MMP and extremal rays

June 20–June 24, 2011
RIMS, Kyoto University, Room 420

June 20 (Monday)

- 10:00–11:00: J. M^{c}Kernan (Massachusetts Institute of Technology), The importance of being big
- 11:30–12:30: C. Birkar (Cambridge University), Existence of log canonical flips and a special LMMP
- 14:00–15:00: J. A. Chen (National Taiwan University), Explicit three-dimensional minimal model program
- 16:00–17:00: M. Mustaţă (University of Michigan), Asymptotic invariants and valuations

June 21 (Tuesday)

- 9:30–10:30: V. V. Shokurov (Johns Hopkins University/Steklov Institute of Mathematics), Mobility and rigidity in birational algebraic geometry
- 10:45–11:45: C. Xu (Massachusetts Institute of Technology), Boundedness of algebraic varieties
- 13:30–14:30: A. Corti (Imperial College London), Extremal Laurent Polynomials and Fano Manifolds
- 14:45–15:45: V. Lazić (Imperial College London), MMP revisited
- 16:15–17:15: M. Kawakita (RIMS), Ideal-adic semi-continuity problem for minimal log discrepancies

June 22 (Wednesday)

- 10:00–11:00: H. Clemens (Ohio State University), The intersection number for pairs of (topological) normal functions and a height pairing for (disjoint, homologically trivial) submanifolds
- 11:30–12:30: Y. Miyaoka (University of Tokyo), Canonical degree of nodal curves on a surface of general type
- 14:00–15:00: J. Kollár (Princeton University), MMP and moduli of varieties
- 16:00–17:00: S. Mori (RIMS), Threefold $\mathbb{Q}$-conic bundles and a personal review of extremal rays

June 23 (Thursday)

- 9:30–10:30: V. Alexeev (University of Georgia), Extending Torelli maps
- 10:45–11:45: S. Kovács (University of Washington), Deformations of Du Bois singularities
- 13:30–14:30: H. Takagi (University of Tokyo), Mirror symmetry and projective geometry of Reye congruence
- 14:45–15:45: J.-M. Hwang (Korea Institute for Advanced Study), Varieties of minimal rational tangents of codimension 1
- 16:15–17:15: Y. Namikawa (Kyoto University), Equivalence of symplectic singularities

June 24 (Friday)

- 9:30–10:30: Y. Prokhorov (Moscow University), On the classification of singular Fano threefolds
- 10:45–11:45: M. Reid (University of Warwick), Rings and varieties

Organizers: Osamu Fujino (Kyoto Univesity), János Kollár (Princeton University), Atsushi Moriwaki (Kyoto University), Shigeru Mukai (RIMS), Noboru Nakayama (RIMS)

Titles and Abstracts

V. Alexeev (University of Georgia)
Title: Extending Torelli maps
Abstract: When does a Torelli-type map to a moduli space of abelian varieties extend to a regular map between compactifications? The answer of course depends on the specific Torelli-type problem and the compactifications considered. I will consider several situations, and give a complete answer for the classical Torelli map and the three compactifications of the moduli of PPAVs: perfect, 2nd Voronoi, and Igusa blowup.

C. Birkar (Cambridge University)
Title: Existence of log canonical flips and a special LMMP
Abstract: I explain the proof of the LMMP for lc pairs in some special situations related to the existence of lc flips and a conjecture of Kollár.

J. A. Chen (National Taiwan University)
Title: Explicit three-dimensional minimal model program
Abstract: Inspired by the work of Prof. Mori, we would like to give a brief survey of recent works on understanding three dimensional minimal model program explicitly. For example, divisorial contractions to a point of higher index can be realized as a weighted blowups by the work of Hayakawa and Kawakita. Flips, divisorial contractions to a curve, and divisorial contraction to a point with non-minimal discrepancies can be factored into simpler maps. We shall give many examples explicitly and explain the main ideas.

H. Clemens (Ohio State University)
Title: The intersection number for pairs of (topological) normal functions and a height pairing for (disjoint, homologically trivial) submanifolds
Abstract: One strand of the study of algebraic n-cycles on a complex projective manifold W of (complex) dimension $2n$ involves mimicking the work of Solomon Lefschetz in the case $n = 1$, namely slicing W into a one-parameter family of hyperplane sections $\{X_t\}$, associating to each hyperplane section X_t a torus $J(X_t)$ called its 'intermediate Jacobian,' and describing primitive algebraic n-cycles of W as sections called 'normal functions' of the resulting bundle of (intermediate) Jacobians. The step of the analysis that fails when

