Discrete Geometric Analysis Abstracts

August 27-31, 2012, RIMS, Kyoto University

The cut-tree of large Galton-Watson trees and the Brownian CRT

Jean Bertoin
Universität Zürich

This talk is based on a joint work with Grégory Miermont.

Consider the edge-deletion process in which the edges of some finite tree are removed one after the other in the uniform random order. Roughly speaking, the cut-tree then describes the genealogy of connected components appearing in this edge-deletion process. Our main result shows that after a proper rescaling, the cut-tree of a critical Galton-Watson tree with finite variance and conditioned to have size n, converges as $n \to \infty$ to a Brownian CRT in the weak sense induced by the Gromov-Prokhorov topology. This yields a multi-dimensional extension of a limit theorem due to Janson for the number of random cuts needed to isolate the root in Galton-Watson trees conditioned by their sizes.

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Amenability of directed groups

Jeremie Brieussel
Kyoto University

Directed groups acting on a tree of bounded valency are amenable, a result first proved using random walks technics. This provides interesting estimates on probabilistic quantities such as entropy and return probability, but this does not give a description of Folner sets. By using a different method, we will describe explicit such sets, providing a second (independent) proof.

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Random networks and epidemics

Tom Britton
Stockholm University

Random networks may be used to describe the social structure in a community. "On" such a network one can model the spread of an infection. Questions of interest to study are for example: Can a big outbreak occur?, How large will it be?, What is the effect of introducing a given vaccination scheme? In the talk we will make a biased survey of this area, with focus on effects of the underlying random network on the questions formulated above.

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Simple random walks on random graphs in critical regimes

David Croydon
University of Warwick

This talk will survey some recent progress on the asymptotic behaviour of simple random walks on random graphs in critical regimes, with the two principal examples considered being critical Galton-Watson trees and the Erdős-Rényi random graph in the critical window. For both of these models, it is known that the graphs in question can be rescaled to yield a random fractal. It will be described how to extend such results to show that the associated simple random walks satisfy a corresponding scaling limit. Furthermore, general techniques that show the discrete transition densities and mixing times can also be rescaled to yield a continuous equivalent will be discussed.

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Group actions on quasi-trees

Koji Fujiwara
Kyoto University

Bass-Serre theory is a fundamental tool in group theory, which is on groups actions on simplicial trees. It turns out that group actions on quasi-trees is also useful. A quasi-tree is a graph which is quasi-isometric to a simplicial tree. Our study has two parts, one is to construct group actions on quasi-trees, and the other is on application once we have an unbounded group actions on quasi-trees.

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Coarse random geometry

Geoffrey Grimmett
University of Cambridge

When can one Euclidean random process be embedded within another? Various forms of embedding are discussed including quasi-isometry and Lipschitz embedding. Under suitable conditions, a process may be embedded in another of higher dimension. The problem is much harder when the spaces have equal dimensions. The answer in this case is not completely known, although Gacs and Basu/Sly have recently presented solutions in the case of one dimension. Part of the material of this talk is joint with Ander Holroyd.

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2D Ising percolation at high temperatures

Yasunari Higuchi
Kobe University

We summarize results concerning spin percolation of the two-dimensional nearest neighbour Ising model in square lattice $\mathbf{Z}^2$. At each point sits a spin variable distributed under a Gibbs measure: If the temperature is below the critical one T_c, then there are two extremal Gibbs measures μ_+ and μ_- at the zero external magnetic field, and + spins percolate under μ_+, while $-$ spins percolate under μ_-. So we can say that the critical value $h_c(T)$ of the external field for the spin percolation is zero when $T < T_c$ ([1]). This is known to be true when $T = T_c$. On the other hand when $T > T_c$, we know that $h_c(T) > 0$ ([2]). So, we can pose questions about scaling limits of crossing probabilities, whether they exist or not, whether limits can be described by SLE_6 as in the independent case. At the critical point $T = T_c$ with zero external field, this is the main question for the model and Smirnov's result [3, 4] shows that they are actually related to $\mathrm{SLE}_{16/3}$. Unfortunately we can not answer these questions in high temperatures, yet. So far, what we can say is about some arm exponents and the following result.

Theorem 1. (joint work with M. Takei and Y. Zhang) The Kesten's hyperscaling relations hold provided that the one arm exponent and the four arm exponent exist for the 2D Ising percolation at high temperatures.

