

On the book *Shu* housed at Yuelu Academy†

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Joint project with

“The research group for ancient Chinese mathematics” in Japan

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Abstract

The book *Shu* is one of the books of Qin bamboo slips purchased by the Yuelu Academy in December 2007 (see [4] and [5]), and the Academy published a report on the *Shu* in 2011 with photographs of the slips (see [1]). We are investigating the *Shu* based on these photographs (see [6],[7], and [8]) in the same manner as our previous work on the *Suanshu-shu* (see [2]). In this paper, we discuss one issue regarding the order of the slips, and two mathematical problems in the *Shu* comparing them with those in the *Suanshu-shu*.

1. Order of slips

The first slip determined by the Yuelu Academy is the final one of the book *Shu*.

2. A problem similar to the *Quxicheng* (slips No. 32 to 34)

We compared this problem in the *Shu* with the *Quxicheng* in the *Suanshu-shu* and obtained a complete interpretation.

3. *Yufang* (slips No. 67 and 68)

We propose an alternate method of division which is appropriate for the use of the word “*chuiyu zhi zong*.”

1. Order of slips

The Yuelu Academy purchased a set of books written in the *Qin* dynasty in the form of eight large mud blocks. The book *Shu* is one of these books, whose slips were dispersed in seven blocks. According to the Yuelu Academy, the first slip is the following:

(1背) 數 0956

(1 back) *Shu*

(1 正) 爲實、以所得禾斤數爲法、如法一步。 0956

(1 front) ... is the dividend, the weight of grain in *jin* is the divisor, and dividing gives the answer in (square) *bu*.

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In the text, the number in parentheses on the top indicates where the slip falls in the order determined by the Yuelu Academy, and the number on the right end is the initial number of the slip, indicating where the slip was in a block of mud.

The condition of the *Shu* is quite different from that of the *Suanshu-shu*, which was embedded almost sequentially in one mud block. It is therefore difficult to determine the correct order of the *Shu*. Although the Yuelu Academy classified and sorted the slips of the *Shu* according to their contents, this is only tentative.

Similar to the *Suanshu-shu*, the *Shu* has only one slip with letters on the back, so there is no doubt that *Shu* is the title of this mathematical book. In general, Chinese text was written vertically, the slips correspond to rows, and they proceed from right to left. Since the text on the front side of this slip is the end part of a certain mathematical problem, we may say that this slip is the final one of the book *Shu*. Moreover, the phrase “the weight of grain in *jin*” (禾斤數) indicates that the theme of the final problem seems to be the calculation of grain tax or conversion between grains.

2. A problem similar to the *Quxicheng* (slips No. 32 to 34)

In slips No. 32 to 34, hemp taxes are calculated. The text in [1] and its translation are as follows:

(32) 梟兌 (稅) 田十六步、大梟高五尺。三步一束。租八斤五兩八銖。今復租之、三步廿八寸當三步又百九十六分步 0841

(33) 之八十七而一束。租七斤四兩三束【銖】九分銖五。求此之術曰、置一束寸數、藉令相乘也、以一束步數乘之以爲實。 0805

(34) 亦置所新得寸數、藉令相乘也、以爲法。實如法得一兩... 0824

There is taxable farmland of hemp of 16 (square) *bu*, and the harvest is large hemp of 5 *chi* in height. If every 3 (square) *bu* is taxed at 1 bundle, then the total land tax is 8 *jin* 5 *liang* 8 *zhu*. Now taxing it again, “3 *bu* 28 *cun*” corresponds to the contract that each

$3\frac{87}{196}$ (square) *bu* is taxed at 1 bundle, thus the total land tax becomes 7 *jin* 4 *liang*

$3\frac{5}{9}$ *zhu*. The “method for calculating this” states: the length in *cun* at one bundle

multiplied by itself, multiplied by the number of (square) *bu* to produce 1 bundle is the dividend; the new length in *cun* multiplied by itself is the divisor; dividing the dividend by the divisor yields ... of 1 *liang*.

