

# Optimal Execution Problem with Market Impact

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In this talk, we introduce the optimal execution problem with market impact (MI). MI is the effect of the investment behavior of traders on the security prices.

First we consider the optimal execution problem as the stochastic control problem in discrete-time model. Next we derive the value function in continuous-time model as the limit of the ones in discrete-time model. Then we study some properties of the continuous-time value function : continuity and semi-group property. We see that the continuity of the value function is varied according to the “strength” of MI (more concretely, the state of the MI function at the infinity point.) If MI is not large, then the value function is not always continuous at  $t = 0$ . On the other hand, if MI is large, then the value function is always continuous with respect to time  $t$ . This means that a block liquidation makes no sense in this case.

Moreover we characterize the continuous-time value function as a viscosity solution of the corresponding Hamilton-Jacobi-Bellman equation (HJB) and show the uniqueness of viscosity solutions of HJB under suitable assumptions (especially where the divergence speed of the MI function is rapid.)

We have also considered the case that a trader must sell up the entire shares of the security until the time horizon. We found that such a sell-out condition does not influence the (value of) continuous-time value function in our model.

We introduce some examples of this model, which tell us that the forms of the optimal execution strategies change according to the amount of the security holdings (the shares of the security held) if MI function has a quadratic form.

We can develop an extended model where the MI function has noise. We introduce some simple random MI models and derive the continuous-time model as the stochastic control problem with jumps.

In this study we mainly treat a convex MI function, whereas it is found from empirical studies that the form of MI function is concave. But it is interesting to consider the case when MI is convex, and we can observe the very effect of MI which affects the trader’s execution policy.

## References

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