Summer School on Algebraic Geometry

2011 年 6 月 13 日–17 日
京都大学数理解析研究所 420 号室

6 月 13 日（月）

- 9:00–9:30: Registration
- 9:30–10:30: P. Cascini (Imperial College London), On the Minimal Model Program, I.
- 10:45–11:45: P. Hacking (University of Massachusetts Amherst), Moduli spaces of surfaces of general type and their compactifications, I.
- 13:30–14:30: IRSO* hours in Room 103
  (* 数理解析研究所国際研究支援室)
- 14:45–15:30: C.-Y. Chi（Harvard), Pseudonorms and Theorems of Torelli type
- 15:45–16:30: D. Kim (Korea Institute for Advanced Study), Some division problems implied by the openness conjecture for multiplier ideal sheaves

6 月 14 日（火）

- 9:30–10:30: M. Păun (Institut Elie Cartan, Université Henri Poincaré), Extension theorems for pluricanonical forms, I.
- 10:45–11:45: J. Starr (Stony Brook University), Spaces of rational curves on Fano manifolds, I.
- 13:30–14:30: P. Cascini, On the Minimal Model Program, II.
- 14:45–15:30: 權業善範 (東京大学), Log pluricanonical representations and abundance conjecture

- 15:45–16:30: C.-J. Lai (University of Utah), Varieties fibered by good minimal models
- 17:15– : カンフォーラにて懇親会

6 月 15 日（水）

- 9:30–10:30: P. Hacking, Moduli spaces of surfaces of general type and their compactifications, II.
- 10:45–11:45: J. Starr, Spaces of rational curves on Fano manifolds, II.

6 月 16 日（木）

- 9:30–10:30: P. Cascini, On the Minimal Model Program, III.
- 10:45–11:45: M. Păun, Extension theorems for pluricanonical forms, II.
- 13:30–14:30: J. Starr, Spaces of rational curves on Fano manifolds, III.
- 14:45–15:30: 渡辺究 (東京大学), Lengths of chains of minimal rational curves on Fano manifolds of Picard number 1
- 15:45–16:30: 伊藤敦 (東京大学), Seshadri constants and Okounkov bodies

6 月 17 日（金）

- 9:30–10:30: M. Păun, Extension theorems for pluricanonical forms, III.
- 10:45–11:45: P. Hacking, Moduli spaces of surfaces of general type and their compactifications, III.
- 13:30–14:15: 松村慎一 (東京大学), Asymptotic cohomology vanishing and a converse of the Andreotti–Grauert theorem on surfaces

- 14:30–15:15: 岡田拓三 (京都大学), On the Mori fiber structures of $\mathbb{Q}$-Fano 3-fold weighted complete intersections

世話人: 川北真之（数理解析研究所）、中山昇（数理解析研究所）、早川貴之（金沢大学）、藤野修（京都大学）

Titles and Abstracts of series of lectures

P. Cascini (Imperial College London)
Title: On the Minimal Model Program
Abstract: We will talk about old and new techniques in the Minimal Model Program. We will show how to prove finite generation of adjoint rings and we will discuss new possible applications of these techniques.

P. Hacking (University of Massachusetts Amherst)
Title: Moduli spaces of surfaces of general type and their compactifications
Abstract: The moduli space of surfaces of general type admits a compactification due to Kollár and Shepherd-Barron which is analogous to the Deligne–Mumford compactification of the moduli space of curves of genus $g > 1$. The construction uses the 3-fold minimal model program to determine a canonical "limit"of a one parameter family of smooth surfaces. We will give an introduction to these moduli spaces with a strong emphasis on examples. Topics will include computing limits of one parameter families, infinitesimal study of the moduli space (deformation theory), and identifying codimension one components of the boundary. We will also review aspects of the Deligne–Mumford compactification of the moduli space of curves as a warm-up.

M. Păun (Institut Elie Cartan, Université Henri Poincaré)
Title: Extension theorems for pluricanonical forms
Abstract: We will present in detail a few extension results for pluricanonical forms. More precisely, we will be concerned with the results in the field which do not require strict positivity hypothesis.

J. Starr (Stony Brook University)
Title: Spaces of rational curves on Fano manifolds
Abstract: Following the revolutionary work of Mori, it is now known that the rational curves on a projective variety profoundly influence the geometry of the variety. These lectures will survey the geometry of the

parameter spaces of rational curves on Fano manifolds and more generally on "rationally connected manifolds"of Kollár–Miyaoka–Mori (those projective manifolds for which every pair of points are connected by a rational curve). This geometry is related to existence of rational points of Fano manifolds defined over a function field, the weak approximation conjecture of Hassett–Tschinkel, the Fano conjecture that many Fano manifolds are non-uniruled, and, quite recently, finiteness of quantum K-theory (following work of A. Buch, et al.).

Lecture 1. Parameter spaces for rational curves. Review of the basic parameter spaces: the Chow variety, the Hilbert scheme and the Kontsevich moduli space. Description of the deformation theory, nonemptiness, bounds on the dimension, and irreducibility.

