

Workshop “ p -adic Arithmetic Geometry”

Abstracts

John Coates, Ramdorai Sujatha

Title: Root numbers and Iwasawa theory, I, II

Abstract: One interesting aspect of non-commutative Iwasawa theory is that non-abelian p -adic Lie extensions of number fields tend to have infinite families of self-dual irreducible Artin representations. If one fixes an elliptic curve defined over the number field and twists its complex L -function by these self-dual Artin representations, one obtains in many cases “evident” zeroes of odd multiplicity at $s = 1$, again usually occurring in infinite families. Thanks to the conjecture of Birch and Swinnerton-Dyer, these zeroes predict the existence of many infinitely divisible elements in the Selmer group of the curve. Our lectures will report on joint work with T. Fukaya and K. Kato which studies this phenomena for false Tate curve extensions and GL_2 -extensions, and proves in some cases the desired connexion between these evident zeroes and the the \mathbf{Z}_p -coranks of Selmer groups. We also relate in some cases the behaviour of root numbers to the ranks of the duals of Selmer groups over certain Iwasawa algebras. Our approach is global in nature, and was partly inspired by calculations of root numbers due to V. Dokchitser and D. Rohrlich. One of the big mysteries at present (about which we can unfortunately say nothing) is to find a p -adic approach to local root numbers. There is considerable overlap between our results and some current work of Rubin and Mazur, who use completely different methods.

Otmar Venjakob

Title: On the leading terms of p -adic L -functions in non-commutative Iwasawa theory

Abstract: The p -adic L -functions of Fukaya and Kato satisfy an interpolation formula related to certain special values of complex L -functions. In this talk we firstly describe how to define the leading terms of these ‘non-commutative’ p -adic L -functions and secondly we prove an interpolation formula for the leading terms of the p -adic L -functions that are associated to certain critical motives now involving the leading terms of the complex L -functions. Finally we discuss conditions under which the GL_2 -Iwasawa Main Conjecture implies the Equivariant Tamagawa Number Conjecture by Burns and Flach. This talk reports on joint work with David Burns.

Jacques Tilouine

Title: On a conjecture of H. Yoshida

Abstract: By p -adic approximations, we construct p -adic genus two Siegel modular forms associated to certain abelian surfaces defined over \mathbf{Q} . Guided by the Fontaine-Mazur conjecture, and a conjecture of H. Yoshida (in 1980), we try to address (in a work in progress) the question of the classicality of the p -adic modular forms so constructed.

Jean-Marc Fontaine

Title: Locally constant φ -modules and p -torsion sheaves

Abstract: I shall discuss a part of a joint program with Uwe Jannsen for constructing and studying a wide class of p -torsion sheaves (for the syntomic topology) in characteristic $p > 0$ and in mixed characteristic (containing the finite and flat commutative group schemes as well as the finite sub-quotient of crystalline representations).

Go Yamashita

Title: p -adic Hodge theory for open varieties

Abstract: We prove semistable conjecture of Fontaine-Jannsen for open varieties (proper semistable model minus relative normal crossing divisor) by Fontaine-Messing-Kato-Tsuji's syntomic cohomology method (we do not use Faltings' almost étale theory). From this, we can derive de Rham conjecture for open varieties and potentially semistable conjecture.

In the proof, we introduce a new method of "hollow log schemes", which are some kinds of tubular neighbourhood of intersections of normal crossing divisors.

I want to talk about an application to p -adic multiple zeta values, if the time admits.

Pierre Colmez

Title: On the p -adic local Langlands correspondence for $GL_2(\mathbf{Q}_p)$

Abstract: I will report on recent developpements towards a p -adic local Langlands correspondence for $GL_2(\mathbf{Q}_p)$.

Fabrizio Andreatta

Title: Relative (φ, Γ) -modules and applications

Abstract: I will define relative (φ, Γ) -modules for Tate affinoid algebras generalizing Fontaine's construction for local fields. I will then discuss joint works with O. Brinon and with A. Iovita providing applications to the computation of étale cohomology (both local and global).

Shinichi Mochizuki

Title: A Survey of Absolute p -adic Anabelian Geometry

Abstract: The absolute Galois groups of nonarchimedean local fields differ from those of number fields in that they admit automorphisms that do not arise from automorphisms of fields. As a result of this difference, unlike the case with number fields, there is a substantial gap between the [already well understood] relative and [not so well understood] absolute versions of anabelian geometry for hyperbolic curves over nonarchimedean local fields. The original motivation, from the point of view of the lecturer, for studying the absolute anabelian geometry of hyperbolic curves over nonarchimedean local fields came from the lecturer's work on the ABC Conjecture. After briefly discussing these motivating aspects of the theory, it is my intention to survey various related results obtained during the last six years by A. Tamagawa and myself. Among the topics I intend to discuss in this survey are the following:

- (1) canonical curves from p -adic Teichmüller theory;
- (2) hyperbolic curves that may be related to Belyi maps;
- (3) configuration spaces associated to hyperbolic curves;
- (4) aspects of the theory that are related to p -adic Hodge theory, and, in particular, the lecturer's previous work on automorphisms of Galois groups of nonarchimedean local fields that preserve the higher ramification filtration.

Pierre Berthelot

Title: On Witt vector cohomology for singular varieties

Abstract: For separated schemes of finite type over a perfect field of characteristic p , the construction of Witt vector cohomology introduced by Serre can be extended so as to yield a cohomology with compact supports. In this context, it is then possible to generalize the classical isomorphism between the slope < 1 part of crystalline cohomology and Witt vector cohomology (for proper and smooth varieties) as a similar isomorphism involving the slope < 1 part of rigid cohomology. Applications will be given to congruences mod q on the number of rational points of certain algebraic varieties over a finite field with q elements.

Takeshi Tsuji

Title: On log crystalline cohomology and arithmetic D -modules

Abstract:

Atsushi Shiho

Title: Relative log convergent cohomology and relative rigid cohomology

Abstract: We introduce a relative version of log convergent cohomology. We prove a comparison theorem with log crystalline cohomology and we relate relative log convergent cohomology to relative rigid cohomology. As a consequence, we prove the coherence and the overconvergence of relative rigid cohomology in the case where a given morphism admits a nice log smooth compactification to which the coefficient extends logarithmically.

Nobuo Tsuzuki

Title: On the log-growth and Frobenius slopes of F -isocrystals on a curve

Abstract: The log-growth of p -adic differential equations were introduced by Dwork, Robba and Christol. Dwork conjectured the behavior of log-growth is similar to that of Frobenius slopes. We give a comparison between the log-growth filtration and the slope filtration at both generic and special points for F -isocrystals of rank 2 on a curve. In this case Dwork's conjecture holds. This is a joint work with Bruno Chiarellotto.