex operators

We introduce an elliptic quantum algebra of type $B_N^{(1)}$. By using the elliptic analogue of the Drinfeld currents, we construct the L-operator, which satisfies the dynamical RLL relation characterizing the elliptic quantum algebra $U_{q,p}(B_N^{(1)})$. We then derive the two types of vertex operators of arbitrary level $U_{q,p}(B_N^{(1)})$-modules. As an example, we give a level one free field representation and show the commutation relations which the vertex operators satisfy.

This is a joint work with Hitoshi Konno.

Yoko Shigyo (Tsudajuku)

On addition formulae of KP, mKP and BKP hierarchies

The KP hierarchy is formulated as an infinite system of bilinear equations for one unknown function, the tau function. The addition formulae of the KP hierarchy had been derived by Sato. It is surprising that the KP hierarchy is equivalent to the simplest equation of the addition formulae. This result had been proved by Takasaki and Takebe by way of the wave function. I give an alternative and direct proof of this result for the KP hierarchy. For the modified KP (mKP) and the BKP hierarchies, similar results hold. The method of the proof in the case of the KP hierarchy can be applied to the mKP and BKP hierarchies. In the case of the mKP hierarchy the result had been proved by Noumi and Takebe. In this talk I will mainly explain about mKP hierarchy.

Shu Kato (Kyoto)

An algebraic study of extention algebras

Note: Affiliations as of 2013.

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