CATEGORICAL ASPECTS OF ALGEBRAIC GEOMETRY IN MIRROR SYMMETRY

ABSTRACTS

Alexei Bondal (Steklov/RIMS)
Derived categories of complex-analytic manifolds

TBA.

Alexender Kuznetsov (Steklov)
Categorical resolutions of singularities

I will give a definition of a categorical resolution of singularities and explain how such resolutions can be constructed.

Kentaro Hori (Toronto)
Phases of N=2 theories in 1+1 dimensions with boundary

TBA.

Yukinobu Toda (Tokyo)
Stability conditions and counting invariants of semistable objects

The notion of stability conditions on triangulated categories is formulated by T. Bridgeland, in order to give the mathematical framework of M. Douglas's Π-stability. In this talk, I consider the moduli problem of semistable objects on some triangulated categories, and show that they are Artin stacks of finite type. Furthermore I introduce the counting invariants of semistable objects, following recent work of D. Joyce, and investigate the wall-crossing formula of the invariants.

Dmitry Kaledin (Steklov/Tokyo)
Deligne conjecture and the Drinfeld double

Deligne conjecture (already proved 10 times or so) describes a structure which exists on the Hochschild cohomology complex of an associative algebra. Braid groups lurk in the background in this story – for instance, the natural structure on Hochschild cohomology is that of a Gerstenhabe algebra, and the operad which controls it is given by homology of the braid group – but they never appear explicitly. We will describe one more proof of Deligne conjecture where braid groups play a clear and major role, with the help of braided tensor categories and the notion of a Drinfeld double. If time permits, we will also describe relation to Hodge-to-de Rham degeneration.

Kentaro Nagao (Kyoto)

Braid group actions on K-groups of quiver varieties

Quantum envelopping algebras act on equivariant K-groups of quiver varieties. Actions of the generators are induced by Fourier-Mukai type functors on derived categories of equivariant coherent sheaves on quiver varieties. Braid groups act on quantum envelopping algebras and some representations of quantum envelopping algebras. So it is expected that there exist braid group actions on derived categories of equivariant coherent sheaves on quiver varieties. We have candidates for functors corresponding to the generators of braid groups. In this talk I will explain how to find these functors. They actually satisfy braid relations on K-theory level.

Emanuele Macri (MPI)

Derived equivalences of K3 surfaces and orientation

We present a proof of the orientation-preserving property for equivalences between derived categories of coherent sheaves of smooth and projective K3 surfaces. This leads to a complete description of the action of the group of all autoequivalences on integral cohomology very much like the classical Torelli Theorem for the automorphism group of K3 surfaces. These results are a joint work with D. Huybrechts and P. Stellari (arXiv:0710.1645).

Hokuto Uehara (Tokyo Metro.)

Derived equivalence for stratified Mukai flop on G(2,4) via tilting generators (with Y. Toda)

We construct tilting generators on some algebraic varieties. As its application, we show derived equivalence for stratified Mukai flop on G(2,4), which was proved in a completely different method by Kawamata several years ago.

Bertrand Toen (Toulouse)
Chern character, loop spaces and derived algebraic geometry

For any scheme X we construct a derived scheme LX, called the "derived loop space of X", which is the derived scheme of maps from the simplicial circle to X. In a first part I will explain the relations between the theory of functions on LX and cyclic homology of X, which will then be used in order to give a construction of the Chern character. In a second part I will explain how this can be generalized in order to construct a "secondary Chern character" defined for certain "sheaves of derived categories" on X (rather than sheaves of vector spaces) and with values in a "secondary cyclic homology". The domain of this secondary Chern character is the Grothendieck group of sheaves of derived categories on X, which is possibly related to (a part of) elliptic cohomology of X.

Bernhard Keller (Jussieu)
Algebraic K-theory via universal invariants (after Goncalo Tabuada)

This is a report on chapter 3 of G. Tabuada's Ph. D. Thesis. The main result is the interpretation of higher algebraic K-theory as a space of morphisms in the target category of the universal additive invariant of dg categories. Essentially, this invariant is the universal functor from the category of differential graded categories with values in a triangulated category which transforms derived Morita equivalences into isomorphisms and semi-orthogonal decompositions into direct sums. The correct formulation of this property and the proof of the main theorem are based on the formalism of derivators as developed by Heller, Grothendieck, Maltsiniotis and Cisinski. Analogous constructions for (algebraic) KK-theory are due to Meyer-Nest, Cortinas-Thom and Garkusha.

Osamu Iyama (Nagoya)
Cluster tilting for one-dimensional hypersurface singularities

This is a joint work with I. Burban, B. Keller and I. Reiten.

Let R be a complete local Gorenstein isolated singularity. A maximal Cohen-Macaulay R-module M is called *cluster tilting* if

$$\begin{aligned} \mathrm{add}M &= \{X \in \mathrm{CM}R \mid \mathrm{Ext}^1_R(M,X)=0\} \\ &= \{X \in \mathrm{CM}R \mid \mathrm{Ext}^1_R(X,M)=0\}. \end{aligned}$$

An important property of cluster tilting objects is that endomorphism algebras $\mathrm{End}_R(M)$ have finite global dimension. If $\dim R = 3$, then they satisfy 3-Calabi-Yau property in their derived categories, and give non-commutative crepant resolutions of R in the sense of Van den Bergh.

We give a classification of cluster tilting objects for one-dimensional reduced hypersuface singularities $R = k[[x,y]]/(f)$. We show that R has a cluster tilting object if and only if f is a product $f_1 f_2 ... f_n$ of power series satisfying $f_i \in (x,y)\backslash(x,y)^2$. In this case R has exactly $n!$ cluster tilting objects and exactly $2^n - 2$ indecomposable rigid objects.

Igor Burban (Mainz)

Vector bundles on cubic curves and Yang-Baxter equations

in my talk based on a joint work with Bernd Kreussler (arXiv:0708.1685) I am going to speak about applications of the theory of vector bundles on elliptic curves and their degenerations to the classical and quantum Yang-Baxter equations. This unexpected connection was recently discovered by Polishchuk in the framework of the homological mirror symmetry and his study of A-infinity structures on derived categories of coherent sheaves.

By a classical result of Belavin and Drinfeld there are three types of solutions of the classical Yang-Baxter equations: elliptic, trigonometric and rational. It turns out that this trichotomy corresponds exactly to three types of projective curves with trivial canonical bundle: elliptic curves, Kodaira cycles and curves with more complicated singularities like a cuspidal cubic curve.

We carry out some explicit calculations of r-matrices and their quantizations coming from singular curves. This approach gives a new insight to a study of degenerations of solutions of the classical and quantum Yang-Baxter equations.

Akira Ishii (Hiroshima)

Brane tilings and crepant resolutions of some three-dimensional toric singularities

This is a joint work with Kazushi Ueda. A brane tiling is a bipartite graph on a real two-torus which encodes the information of a quiver with relations. If the tiling consists of hexagons, the quiver is the McKay quiver associated with a finite abelian subgroup of SL(3, C). The McKay correspondence is expected to be generalized to the case of general brane tilings satisfying certain consistency conditions. In this talk, I will explain what is expected and state that the moduli space of quiver representations is a crepant resolution of corresponding three-dimensional toric singularity under some milder conditions. We also discuss the variation of the moduli spaces according to the stability parameters.

Dmitri Orlov (Steklov/Osaka)
TBA

TBA.