The first task in this problem is the calculation of the land tax for farmland of hemp of 16 square *bu*. As described in slip No. 16, “(to convert to *liang* in weight,) multiply by 5 for thick hemp, multiply by 6 for hemp of middle thickness, multiply by 7 for thin hemp,” and the harvest in this problem is thick hemp, thus the number of bundles

multiplied by height is multiplied by 5 to obtain the number of *liang* in weight. Therefore, the total amount of land tax is $(16 \div 3)(\text{number of bundles}) \times 5(\text{chi}) \times 5 = \frac{400}{3}(\text{liang}) = 8(\text{jin}) 5(\text{liang}) 8(\text{zhu})$ in weight.

But the phrase "3 bu 28 cun " was not comprehensible in [1]. The Yuelu Academy assumed that this phrase was redundant. If the contract is changed as in the following phrase, the recalculation of tax is correct, that is, if every $3\frac{87}{196}$ (square) *bu* is taxed at 1

bundle, the total amount of land tax is $\left(16 \div 3\frac{87}{196}\right)(\text{number of bundles}) \times 5(\text{chi}) \times 5 =$

$\frac{78400}{675}(\text{liang}) = 7(\text{jin}) 4(\text{liang}) 3\frac{5}{9}(\text{zhu})$ in weight. However, the following sentences

regarding the method were also incomprehensible. In this context, the tax is calculated via *liang* in weight, and that is why the final, undecipherable character was assumed to be "*liang*."

We compared this problem with the *Quxicheng* in the *Suanshu-shu* and obtained a complete interpretation. The text and its translation in [3] are as follows:

取梟程 取梟程十步三圍束一、今乾之廿八寸、問幾何步一束。術曰、乾自乘為法、生自乘又以生一束步數乘之為實、實如法得十一步又九十八分步四十七而一束。

The norm for obtaining hemp is that 10 (square) *bu* of land yield 1 bundle 3 *wei* = 30 *cun* in circumference. Drying it then yields a bundle 28 *cun* in circumference. The question is how many (square) *bu* will yield 1 bundle [of dried hemp 30 *cun* in circumference]? The method states: the (amount of) dried (hemp) multiplied by itself is the divisor; the (amount of) fresh (hemp) multiplied by itself multiplied by the number of (square) *bu* to produce 1 bundle of fresh hemp is the dividend; dividing the dividend by the divisor yields $11\frac{47}{98}$ (square) *bu* to produce 1 bundle [of dried hemp].

The purpose of the *Quxicheng* in the *Suanshu-shu* is to calculate the area of farmland, say x square *bu*, that will yield 1 bundle of dried hemp 30 *cun* in circumference. The area of farmland has a constant ratio to that of a cross section of a bundle of dried hemp, that is, $10 : 28^2 = x : 30^2$ (see Figure 1), thus people in ancient China calculated

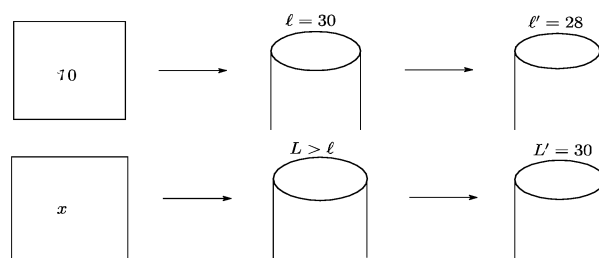


Figure 1

$$x = (30^2 \times 10) \div 28^2 = \frac{1125}{98} = 11 \frac{47}{98} \text{ square } bu.$$

In this problem in the *Shu*, the assumption that taxing 1 bundle of dried hemp is 30 *cun* in circumference is hidden. Our revised text and its translation are as follows:

(32) 梟兌 (稅) 田十六步、大梟高五尺。三步一束。租八斤五兩八銖。今 [乾之廿八寸一束]。復租之、三步廿八寸當三步又百九十六分步 0841

(33) 之八十七而一束。租七斤四兩三束【銖】九分銖五。求此之術曰、置一束寸數、藉令相乘也、以一束步數乘之以爲實 0805

(34) 亦置所新得寸數、藉令相乘也、以爲法。實如法得一 步 . . . 0824

There is taxable farmland of hemp of 16 (square) *bu*, and the harvest is large hemp of 5 *chi* in height. If each 3 (square) *bu* is taxed at 1 bundle (of 30 *cun* in circumference), then the total land tax is 8 *jin* 5 *liang* 8 *zhu*. Bundles of hemp shrink to 28 *cun* in circumference after drying. We therefore tax the farmland again (to obtain bundles 30 *cun* in circumference). Since each 3 (square) *bu* is taxed at 1 bundle of dried hemp 28 *cun* in circumference, the ratio corresponds to the ratio that each $3 \frac{87}{196}$ (square) *bu* is taxed at