References

[1] Coniglio, A., Nappi, C. R., Peruggi, F. and Russo, L., *Commun. Math. Phys.*, **51**, 315–323 (1976).

[2] Higuchi, Y., *Probab. Theory Relat. Fields*, **97**, 1–33 (1993).

[3] Smirnov, S., *Ann. Math.*, **172**, 1435–1467 (2010).

[4] Duminil-Copin, H. and Smirnov, S., arXiv:1109.1549v1[math.PR] 7 Sep. 2011.

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Discrete convexity and polynomial solvability in minimum 0-extension problems

Hiroshi Hirai
University of Tokyo

The minimum 0-extension problem on graph G (0-Ext[G]) includes a number of basic combinatorial optimization problems, such as minimum (s,t)-cut problem, multiway cut problem, and so on.

Karzanov proved the polynomial solvability for a large class of a certain modular graphs in connection with multicommodity flow problems, and raised the question: What is the graph G for which 0-Ext[G] can be solved in polynomial time ? He also proved that 0-Ext[G] is NP-hard if G is not modular or not orientable (in a certain sense).

In this paper, we prove the converse: if G is orientable modular, then 0-Ext[G] can be solved in polynomial time. This result achieves the complete classification of 0-Ext-tractable graphs. To prove this result, we develop a theory of discrete convex functions on orientable modular graphs.

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Globally rigid graphs and frameworks

Tibor Jordán
Eötvös University

A bar-and-joint framework (or geometric graph) is said to be globally rigid in some fixed dimension if its edge lengths uniquely determine the framework up to congruence. We give a survey of the numerous recent results on globally rigid frameworks and their underlying graphs.

We focus on the combinatorial and algorithmic aspects of global rigidity. Potential applications in localization problems of sensor networks, structural problems of molecules, formations of autonomous agents, and in other areas of discrete geometry will also be discussed.

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Invariance and unimodularity

Vadim Kaimanovich
University of Ottawa

There are two classes of measures on the space of (isomorphism classes of) infinite rooted graphs: measures invariant with respect to the associated root moving equivalence relation, and so-called unimodular measures. Invariance is natural from ergodic or geometrical point of view (being the counterpart of holonomy invariance for foliations), whereas unimodularity has been introduced and studied by probabilists. Although for measures concentrated on graphs with no symmetries these two notions coincide, they are different in general. In the talk we shall clarify the general relationship between these two classes of measures.

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Phase transition in random discrete structures

Mihyun Kang
Technische Universität Graz

Random discrete structures have been extensively studied during the last few decades and have become one of the central themes of contemporary mathematics. This is partly because they are useful for modelling, analysing and solving structural and algorithmic problems arising from mathematics, theoretical computer science and natural sciences, and they provide a wide potential range of applications.

The phase transition and its critical behaviour is a fascinating phenomenon observed in various contexts. The phase transition deals with an abrupt change in the properties of a large structure by altering critical parameters. The phase transition in random discrete structures has captured the attention of many scientists, and its intense study has brought together different fields such as discrete mathematics, probability theory and theoretical computer science as well as statistical physics.

In this talk we discuss phase transitions in random discrete structures including Ising model, percolation, random graphs and random walks.

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Geometric group theory, large scale geometry, and discrete analysis

Bruce Kleiner
Courant Institute

The lecture will survey some of the roles that discrete analysis plays in geometric group theory, emphasisizing connections with (coarse) negative curvature and uniformization/geometrization problems. The lecture will be aimed at those who are not familiar with geometric group theory.

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Nonlinear spectral gap and fixed point property

Takefumi Kondo
Tohoku University

A remarkable result by Zuk gave a criterion for providing Kazhdan's property(T) by means of spectral gap of finite graphs. This result was generalized by Izeki-Nayatani and Gromov for the class of CAT(0) spaces. We generalize these criteria to the class of uniformly convex spaces and give some estimation of nonlinear spectral gap.

