Towards a nonlinear Schwarz list

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I will describe what is known of the (non-Riccati) algebraic solutions of the sixth Painleve equation. This is the explicit form of the simplest non-abelian Gauss-Manin connection, so its algebraic solutions should constitute the nonlinear analogue of Schwarz's list of algebraic solutions of the Gauss hypergeometric equation, which is the explicit form of the simplest abelian *Gauss*-Manin connection.

Particular emphasis will be placed on:

a) the role of Okamoto symmetries (which relate many solutions sometimes thought to be "new")

b) the fact the almost all solutions discovered so far have finite linear monodromy group (in some representative of the deformation/Okamoto equivalence class)

c) that the resulting list is quite simple, namely: 4 continuous families, 1 discrete family plus a finite number of exceptional solutions

d) methods used to construct the solutions

The hope is that the current list comprises all algebraic solutions (although this is only conjecture) but in any case it should facilitate the recognition of genuinely new solutions if they arise in the future.