1 bundle 30 *cun* in circumference, thus the total land tax becomes 7 *jin* 4 *liang* $3 \frac{5}{9}$ *zhu*.

The method to calculate the latter ratio states: the length in *cun* of a fresh bundle multiplied by itself, multiplied by the number of (square) *bu* to produce 1 bundle is the dividend; the new length in *cun* of a dried bundle multiplied by itself is the divisor; dividing the dividend by the divisor yields ... of 1 *bu*.

In the latter half of this problem, the calculation of the area of farmland which yields one bundle of dried hemp 30 *cun* in circumference is described as in the *Quxicheng* of the *Suanshu-shu*. Since the area has a constant ratio to that of the a cross section of a bundle of

$$\text{dried hemp, } 3 : 28^2 = y : 30^2$$

(see Figure 2), thus we have

$$y = (30^2 \times 3) \div 28^2 = \frac{675}{196} = 3 \frac{87}{196}$$

square *bu*, which is the same as described in the method. Moreover, since the method explains how to calculate not the amount of tax, but the area of farmland, the final character is not “*liang*,” but (square) “*bu*.”

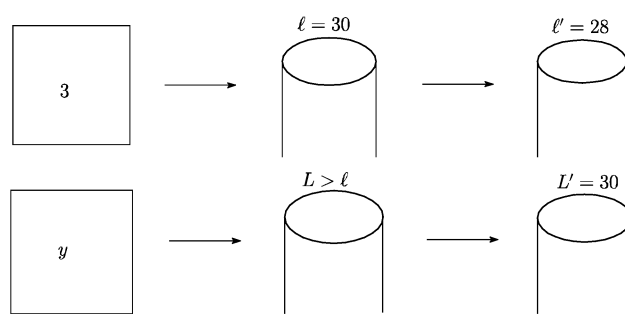


Figure 2

3. *Yufang* (slips No. 67 and 68)

In this problem, a square parcel of land is divided equally between three people, with the exclusion of a small path that crosses the land. The text in [1] and its translation are as follows:

(六七)宇方百步、三人居之。巷廣五步、問宇幾何。其術曰、除巷五步、餘九十五步。以三人乘之、以爲法。以百乘九十 0884

(六八)五步者、令如法一步、即墜宇之縱也。 0825

Let *yu* (a housing site) be a square parcel of land with each side 100 *bu* in length. There are 3 people and they divide the housing site equally, with the exclusion of a small path with a width of 5 *bu* that crosses the site. The question is: how large is each person's portion? The method states: subtract the 5 *bu* width of the path (from the side of 100 *bu*) leaving a remainder of 95 *bu*. "This" multiplied by 3 persons is the divisor. Dividing the dividend by the divisor yields the length in *bu*, that is, the vertical [sic] length of each parcel of land.

The word *yu* refers to a housing site, and each side is 100 *bu* = 6000 *cun* \doteq 147m. There are no answers providing the size of a portion in this problem. The method explains how to calculate "*chuiyu zhi zong*," but the meaning of "*chuiyu*" is not clear. Note that "*chui*" means a "margin" or "border."