$n > 1$ derives from the fact that, unlike the $n = 1$ case, not all points of $J(X_t)$ come from homologically trivial algebraic $(n-1)$-cycles on X_t. In the hope that it may shed some light on this obstruction, we explore the much easier topological problem. We define differentiable normal functions associated to any primitive integral $2n$-cohomology class of W and show the intersection number of any two such classes is given by the first Chern class of the restriction of the (topological) Poincaré bundle to the graph of the corresponding pair of normal functions in $J(X_t) \times J(X_t)$. Borrowing from the theory of differential characters, we parametrize $J(X_t)$ with (sums of) homologically trivial differentiable sub-manifolds of X_t of real dimension $2n-2$. We define a 'height pairing' for disjoint pairs of such cycles and, generalizing work of R. Hain in the algebraic case, interpret the height pairing as a lifting of the corresponding point in $J(X_t) \times J(X_t)$ to a point in the fiber of the Poincaré bundle. This is joint work with Mirel Caibar.

A. Corti (Imperial College London)

Title: Extremal Laurent Polynomials and Fano Manifolds

Abstract: A Laurent polynomial is extremal if the Picard–Fuchs system has smallest possible ramification. I explain how extremal Laurent polynomials (ELP) mirror the quantum cohomology of Fano manifolds. I describe a class of ELP called Minkowski polynomials, and I show that the list of Minkowski polynomials in three variables allows one to recover the classification of Fano 3-folds of Fano, Iskovskikh and Mori–Mukai. I discuss a project to list Minkowski polynomials in four variables and implications for the classification of Fano 4-folds. This is experimental work in progress with Tom Coates, Sergei Galkin, Vasily Golyshev and Al Kasprzyk.

J.-M. Hwang (Korea Institute for Advanced Study)

Title: Varieties of minimal rational tangents of codimension 1

Abstract: Given a smooth projective variety X, the variety of minimal rational tangents (VMRT) at a general point x is a subvariety of the projectivized tangent space $PT_x(X)$, defined as the union of tangent directions to rational curves of minimal degree through x. The result of Cho, Miyaoka and Shepherd-Barron says that if the VMRT at x is $PT_x(X)$, then X is projective space. We show that if the VMRT at x is a smooth hypersurface in $PT_x(X)$, then X is rational.

M. Kawakita (RIMS)
Title: Ideal-adic semi-continuity problem for minimal log discrepancies
Abstract: De Fernex, Ein and Mustaţă, after Kollár, proved the ideal-adic semi-continuity of log canonicity to obtain Shokurov's ACC conjecture for log canonical thresholds on l.c.i. varieties. I discuss its generalisation to minimal log discrepancies, proposed by Mustaţă.

J. Kollár (Princeton University)
Title: MMP and moduli of varieties
Abstract: We discuss applications of MMP to the moduli of varieties of general type.

S. Kovács (University of Washington)
Title: Deformations of Du Bois singularities
Abstract: This is a report on work in progress joint with Karl Schwede. We prove an inversion of adjunction type statement for Du Bois singularities: Let X be a scheme of finite type over the complex numbers and H an effective Cartier divisor. If H has Du Bois singularities, then X has Du Bois singularities near H. This easily implies that small deformations of Du Bois singularities are also Du Bois. As an application we obtain a restriction theorem for certain non-lc ideals.

J. M$^{\text{c}}$Kernan (Massachusetts Institute of Technology)
Title: The importance of being big
Abstract: We review some recent results in Mori theory which highlight the importance of big divisors.

V. Lazić (Imperial College London)
Title: MMP revisited
Abstract: I will talk about a recent joint work with P. Cascini in which we prove finite generation of the canonical ring without Mori theory.

Y. Miyaoka (University of Tokyo)
Title: Canonical degree of nodal curves on a surface of general type

Abstract: A conjecture of Green and Lang asserts that there should be only finitely many rational/elliptic curves on a given surface of general type. Caporaso–Harris–Mazur pointed out, however, that the above conjecture implies a stronger assertion to the effect that the canonical degree of rational/elliptic curves on a minimal surfaces of general type with given numerical invariants is universally bounded by a certain function of the invariants. The universal boundedness of the canonical degree has been established when 1) we restrict the curves in question to the ones without nodes and ordinary triple points, or 2) $K_X^2 > c_2(X)$ with explicit upper bound. In the talk, we try to give a universal bound for curves with many ordinary double points, thereby extending our previous results to general rational/elliptic curves on a given surface.

S. Mori (RIMS)
Title: Threefold $\mathbb{Q}$-conic bundles and a personal review of extremal rays
Abstract: I will give a survey of my ongoing project with Prokhorov on threefold $\mathbb{Q}$-conic bundles as well as a personal review of extremal rays in retrospect. The joint project concerns a proper morphism $f : (X, C) \to (Z, o)$ from an analytic threefold X with at most terminal singularities to a germ of a normal analytic space (Z, o) such that $-K_X$ is f-ample and C, the reduced part of the fiber of f over o, is an irreducible curve. They have been divided into several types. We have classified $\mathbb{Q}$-conic bundles, the case of two dimensional Z, for some of the types. In the course of our treatment, we also classified some of the divisorial cases, where f is a bimeromorphic morphism with an exceptional divisor.