Lecture 2. Rational connectedness of space of rational curves. Investigation of the problem when parameter spaces of rational curves are rationally connected together with applications to existence of rational points, weak approximation, and finiteness of quantum K-theory.

Lecture 3. Non-uniruledness of spaces of rational curves. Investigation of the Kodaira dimensions and non-uniruledness of parameter spaces of rational curves motivated by the conjecture that there exist non-unirational Fano manifolds.

Titles and Abstracts of talks

C.-Y. Chi (Harvard)
Title: Pseudonorms and Theorems of Torelli type
Abstract: For projective manifolds M and M', we prove that if they have isometric canonically pseudonormed pluricanonical spaces

$$\big(H^0(M, mK_M), \langle\!\langle\ \rangle\!\rangle_m\big) \simeq \big(H^0(M', mK_{M'}), \langle\!\langle\ \rangle\!\rangle_m\big)$$

for a single positive integer m which is sufficiently large and sufficiently divisible, then the images of their m-pluricanonical maps have isomorphic closure.

Y. Gongyo (University of Tokyo)
Title: Log pluricanonical representations and abundance conjecture
Abstract: In this talk, I will talk about the finiteness of log pluricanonical representations and the abundance conjecture for semi-log canonical pairs. Actually we prove the finiteness of log pluricanonical representations for projective log canonical pairs with semi-ample log canonical divisor. As a corollary, we obtain that the log canonical divisor of a projective semi log canonical pair is semi-ample if and only if so is the log canonical divisor of its normalization. This is joint work with O. Fujino.

A. Ito (University of Tokyo)
Title: Seshadri constants and Okounkov bodies
Abstract: Seshadri constant is an invariant which measures the local positivity of a line bundle. The Okounkov body is a closed convex set defined for a big line bundle, and it has much information of the line bundle. In this talk, I explain a relation of Seshadri constants and Okounkov bodies.

D. Kim (Korea Institute for Advanced Study)
Title: Some division problems implied by the openness conjecture for multiplier ideal sheaves
Abstract: Plurisubharmonic (psh) functions appear as local weight functions of a singular hermitian metric of a pseudoeffective line bundle in

algebraic geometry. The multiplier ideal sheaf of a general psh function is conjectured to have a certain common behavior with the special case of an 'algebraic' psh function (essentially the openness conjecture of Demailly and Kollár). We will discuss some new Skoda type division results for which the openness conjecture had been an obstacle. One of them is the exactness of general Skoda complexes, which can be used to show that not every integrally closed ideal is an analytic multiplier ideal, following the work of Lazarsfeld and Lee.

C.-J. Lai (University of Utah)
Title: Varieties fibered by good minimal models
Abstract: Let $f : X \to Y$ be an algebraic fiber spaces such that the general fiber has a good minimal model. We show that if f is the Iitaka fibration, then X has a good minimal model. In particular, the minimal model conjecture (for smooth projective varieties) is reduced to the case of varieties with Kodaira dimension zero (and the abundance conjecture).

S. Matsumura (University of Tokyo)
Title: Asymptotic cohomology vanishing and a converse of the Andreotti–Grauert theorem on surfaces
Abstract: The purpose of this talk is to study a relation between the curvature positivity and the asymptotic behavior of higher cohomology groups for tensor powers of a holomorphic line bundle. A theorem of Andreotti and Grauert asserts that a partial curvature positivity implies asymptotic vanishing of certain higher cohomology groups. In this talk, we discuss the converse implication under various situations. The problem is related with holomorphic Morse inequalities. We prove the converse implication on smooth projective surfaces without any assumption on line bundles.

T. Okada (Kyoto University)
Title: On the Mori fiber structures of $\mathbb{Q}$-Fano 3-fold weighted complete intersections
Abstract: We will talk about the work which aims to determine the Mori

fiber structures of $\mathbb{Q}$-Fano 3-fold WCI of codimension 2 with terminal singularities using the method of maximal singularities. There are 85 families of those varieties in the Fletcher's list and some of the families are known to be birationally rigid or bi-rigid. We will explain which (remaining) family is birationally rigid or not.

K. Watanabe (University of Tokyo)
Title: Lengths of chains of minimal rational curves on Fano manifolds of Picard number 1
Abstract: We consider a natural question how many minimal rational curves are needed to join two general points on a Fano manifold X of Picard number 1. As an application, we give a better bound on the degree of Fano 5-folds of Picard number 1. If we have time, I also talk about a bound of the minimal length under mild conditions on the variety of minimal rational tangents. The contents of my talk concern with my papers entitled "Lengths of chains of minimal rational curves on Fano manifolds", which was published from Journal of Algebra (Volume 325, Issue 1, 1 January 2011, Pages 163-176), and "A bound of lengths of chains of minimal rational curves on Fano manifolds of Picard number 1"(http://arxiv.org/abs/1010.2005) (to appear in J. Algebra).