In [1], the division was interpreted as in Figure 3, and "*chuiyu zhi zong*" is the side of each person's portion of land that adjoins the path. The calculation was as follows: after removing the path, the area of the resulting site is 100 times 95 square *bu*, thus the area of each person's portion of land is $\frac{100 \times 95}{3}$ square *bu*. Divided by the side $(100 - 5) = 95$ *bu* perpendicular to the path, we have the length of the side of each person's portion of land that adjoins the path. The calculation is

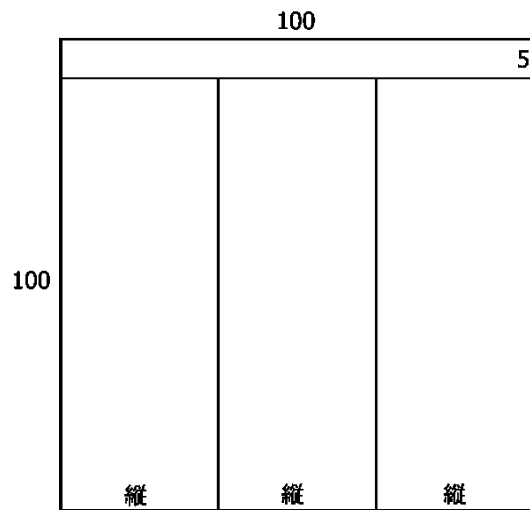


Figure 3

$$(100 \times 95) \div ((100 - 5) \times 3) = \frac{100}{3} = 33\frac{1}{3}.$$

We questioned this interpretation from the following viewpoints.

1. Why not simply divide the side of 100 *bu* along the path of the resulting site by 3, that

$$\text{is, } 100 \div 3 = \frac{100}{3} = 33\frac{1}{3}?$$

2. In the *Suanshu-shu*, the word “*zong*” refers primarily to the vertical side of a rectangle. In this problem, it is natural to suppose that the author is standing on the path. We wonder if the answer should be expressed using “*quang*” (廣), which refers primarily to the horizontal side of a rectangle.

In this paper, we propose an alternate method of division as in Figure 4: though the areas of the three portions are the same, their shapes are not the same. The path crosses through the middle of the land, and after excluding the path, the separated portion on top is called “*chuiyu*.”

We interpret “*chuiyu zhi zong*” to mean the side of “*chuiyu*” perpendicular to the path, so the use of the word “*zong*” here is appropriate. Since this portion is separated from the other two, the use of the word “*chui*” here is also appropriate. Note that, in order to adopt this interpretation, we must assume that the phrase “the side 100 *bu*” has been omitted before “this.”

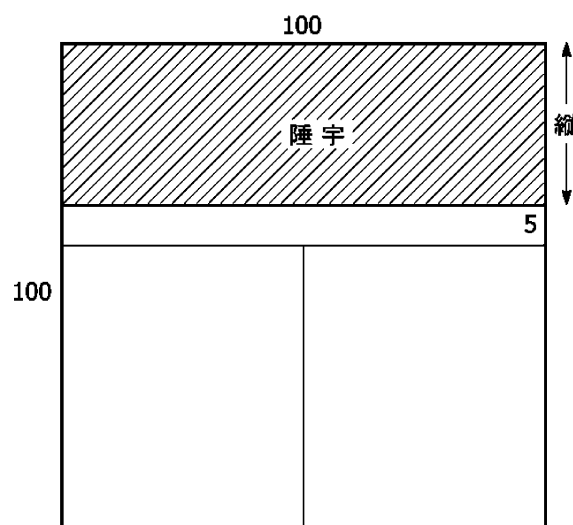


Figure 4

We therefore propose the following revised text and translation:

(六七) 宇方百步、三人居之。巷廣五步、問宇幾何。其術曰、除巷五步、餘九十五步。【方百步】以三人乘之、以爲法。以百乘九十 0884
 (六八) 五步者、令如法一步、即睡宇之縱也。 0825

Let *yu* (a housing site) be a square with each side 100 *bu* in length. There are 3 people and they divide the housing site equally, with the exclusion of a small path with a width of 5 *bu* that crosses the site. The question is: how large is each person’s portion? The method states: subtract the 5 *bu* width of the path (from the side of 100 *bu*) leaving a remainder of 95 *bu*. The side of 100 *bu* multiplied by 3 persons is the divisor. Dividing the dividend by the divisor yields the length in *bu*, that is, the vertical length of the portion separated by the path from the other two portions.

In our proposal, the method is as follows: after removing the path, the area of the resulting site is 100 times 95 square *bu* as in [1]. Since the areas of the three portions are the same, one third of the resulting area is the area of the separated portion. Therefore, the latter area divided again by the side along the path is the other side, that

is “*chuiyu zhi zong*.” The calculation is $(100 \times 95) \div (100 \times 3) = \frac{95}{3} = 31\frac{2}{3}$.