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A Mathematical challenge to materials science

Motoko Kotani
Tohoku University

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The shape of (2+1)-dimensional SOS

Eyal Lubetzky
Microsoft, Seattle

We present new results on the (2+1)-dimensional Solid-On-Solid model at low temperatures (also known as the Onsager-Temperley sheet, introduced in the 1950's). Bricmont, El-Mellouki and Froehlich (1986) showed that in the presence of a floor there is an entropic repulsion phenomenon, lifting the surface to a height which is logarithmic in the side of the box. We refine this and establish that the typical height of the SOS surface is precisely the integer part of $[1/(4\beta)\log L]$, where L is the side-length of the box and β is the inverse-temperature. Moreover, with high probability the surface is a plateau with cube-root fluctuations from the side boundaries. As a consequence of these results, and in contrast to the 1D behavior, the Glauber dynamics for SOS is exponentially slow, as it passes through a series of meta-stable states in order to rise from an initially flat configuration to its final height.

Based on joint works with Pietro Caputo, Fabio Martinelli, Fabio Toninelli and Allan Sly.

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Random discrete Morse theory and the complicatedness of triangulations

Frank H. Lutz
TU-Berlin

Discrete Morse theory is extensively used as a preprocessing step for homology computations. Although it is NP-hard to find optimal discrete Morse functions, most data appears to be easy and it is hard to construct complicated examples.

In this talk, we introduce a measure for the *complicatedness* of triangulations. For this, we define the *discrete Morse spectrum* of a simplicial complex to be the distribution of discrete Morse vectors that are obtained by choosing free faces for collapses and critical faces uniformly at random. The complicatedness then is the expected number of critical cells.

It is hopeless to compute the discrete Morse spectrum for larger complexes but it can easily be explored by random experiments. Our approach works well for manifolds and allows to compute optimal discrete Morse vectors in many cases. In particular, we showed collapsibility of a triangulated 5-manifold different from a 5-ball with the example having face vector $(5013, 72300, 290944, 495912, 383136, 110880)$.

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Laplacian-based centrality in directed graphs

Naoki Masuda
University of Tokyo

The PageRank is a dominant definition of centrality (i.e., real value attached to each vertex, representing the importance of the vertex) in directed graphs. I would like to discuss the utility of an alternative centrality based on the graph Laplacian. The Laplacian-based centrality value for a vertex is in fact equal to the probability that a new type introduced at the vertex takes over the entire graph (i.e., consensus) in the voter model dynamics. In addition, the Laplacian-based centrality captures importance of vertices in various other dynamics on graphs including synchronization. Finally, by using spanning trees, I derive the approximation scheme for the Laplacian-based centrality when graphs possess community structure (i.e., group structure).

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Introduction to discrete convex analysis

Kazuo Murota
University of Tokyo

Discrete convex analysis is a theory that aims at a discrete analogue of convex analysis for nonlinear discrete optimization. Technically it is a nonlinear generalization of matroid/submodular function theory; matroids are generalized to M-convex functions and submodular set functions to L-convex function. Fundamental concepts and theorems in discrete convex analysis are explained, including conjugacy and duality.

References

[1] S. Fujishige: Submodular Functions and Optimization, 2nd ed., Elsevier, 2005.

[2] K. Murota: Discrete Convex Analysis, SIAM, 2003.

[3] K. Murota: Recent developments in discrete convex analysis, in: W. Cook, L. Lovasz and J. Vygen, eds., Research Trends in Combinatorial Optimization, Springer, 2009, Chapter 11, 219-260.

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Representations of convex sets and conic matrix factorizations

Pablo A. Parrilo
MIT

In optimization one often represents convex sets in terms of convex cones. Such representations or 'lifts' of a convex set are especially useful if the cone admits efficient algorithms for linear optimization over its affine slices, as in the classical cases of linear and semidefinite programming. Despite the fact that these techniques are widely used, there are many aspects (particularly, existence and efficiency) that are still poorly understood. In this talk we discuss the relationship between conic representations of convex sets, and a special "conic" factorization of an operator associated to the convex set, generalizing earlier results of Yannakakis on polyhedral lifts of polytopes and nonnegative factorizations. When the cones live in a family, our results lead to the definition of the rank of a convex set with respect to this family (e.g., psd rank of a convex set). We will provide a gentle introduction to these techniques, emphasizing geometric intuition, open questions as well as recent results. Based on joint work with Joao Gouveia and Rekha Thomas.

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Random walks on groups driven by low moment measures

Laurent Saloff-Coste
Cornell University

(Joint work with A Bendikov, to appear in Annals of Probability)

On any finitely generated group G, the large n rough asymptotics of the probability of return of the random walk driven by a symmetric finitely supported measure q with generating support is, in fact independent of q. It is a group invariant.