M. Mustaţă (University of Michigan)
Title: Asymptotic invariants and valuations
Abstract: There are analogues of familiar invariants of singularities, such as the log canonical threshold, in an asymptotic setting. They are attached to graded sequences of ideals and to plurisubharmonic functions. I will discuss some results and open questions related to such invariants. This is joint work with Mattias Jonsson.

Y. Namikawa (Kyoto University)
Title: Equivalence of symplectic singularities

Abstract: We will discuss the uniqueness problems of symplectic structures on a symplectic singularity X having a $\mathbb{C}^*$-action with positive weights. A symplectic structure determines a Poisson structure on X and a contact orbifold structure on the associated projective variety $P(X)$. These structures will play important roles in the uniqueness problem.

Y. Prokhorov (Moscow University)
Title: On the classification of singular Fano threefolds
Abstract: We will present some recent progress in the classification problem of singular Fano threefolds. One of our main goal is a partial classification of such threefolds having sufficiently many anticanonical sections.

M. Reid (University of Warwick)
Title: Rings and varieties
Abstract: Many varieties of interest in the classification of varieties are obtained as Spec or Proj of a Gorenstein ring. In codimension ≤ 3, the well known structure theory provides explicit methods of calculating with Gorenstein rings. In contrast, there is no useable structure theory for rings of codimension ≥ 4. Nevertheless, in many cases, Gorenstein projection (and its inverse, Kustin–Miller unprojection) provide methods of attacking these rings. These methods apply to sporadic classes of canonical rings of regular algebraic surfaces, and to more systematic constructions of $\mathbb{Q}$-Fano 3-folds, Sarkisov links between these, and the 3-folds flips of Type A of Mori theory. See for example

Gavin Brown, Michael Kerber and Miles Reid, Fano 3-folds in codimension 4, Tom and Jerry unprojection constructions of $\mathbb{Q}$-Fano 3-folds, arXiv:1009.4313

Shigefumi Mori, On semistable extremal neighborhoods, in Higher dimensional birational geometry (Kyoto, 1997), Adv. Stud. Pure Math. **35**, Math. Soc. Japan, Tokyo, 2002, pp. 157–184

my website + 3-folds

V. V. Shokurov (Johns Hopkins University/Steklov Institute of Mathematics)
Title: Mobility and rigidity in birational algebraic geometry
Abstract: This talk is about advances and limitations of modern methods in birational algebraic geometry.

H. Takagi (University of Tokyo)
Title: Mirror symmetry and projective geometry of Reye congruence
Abstract: This is a joint work with Shinobu Hosono.

It is well-known that the projective dual of the second Veronese variety $v_2(\mathbb{P}^n)$ is the symmetric determinantal hypersurface H. However, in the context of homological projective duality after Kuznetsov, it is natural to consider that the $\mathrm{Chow}^2\mathbb{P}^n$ and H are dual (note that $\mathrm{Chow}^2\mathbb{P}^n$ is the secant variety of $v_2(\mathbb{P}^n)$). Though we did not yet formulate what this duality exactly means in full generality, we show some results in this context for $n = 4$. We consider $\mathrm{Chow}^2\mathbb{P}^4$ in $\mathbb{P}(S^2V)$ and H in $\mathbb{P}(S^2V^*)$, where V is the vector space such that $\mathbb{P}^4 = \mathbb{P}(V)$. Take a general 4-plane P in $\mathbb{P}(S^2V^*)$ and let P' be the orthogonal space to P in $\mathbb{P}(S^2V)$. Then $X := \mathrm{Chow}^2\mathbb{P}^4 \cap P'$ is a smooth Calabi–Yau 3-fold, and there exists a natural double cover $Y \to H \cap P$ with a smooth Calabi–Yau 3-fold Y. It is easy to check that X and Y are not birational each other. Our main result asserts the derived equivalence of X and Y. This derived equivalence is given by the Fourier-Mukai functor $D(X) \to D(Y)$ whose kernel is the ideal sheaf in $X \times Y$ of a flat family of curves on Y parameterized by X. Curves on Y in this family have degree 5 and arithmetic genus 3, and these have a nice interpretation by a BPS number of Y.

C. Xu (Massachusetts Institute of Technology)
Title: Boundedness of algebraic varieties
Abstract: I will discuss the recent joint work with (Hacon and M$^{\mathrm{c}}$Kernan), which established certain boundedness type results. I will also explain why we need it, mainly by focusing on its application to the moduli space of higher dimensional stable varieties.