References

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中文要旨

一、关于排列顺序

在《岳麓书院藏秦简(贰)》(即《数》书)里,整理者把正面写有“为实,以所得禾斤数为法,如法一步”,背面写有“数”的简作为开头的第一简。虽然这个“数”字毫无疑问是这批简的书名,但我们不能同意把这支简作为开头的第一简。试述理由如下:

- 1) 因为第一简正面的内容是某一算题的结尾,所以在这支简之前应该还有其他的简。所以这支简不应该是第一支简。
- 2) 张家山汉简《算数书》的书名是写在《相乘》题最后的第六简背面的。根据“出土示意图”可知,《算数书》是从《少广》题开始,以《相乘》题为结尾,并在结尾算题的最后一简背面写上书名的。《数》应该也是一样的。从“禾斤数”一句来看,该简的内容或属于“税田”,或属于“谷物换算”一类的算题。而《数》的结尾部分正是这类算题。我们认为这批简的书名是写在最后一简背面的。

二、阐明(三二+三三+三四)简

对于三二+三三+三四简,整理者作了以下释文:

泉兑(税)田十六步、大泉高五尺。三步一束。租八斤五两八铢。今复租之、三步廿八寸当三步又百九十六分步 0841
之八十七而一束。租七斤四两三束【铢】九分铢五。求此之術曰、置一束寸數、藉令相乘也、以一束步數乘之以爲實。 0805
亦置所新得寸數、藉令相乘也、以爲法。實如法得一[兩]· · · 0824

算题开头部分的“泉兑(税)田十六步、大泉高五尺。三步一束。租八斤五两八铢”的意思是:16平方步的税田里有高5尺的大泉,如果每3平方步课税1束的话,问税的重量是多少?(一六)简里有“大泉五之,中泉六之,细泉七之”的句子,是说把大泉的束数换成重量两时,其换算计数是5。因而计算税重量的公式为:

$$(16 \div 3) (\text{束}) \times 5 (\text{尺}) \times 5 = \frac{400}{3} (\text{两}) = 8 (\text{斤}) 5 (\text{两}) 8 (\text{铢})$$

计算结果与“租八斤五两八铢”一致。开头部分的文章是没有问题的。令人费解的是下面“三步廿八寸当三步又百九十六分步之八十七而一束”之中的“三步廿八寸当”六个字。“三步又百九十六分步之八十七而一束”的意思是说:每 $3\frac{87}{196}$ 平方步课税1束时,求其税重量的计算是:

$$\left(16 \div 3\frac{87}{196}\right) (\text{束}) \times 5 (\text{尺}) \times 5 = \frac{78400}{675} (\text{两}) = 7 (\text{斤}) 4 (\text{两}) 3\frac{5}{9} (\text{铢})$$

计算结果与“租七斤四兩三束【銖】九分銖五”一致。可是，对于“三步廿八寸當”六字，整理者注释说：“‘三步廿八寸當’似为衍文，或者‘三步廿八寸當’处有脱文。”这个注释表明整理者无法解释这六个字。以致整理者未能阐明“求此之術曰”以下的內容。

《算数书》里有与本算题类似的内容,即【三四】「取泉程」题:

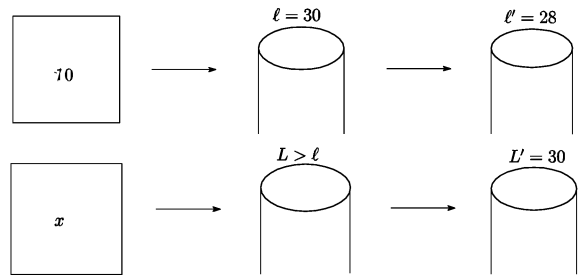
取泉程 十步三圍束一、今乾之廿八寸、問幾何步一束。術曰、乾自乘為法、生自乘又以生一束步數乘之為實、實如法得十一步又九十八分步四十七而一束。

取泉程的意思是:每 10 平方步得 3 围 (30 寸) 之泉 1 束。现在, 使其 (周长 3 围之新鲜泉 1 束) 干燥为 28 寸。问: 几平方步可得 3 围之干燥泉 1 束?