Given a "finite moment condition" (e.g., given by a low power of the word norm $|g|$ associated with a finite symmetric generating set), we can ask:

What can be said of the large n rough asymptotics of the probability of return of a random walk driven by a symmetric measure satisfying this fixed moment condition?

For instance, on the integers, what can we say about the probability of return of the random walk associated with a symmetric measure with finite (1/2)-moment?

In general, one might hope to derive a lower bound on this probability of return. Recent results in this direction will be described.

From a geometric perspective, it would be interesting to understand whether or not the problem described above leads to new "group invariants" associated with the concept of random walk. This appears to be a difficult question.

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Non-intersecting random walks in low dimensions

Daisuke Shiraishi
Kyoto University

We consider two random walks conditioned "never to intersect" in $\mathbb{Z}^2$. We show that each of them has infinitely many *global* cut times with probability one. In fact, we prove that the number of global cut times up to n grows like $n^{\frac{3}{8}}$. Next we consider the union of their trajectories to be a random subgraph of $\mathbb{Z}^2$ and show the subdiffusivity of the simple random walk on this graph.

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Coincidence symmetry groups

Toshikazu Sunada
Meiji University

A mathematical view to the theory of coincidence-site lattices (CSL) initiated by material scientists is given. Especially coincidence symmetry groups (a generalization of point groups in crystallography) are discussed in connection with a certain Diophantine problem.

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A local ergodic theorem for an infinite tower of coverings

Ryokichi Tanaka
Tohoku University

A local ergodic theorem is one of key step to obtain the hydrodynamic-type scaling limit for stochastic particle systems. It enables us to replace a local space-time average to a global one in appropriate accuracy. For a tower of abelian covering graphs, we establish a local ergodic theorems and apply it for the scaling limit to obtain a nonlinear heat equation on a torus. We formulate the local ergodic theorem by introducing the notion of local function bundles. As a possible generalization, we will also mention other residually finite amenable group action cases.

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Matroids of group-labeled graphs in applied discrete geometry

Shin-ichi Tanigawa
Kyoto University

A Γ-gain graph (also called a group-labeled graph) is a graph whose oriented edges are labeled invertibly from a group Γ. Zaslavsky proposed two matroids on Γ-gain graphs, called gain matroids and lift matroids, and investigated their linear representations. Each matroid has a canonical representation over a field $\mathbf{F}$ if Γ is isomorphic to a subgroup of $\mathbf{F}^{\times}$ in the case of gain matroids or Γ is isomorphic to an additive subgroup of $\mathbf{F}$ in the case of lift matroids. The canonical representations of the gain matroid of a complete graph is also known as a Dowling geometry, as it was first introduced by Dowling for finite groups Γ.

In this talk, we extend these matroids in two ways. The first one is extending the rank function of each matroid, based on submodular functions over Γ, and another one is extending the canonical linear representation of the union of d copies of a gain matroid or a lift matroid, based on linear representations of Γ on a d-dimensional vector space. We show that linear matroids of the latter extension are indeed special cases of the first extensions, as in the relation between Dowling geometries and gain matroids.

This work is motivated from recent research on the combinatorial rigidity of graphs with symmetries. As applications, we give new results on this topic, including combinatorial characterizations of parallel re-drawability of graphs with a point group or a crystallographic symmetry and characterizations of the generic rigidity of body-bar frameworks with a point group or a crystallographic symmetry.

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Upper bounds for packings of spheres of several radii

Frank Vallentin
TU Delft

How densely can one pack given objects into a given container? Problems of this sort, generally called packing problems, are fundamental problems in geometric optimization. An important example having a rich history is the sphere packing problem where one packs equal-sized spheres into Euclidean space. From a physical point of view, packings of spheres of different sizes are relevant as they can be used to model chemical mixtures which consist of multiple atoms or, more generally, to model the structure of composite material.

In the talk I give a theorem that can be used to upper bound the densities of packings of translates of different convex bodies in Euclidean space. Then I report on explicit computations (using semidefinite optimization, harmonic analysis and polynomial optimization), obtaining new bounds for single-size and multiple-size sphere packings.

Joint work (`http://arxiv.org/abs/1206.2608`) with David de Laat and Fernando Mario de Oliveira Filho.