Isomonodromic deformation of Lamé connections

Frank Loray
CNRS – Univ. de Rennes

Isomonodromic deformations of rank 2 logarithmic connections with singular points 0, 1, t and ∞ over the Riemann sphere are parametrized by the solutions $q(t)$ of Painlevé VI equation. Some discrete groups of symmetries of P_{VI} equation naturally arise from the birational geometry of logarithmic connections. An extra symmetry was found by K. Okamoto by direct computations. Here, we provide a geometric interpretation of this symmetry. After lifting conveniently the connection over the elliptic curve $E_t : \{y^2 = x(x-1)(x-t)\}$, the variation of the underlying vector bundle (along isomonodromic deformation) provides a new solution $\tilde{q}(t)$ of P_{VI} equation, namely the Okamoto symmetric of $q(t)$. In particular, isomonodromic deformations of the Lamé connection over E_t arise as a particular case of our construction and we recover in a more natural way recent results of S. Kawai and Levin-Olshanetsky.