其计算是根据圆柱体断面面积与圆周自乘

$10 : 28^2 = x : 30^2$ 的比例关系, 求得

$$x = (30^2 \times 10) \div 28^2 = \frac{1125}{98} = 11 \frac{47}{98}$$



本算题也跟「取泉程」题一样,问当周长 3 围(30 寸)的新鲜泉干燥而减为 28 寸时,为了得到周长 3 围(30 寸)的干燥泉,需要几平方步? 也就是说,在本算题里“今”字后面省略了“乾之廿八寸”一句。“三步廿八寸当三步又百九十六分步之八十七而一束”的意思是:每 3 步得周长

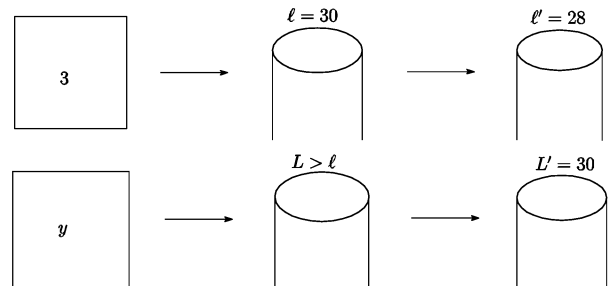
28 寸的泉相当于每 $3 \frac{87}{196}$ 步得(周长 30 寸)1 束的泉。

计算方法为根据 1 束的步数与干燥泉的断面面积之间 $3 : 28^2 = y : 30^2$ 的比例关系, 求得

$$y = (30^2 \times 3) \div 28^2 = \frac{675}{196} = 3 \frac{87}{196}$$

“求此之術”以下的內容是说:“置一束寸数 (30 寸),此数自乘之后乘以一束的步数(3 步)为

实。置所新得的寸数(28 寸),使这个数自乘为法”。这与上面的计算是完全一致的。其算法是求 y, 而不是直接求税重量的。因而(三四)简文“实如法得一”后面的一字并不是“两”字,而是“步”字。



三, 关于「宇方」题

「宇方」题如下:

(六七)宇方百步、三人居之。巷廣五步、問宇幾何。其術曰、除巷五步、餘九十五步。以三人乘之、以為法。以百乘九十

0884

(六八)五步者、令如法一步、即陞宇之縱也。

0825

算题中的“宇”指宅地,“巷”指小路。我们同意整理者的看法,认为“陲宇”的“陲”是边缘的意思。

本算题没有答案,只有计算法。算题的内容是:

从一边 100 步的正方形土地先消除宽幅 5 步的小路,然后把剩下的土地平分给 3 个人时,求“陲宇之纵”的。为了解答这个算题,最重要的是如何解释“陲宇之纵”。

整理者的解释如下:

把土地用向小路垂直的线平分成 3 个,这样划分时,“陲宇之纵”相当于面临小路的一边。解法是把消除小路面积的剩余土地面积除以 3,然后继续除以已知一边。计算式是:

$$(100 \times 95) \div ((100 - 5) \times 3) = \frac{100}{3} = 33\frac{1}{3}$$

我们不能同意这种看法。理由有二:

第一,为什么不直接将面临小路的一边 100 步平分成 3?

第二,这样求得的一边,从置身于小路的人来说,应该称“广”,不能称“纵”。

关于平分土地的办法,我们认为,虽然划分给 3 人的土地面积是相同的,但其形状却是不一样的。参考下图:

如此解释的话,“陲宇”相当于沿着小路的土地,即“陲宇”是与“宇”共边的矩形。“陲宇之纵”表示与“宇”有一条共边的矩形之纵。位于小路看此矩形时,可称矩形的短一边为“纵”。如此,算题中的“以三人乘之”的“之”不是指着“餘九十五步”,而是指“宇方百步”。计算方法是:先把消除小路面积的剩余面积平分成 3 个,然后把一个除以 100 步,就能求得解答。计算公式即:

$$(100 \times 95) \div (100 \times 3) = \frac{95}{3} = 31\frac{2}{3}$$

